Quantitative Finance and Investment Portfolio Management Formula Sheet

Fall 2019 and Spring 2020

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas, candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula sheet was developed sequentially by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

The Handbook of Fixed Income Securities, 8th ed., F. Fabozzi
Chapter 67

Page 1588 $$A(\bar{S},T) = \frac{1 - \exp\left(-\left(W + \dfrac{\bar{S}}{(1-R)}\right)T\right)}{W + \dfrac{\bar{S}}{(1-R)}} \times \frac{365}{360}$$

Page 1590 $$V^{Index} = \frac{1}{P}\sum_{p=1}^{P}(C_I(T) - S_p(T))A_p(T) = (C_I(T) - \bar{S}_I(T))\bar{A}_I(T)$$

Default Risk and the Effective Duration of Bonds, Babbel, Merrill, Panning (QFIP 130-19)

(A.1) $$\begin{aligned} 0 = &\tfrac{1}{2}B_{rr}\sigma^2 r + \tfrac{1}{2}B_{SS}\sigma_S^2 S^2 + B_{rS}\rho\sigma\sigma_S\sqrt{r}S \\ &+B_r\kappa(\mu - r) + B_S(r - c_S)S - B_\tau + c_B - rB \\ &-B_r\sigma\sqrt{r}\lambda(r,t) \end{aligned}$$

(A.2) $\lim_{r\to\infty} B(r,S,\tau;c_B) = 0; \quad \tau > 0$

(A.3) $\lim_{S\to\infty} B(r,S,\tau;c_B) = G(r,\tau;c_B)$ (There is an error in the paper.)

(A.4) $$B(r,0,\tau;c_B) = \min\left[\delta(\tau)G(r,\tau;c_B), V_R\left(r,\tau;\sum_{i=1}^{n} C_i^S\right)\right]$$

Managing Investment Portfolios, a dynamic process, Maginn, et al
Chapter 8

Page 523 $TRCI = CR + RR + SR$

Page 553 $RR_{n,t} = (R_t + R_{t-1} + R_{t-2} + \ldots + R_{t-n})/n$

Page 554 $$DD = \sqrt{\frac{\sum_{i=1}^{n}[\min(r_i - r^*, 0)]^2}{n-1}}$$

Page 555 $$\text{Sharpe Ratio} = \frac{ARR - rf}{SD}$$

Page 556 $$\text{Sortino Ratio} = \frac{ARR - rf}{DD}$$

The Secular and Cyclic Determinants of Capitalization Rates: The Role of Property Fundamentals, Macroeconic Factors, and "Structural Changes," Chervachidze, Costello, Wheaton (QFIP 113-13)

(1) $$\begin{aligned} \text{Log}(C_{j,t}) = &a_0 + a_1\log(C_{j,t-1}) + a_2\log(C_{j,t-4}) + a_3\log(RRI_{j,t}) + a_4 RTB_t + a_7 Q2_t \\ &+a_8 Q3_t + a_9 Q4_t + a_{10} D_j \end{aligned}$$

(1.1) $RRI_{j,t-s} = \text{Real Rent}_{j,t}/\text{Mean}(\text{Real Rent}_j)$

(2) $$\begin{aligned} \text{Log}(C_{j,t}) = &a_0 + a_1\log(C_{j,t-1}) + a_2\log(C_{j,t-4}) + a_3\log(RRI_{j,t-s}) + a_4 RTB_t \\ &+a_5 SPREAD_t + a_6 DEBTFLOW_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t + a_{10} D_j \end{aligned}$$

(2.1) $DEBTFLOW_t = \text{Total Net Borrowing and Lending}_t / GDP_t$

(3) $$\text{Log}(C_{j,t}) = a_0 + a_1 \log(C_{j,t-1}) + a_2 \log(C_{j,t-4}) + a_3 \log(RRI_{j,t-s}) + a_4 RTB_t + a_5 SPREAD_t + a_6 DEBTFLOW_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t$$

(4) $$\text{Log}(C_{j,t}) = a_0 + a_1 yearq + a_2 \log(C_{j,t-1}) + a_3 \log(C_{j,t-4}) + a_4 \log(RRI_{j,t-s}) + a_5 RTB_t + a_6 SPREAD_t + a_7 DEBTFLOW_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t + a_{10} D_j$$

What is an Index? Lo (QFIP 132-19)

(1) $$\tilde{R}_t = \kappa_t R_t, \quad \kappa_t = \text{Min}\left[\frac{\sigma_o}{\hat{\sigma}_{t-q}}, \tilde{l}\right]$$

$$\hat{\sigma}^2_{t-q} = \frac{1}{n-1} \sum_{j=q}^{q+k-1} \left(R_{t-j} - \hat{\mu}_{t-q}\right)^2, \quad q, \tilde{l} \geq 1$$

(2) $$\tilde{R}_t = \kappa_t R_t, \quad \kappa_t = \text{Min}\left[\frac{\sigma_o}{\hat{\sigma}_{t+2}}, \tilde{l}\right]$$

$$\hat{\sigma}^2_{t+2} = \frac{1}{n-1} \sum_{j=0}^{k-1} \left(R_{t+2-j} - \hat{\mu}_{t-q}\right)^2, \quad \tilde{l} \geq 1$$

Modern Investment Management: An Equilibrium Approach, B. Litterman (QFIP 140-19)

Chapter 7

(7.3) $$\mu^* = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q]$$

Modern Investment Management: An Equilibrium Approach, B. Litterman (QFIP 142-19)

