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MULTIVARIATE MODELING OF ASSET RETURNS FOR INVESTMENT GUARANTEES VALUATION

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Introduction

- Canadian insurance industry has widely accepted the regime-switching model for valuation purposes
- However, different funds are offered and dependence between them must be accounted for
- Objectives
  - Present multivariate models that can be used for valuation purposes
  - Compare the CTE provisions for each model
  - Analyze the impacts of a multivariate estimation on the CTE provision

Outline

1. Introduction
2. Models
3. Data
4. Estimation
5. Results
6. Monte Carlo experiment

Models – Multivariate modeling

- Families of models considered
  - Multivariate regime-switching
  - Multivariate GARCH
- Desired features
  - Time-varying volatility and dependence
  - Manageable number of parameters
  - PSD covariance matrix
- Need to specify the dependence structure
  - Copulas or multivariate error distribution

Models – Regime-switching

- Types of regimes
  - Global regimes
    - Each market is influenced by a common process
    - Each market is simultaneously in the same regime
  - Local regimes
    - Each market has its own set of regimes
    - Captures specific market conditions
    - Transitions are independent between markets
  - Local-global regimes
    - Similar to local regimes
    - Transitions are dependent between markets

Models – GARCH

- VECH model (or diagonal VECH)
  - **Idea**: use a univariate GARCH(1,1) for each element of the covariance matrix
  - Covariance matrix:
    - Not PSD unless we impose PSD parameter matrices
    - PSD if we redefine the PSD parameter matrices

    ➔ Model known as PSD VECH
- BEKK model
  - Matrix quadratic form: PSD covariance matrix
  - Features volatility and dependence diffusion

Models – GARCH

- Dynamic Conditional Correlation (DCC)
  - **Idea**: modeling the dynamics of the conditional correlation of returns is equivalent to modeling conditional covariance of standardized returns.
  - 2 steps modeling/estimation
    (1) Volatility
    (2) Correlation
  - Can use many models for the correlation dynamics
  - PSD covariance matrix if correlation matrix is PSD
  - Particular case: **constant correlation model** (CCORR or CCC)

Data

- Monthly data from January 1956 to September 2005
- 4 different markets
  - Canada: S&P TSX total return index
  - U.S.: S&P 500 total return index
  - U.K.: monthly historical return of
    - Actuaries Investment Index (01/1956 – 04/1962)
    - FTSE All Shares Index (05/1962 – 09/2005)
  - Japan : monthly historical return of TOPIX

Estimation – Regime-switching

- Different pairs of markets considered
  - Canada – U.S. (high correlation)
  - U.S. – Japan (low correlation)
- Global regime is the most parsimonious
  - Even for low correlated markets
  - Global-local approach has too much parameters to be chosen
  - Global regime approach only will be considered

Estimation – Regime-switching

- Different global regime models considered
  - Gaussian or Student copula
  - 2 or 3 regimes
  - 1 or 2 correlation matrices
  - Compared with constrained univariate model used in practice
- Constrained univariate model – 3 steps

(1) Transition probabilities: univariate estimation on some arbitrary market or portfolio

(2) Means and variances: given (1), perform constrained univariate estimation on each market

(3) Correlation matrix: estimated empirically

Estimation

- Ranked parsimonious models (SBC)

Rank	Model	Param.	Log-like.	SBC
1	CCORR GARCH	22	4424.38	4354.08
2	RS2LN w/ 1 corr. matrix	24	4423.72	4347.04
3	Same as (2) w/ Student copula	25	4426.50	4346.62
Last	Constrained Univ. RS2LN	24	4321.89	4245.20

- Constant correlation GARCH model is the most parsimonious
- Consistent with the regime-switching model with one correlation matrix

Results - Assumptions

- CTE provision with 3% annual MER (continuously compounded)
- Different CTE levels (60%, 80%, 95%)
- Initial ratio of MV/GV = 100%
- We consider:
  - CTE computation of an equally weighted portfolio in the 4 markets
  - Comparison with univariate models: Canadian market

Results – 4 markets

Model	60%-CTE	80%-CTE	95%-CTE
CCORR GARCH	3.3%	6.6%	22.8%
RS2LN w/ 1 corr. matrix	5.6%	11.2%	33.8%
Same as (2) w/ Student copula	4.5%	8.9%	29.1%
Constrained Univ. RS2LN	6.5%	13.0%	38.1%

- Significant variation between RS models
- Less variation between GARCH models (not shown)
- Largest CTE provisions with constrained univariate RS model (for this portfolio) but it is not always the case

Results – 1 market

Model	60%-CTE	80%-CTE	95%-CTE
RS2LN	8.9%	17.8%	47.0%
MRS2LN	8.2%	16.5%	43.9%
GARCH	2.7%	5.3%	20.9%
CCORR	4.8%	9.7%	30.6%

Univariate model

- Univariate vs multivariate
  - RS: not clear whether there is an increase in CTE
  - GARCH: significant increase in CTE
- Regime-switching vs GARCH
  - RS: CTE provisions are greater than with GARCH
  - True for all other portfolios

