GI ADV Model Solutions
Spring 2024

1. **Learning Objectives:**

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5e) Calculate the price for a proportional treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question tested a candidate's ability to analyze aspects of a pricing analysis for a proportional treaty. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the probability of a combined ratio of more than 100%.

Commentary on Question:

Note that the question did not indicate whether the loss distribution provided was discrete or continuous. Assuming either form for the distribution was acceptable and would earn full credit if answered correctly with that assumption. The model solution assumes that the loss distribution is continuous. Because of this assumption, x is set to the midpoint of loss amounts. Noter that if the assumption was a discrete loss distribution, x would be set to the loss amount.

1. Set up columns for Loss, x, F(x) and p(x). F(x) is the cumulative probability. Then p(x), the probability of x, is the successive differences of the cumulative probability.

2. Set up columns for the reinsurance loss ratio (LR) and the reinsurance combined ratio (CR). The LR(x) is x times one million divided by the reinsurance premium of eight million. The CR(x) is the LR(x) plus the ceding commission (30%) plus brokerage fees (5%) plus other expenses (2%).

1. Continued

3. Find the CR just above and just below 100%, and its associated cumulative probability for the *x* that produces it.

Reins. CR	F(*x*)
93.3%	0.7191
105.8%	0.8182

4. Using interpolation between the two points in step 3 above:

Reins. CR	F(*x*)
100.0%	0.7726

Therefore, the probability is 22.7% (= 1 – .7726 in % form).

(b) Calculate the expected loss ratio after the loss corridor.

Commentary on Question:
This is based upon the assumption from part (a). Using the alternative assumption would provide a different acceptable answer.

1. Set up columns for the losses in the loss corridor at *x* and the associated revised reinsured LR(*x*).

   Losses in the corridor at *x* equal the maximum of 0% and (LR(*x*) from part (a) minus 60%) capped at 40% (i.e., the full amount in the corridor from 60% to 100% for LR(*x*) greater than 100%).

   The revised reinsured LR(*x*) is LR(*x*) from part (a) minus 75% of the losses in the corridor at *x*.

2. The expected LR is the sum of the revised reinsured LR(*x*) times the p(*x*) from part (a) divided by the sum of p(*x*) for all *x*.

The expected LR is 45.0%.

(c) Calculate the expected combined ratio.

Commentary on Question:
This is based upon the assumption from part (a). Using the alternative assumption would provide a different acceptable answer.

1. Continued

1. Set up columns for the commission at *x*, (from the sliding scale based upon the revised LR(*x*) from part (b)) and the revised combined ratio CR(*x*).

   - If the revised LR(*x*) is less than, or equal to 40%, the commission is 35%
   - If the revised LR(*x*) is greater than, or equal to 70%, the commission is 15%
   - If the revised LR(*x*) is between 60% and 70%, the commission is 15% plus (70% minus LR(*x*))
   - If the revised LR(*x*) is between 40% and 60%, the commission is 25% plus one half of (LR(*x*) minus 40%).

   The revised reinsured CR(*x*) is the sum of the revised LR(*x*) from part (b), the commission at *x* as determined by the scale (as noted in the bullet points above), brokerage fees (5%) and other expenses (2%).

2. The expected CR is the sum of the revised reinsured CR(*x*) from step 1 times the p(*x*) from part (a) divided by the sum of p(*x*) for all *x*.

The expected CR is 81.1%.

(d) Assess whether the sliding scale commission is balanced.

Commentary on Question:
This is based upon the assumption from part (a). Using the alternative assumption would provide a different acceptable answer.

The expected commission is the sum of the commission at *x* from part (c) times the p(*x*) from part (a) divided by the sum of p(*x*) for all *x*.

If the expected commission is approximately equal to the provisional commission, it is balanced.

Expected commision	29.1%
Provisional commission	30.0%
Difference (%)	-3.02%

These appear different enough, so the scale may be considered as imbalanced.

2. Learning Objectives:

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6c) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed. (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

Commentary on Question:

This question tested a candidate's understanding of the individual risk rating methods of schedule rating, experience rating and dividend plans.

Solution:

(a) Select one of the options from within the brackets to fill in the blank to make each of the following statements true regarding individual risk rating.