Chapter 10

(10.5) $$RACS_t \equiv \frac{E_t[S_{t+1} - S_t(1+R_f)]}{\sigma_t[S_{t+1} - S_t(1+R_f)]} = \frac{E_t[S_{t+1} - S_t(1+R_f)]}{\sigma_t[S_{t+1}]}$$

(10.A.1) $$R_{L,t} - R_{f,t} = \beta\left(R_{B,t} - R_{f,t}\right) + \varepsilon_t$$

(10.A.2) $$V = Ce^{-r(T-t)}$$

(10.A.3) $$dV = \frac{\partial V}{\partial C}dC + \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial t}dt = V\frac{dC}{C} - (T-t)Vdr + rVdt$$

(10.A.4) $$\frac{dV}{V} = \frac{dC}{C} - (T-t)dr + rdt$$

(10.A.5) $$V = \int_t^\infty C_T e^{-r_T(T-t)}dT$$

(10.A.6) $$dV = -C_t dt + \int_t^\infty \left[dC_T - (T-t)C_T + r_T C_T\right] e^{-r_T(T-t)}dT$$

(10.A.7) $$d\varepsilon_t = \frac{\int_t^\infty dC_T e^{-r_T(T-t)}dT}{\int_t^\infty C_T e^{-r_T(T-t)}dT}$$

(10.A.8) $E_t[d\varepsilon_t] = 0$ (There is an error in the paper.)

(10.A.9) $$E_t[d\varepsilon_t^2] = \sigma_\varepsilon^2 dt$$

(10.A.10) $$S_{t+1} = A_t[\alpha(1+R_{E,t+1}) + (1-\alpha)(1+R_{B,t+1})] - L_t[1 + R_f + \beta(R_{B,t+1} - R_f) + \varepsilon_{t+1}]$$

(10.A.11) $$\frac{S_{t+1}}{A_t} = \alpha(1+R_{E,t+1}) + R_{B,t+1}\left(1 - \alpha - \frac{L_t}{A_t}\beta\right) - \frac{L_t}{A_t}\varepsilon_{t+1} + 1 - \alpha - \frac{L_t}{A_t}[1 + (1-\beta)R_f]$$

(10.A.12) $$\min_\alpha Var_t\left(\frac{S_{t+1}}{A_t}\right) = \alpha^2\sigma_E^2 + \left(1 - \alpha - \beta\frac{L_t}{A_t}\right)^2 \sigma_B^2 + \left(\frac{L_t}{A_t}\right)^2 \sigma_\varepsilon^2 + 2\alpha\left(1 - \alpha - \beta\frac{L_t}{A_t}\right)\rho\sigma_E\sigma_B$$

(10.A.13) $$\alpha\sigma_E^2 + \left(\alpha - 1 + \beta\frac{L_t}{A_t}\right)\sigma_B^2 + \left(1 - 2\alpha - \beta\frac{L_t}{A_t}\right)\rho\sigma_E\sigma_B = 0$$

$$\alpha = \frac{\left(1-\beta\frac{L_t}{A_t}\right)\left(\sigma_B^2-\rho\sigma_E\sigma_B\right)}{\sigma_E^2+\sigma_B^2-2\rho\sigma_E\sigma_B} \tag{10.A.14}$$

$$E_t(S_{t+1}) = E_t\{A_t[\alpha R_{E,t+1}+(1-\alpha)R_{B,t+1}]-L_t[R_f+\beta(R_{B,t+1}-R_f)+\varepsilon_{t+1}]\} \tag{10.A.15}$$

$$\alpha = \frac{\mu_B\left(\beta\frac{L_t}{A_t}-1\right)+\frac{L_t}{A_t}\left[R_f(1-\beta)+\eta\right]}{\mu_E-\mu_B} \tag{10.A.16}$$

$$F_1 = \frac{1}{1-p}F_0(1+R_{x,1})-\frac{p}{1-p} = aF_0(1+R_{x,1})+b \quad a=\frac{1}{1-p}, b=-\frac{p}{1-p} \tag{10.A.17}$$

$$\begin{aligned} F_2 &= aF_1(1+R_{x,2})+b = a[aF_0(1+R_{x,1})+b](1+R_{x,2})+b \\ &= a^2F_0(1+R_{x,1})(1+R_{x,2})+ab(1+R_{x,2})+b \end{aligned} \tag{10.A.18}$$

$$F_t = a^tF_0\prod_{\substack{1\le s\le t\\ s\in N}}(1+R_{x,s})+b\sum_{i=0}^{t-1}a^i\prod_{\substack{1\le j\le i\\ j\in N}}(1+R_{x,t-(j-1)}) \tag{10.A.19}$$

$$E_0(F_t) = a^tF_0E_0\left[\prod_{\substack{1\le s\le t\\ s\in N}}(1+R_{x,s})\right]+b\sum_{i=0}^{t-1}a^iE_0\left[\prod_{\substack{1\le j\le i\\ j\in N}}(1+R_{x,t-(j-1)})\right] \tag{10.A.20}$$

$$E_0\left[\prod_{\substack{1\le s\le t\\ s\in N}}(1+R_{x,s})\right] = \prod_{\substack{1\le s\le t\\ s\in N}}E_0[1+R_{x,s}] = (1+\mu_x)^t \tag{10.A.21}$$

$$E_0\left[F_t\right] = \left(\frac{1+\mu_x}{1-p}\right)^tF_0+p\frac{1-\left(\frac{1+\mu_x}{1-p}\right)^t}{\mu_x+p} \tag{10.A.22}$$