Monte Carlo experiment

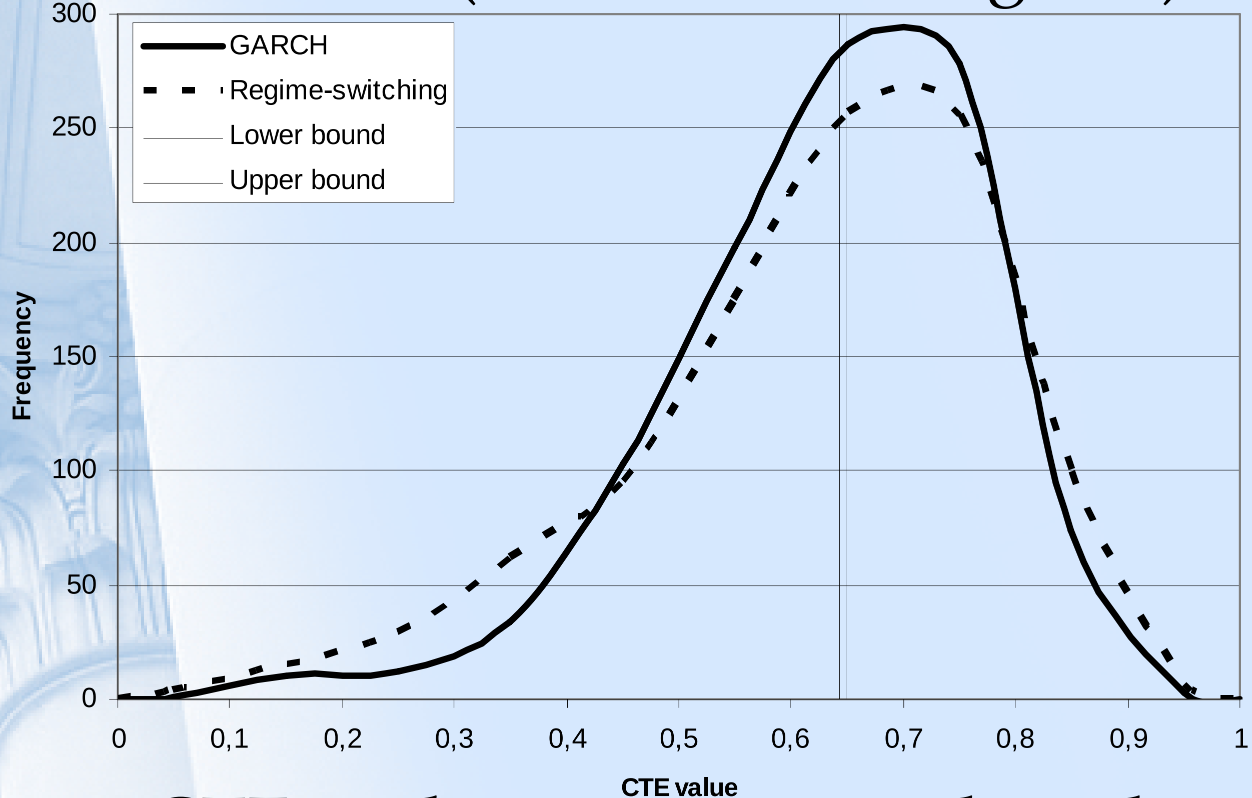
- Market is more complex than GARCH or regime-switching
- Misspecification (and small samples) have impacts on CTE
  - Bias, non-normality
- **Purpose**: obtain the real distribution of the CTE given a complex market model
- Market model: bivariate stochastic volatility model

Monte Carlo experiment

- Market model specifications:
  - Log-volatilities: correlated VAR(1)
  - Errors: constant correlation bivariate normal
  - Parameters: usual range for stochastic volatility
- Assumptions:
  - CTE: 120 months, 95%, 3% MER, MV/GV=1
  - Portfolio allocation: 60%-40%
  - True CTE: approximated empirically using 100 000 paths of 120 months ➔ 100 000 accumulation factors
  - CTE distribution: 1000 samples of 500 observations
- Approximating models used: RS2LN w/ 1 corr. matrix and CCORR GARCH

Monte Carlo experiment

- CTE distribution (smoothed histogram)



- Greater CTE underestimation risk with regime-switching model

Monte Carlo experiment

- Bias

	GARCH	Regime-switching
Mean	0,634167	0,622031
True value	0,645354	
Std dev.	0,137012	0,163142
RMSE	0,137468	0,164801

- Path by path inconsistency
  - Only 3 of the 10 smallest (largest) CTEs are common to both models
  - Correlation of 78% between pairs of CTEs

Conclusion

- With the specific dataset presented:
  - CCORR GARCH model is the most parsimonious model (SBC)
  - Current practitioner's approach provides the worst fit
  - Significant variation in CTEs among models
- Monte Carlo experiment was intended to assess the costs of misspecifying the market model

Conclusion

- Model selection alternative:
  - Choose a model with the lowest CTE RMSE since the target output is the CTE
  - CCORR GARCH has lowest CTE RMSE
- Even if the regime-switching model is widely accepted in a univariate framework
  - Should be reconsidered in a multivariate setting
- Final model selection and CTE computation
  - Depends on the context
  - Should reflect the actuary's judgment

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