TITLE: Safety Mean-Variance Hedging - Systematic Introducing of Convex Duality Method

SPEAKER: **Dian Zhu,** University of Waterloo
CO-AUTHOR(S): **Andrew Heunis**, University of Waterloo

ABSTRACT: We apply conjugate duality method to establish existence of optimal strategy in a mean-variance hedging problem. With the goal of minimizing the final hedging error in a continuous-time market setting, both hedging strategy and portfolio wealth are constrained. Convex constrain for the portfolio strategy covers the cases like no short selling or stocks prohibition, while portfolio wealth lower bounded by a continuous function guarantees a safer hedging strategy away from possible large losses. A systematic procedure of constructing the dual problem is developed as well, since the introducing of dual problem is mysterious among literatures. This is especially demonstrated by a deterministic case, which offers some intuition for the stochastic environment when asset prices are modeled by Ito processes. We synthesize a dual optimization problem and establish a set of optimality relations, similar to the Euler-Lagrange and transversality relations of calculus of variations, giving necessary and sufficient conditions for the given optimization problem and its dual to each have a solution, with zero duality gap. We then resolve these relations to establish existence of an optimal portfolio.