(i) The schedule rating adjustment is typically applied _______ premium discounts. [*after, before*]

(ii) An experience modification factor of _______ is referred to as a credit modification. [*greater than 0, greater than 1, less than 0, less than 1*]

(iii) Increasing the cap applied to claims _______ *the responsiveness of an experience rating formula*. [*decreases, does not affect, increases*]

(iv) D-ratio curves relate to _______ in experience rating. [*application of premium discounts, determination of credibility, limiting of claims*]

(i) before
(ii) less than 1
(iii) increases
(iv) limiting of claims

(b) Select one of the options from within the brackets to fill in the blank to make each of the following statements true regarding insurer dividend plans.

(i) Insurers offer dividend plans to U.S. insureds for _______ coverage. [*commercial automobile, professional liability, workers compensation*]

(ii) Dividend plans closely resemble _______ *rating* plans. [*prospective, retrospective, schedule*]

2. **Continued**

(iii) Dividend plans are also referred to as _______. [*participating policies, predictive rating plans, risk-control plans*]

(iv) Dividend payments may require approval by the _______. [*insurer's board of directors, insurer's shareholders, regulatory authority*]

(v) In a sliding-scale dividend plan, the insured's claims experience _______ *dividend payments.* [*affects, does not affect*]

(vi) An insurer's board of directors may _______ dividend payments for all dividend plan policies. [*not reject, reject*]

(vii) Dividend payments occur after _______. [*the end of the policy period, the filing of the financial statements, settlement of the claims on the policy*]

(viii) A combined dividend plan is a combination of the sliding-scale and _______ dividend plans. [*flat, schedule-rated, split-rated*]

(i) workers' compensation
(ii) retrospective
(iii) participating
(iv) regulatory authority
(v) affects
(vi) reject
(vii) the end of the policy period
(viii) flat

(c) Describe the use of safety groups for U.S. workers compensation dividend plans.

Commentary on Question:
There are several ways to describe these plans. The model solution is an example of a full credit solution.

Safety groups are used to pool insureds' premiums and claims for similar employers. The dividends of a safety group are determined based on the aggregated experience of the group and are not based on an individual member's experience.

3. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question tested a candidate's understanding of the Clark's stochastic LDF model. This question included data and results from a completed analysis (using Clark's stochastic reserving model) in Excel. Candidates were not required to perform any calculations. Candidates were to respond in Word. Each question part presented a report excerpt. Candidates were expected to critique the excerpt based on the data and analysis in Excel. The model solution for each part provides an example of a full credit solution but does not represent the only acceptable full credit response.

Solution:

(a) Critique Part 1.

Part 1: *I have been provided with a loss triangle representing accident years 2016-2023. All possible twelve-month development periods are represented. I noticed that one of the increments is negative. While the algorithm provides results for this case, Clark's procedure should not be used when there are negative increments.*

The model will still work if some actual points show decreasing losses. However, if there is real expected negative development then a different model should be used.

The report is thus incorrect. Negative increments are acceptable, though not in all situations. It should have said that the procedure can be used with negative observed increments provided expected increments are positive.

(b) Critique Part 2.

Part 2: *I have performed maximum likelihood estimation of the two parameters. The calculations are in cells G7:K43 and the maximizing values are in cells H44 and H45. The values that maximize the likelihood function are* $\theta = 37.44$ *and ELR = 0.5506.*

3. Continued

The calculations are incorrect. When obtaining the likelihood value, the increments to be used should be 0, 6, 18, 30, and so on. The report used 0, 12, 24, 36, and so on.

(c) Critique Part 3.

Part 3: *I have calculated the estimate of the scale factor in cells M7:M44 as* $\hat{\sigma}^2 = 309.1$.

The individual numbers are correct as is the sum. However, the sum should be divided by 34 not 33. This is because the number of observations(36) minus the number of estimated parameters (2) is 34.

(d) Critique Part 4.

Part 4: *I have calculated the loss reserve estimate for all years in cell F60 as 31,103. I have further calculated the process standard deviation of the loss reserve in cell F62 as 3,101. Assuming a normal distribution, two standard deviations provide 95% confidence. Therefore, we can be 95% confident that the actual development will be between 24,901 and 37,304.*

The calculations in the report are correct, as is the assumption. However, the process standard deviation only accounts for one aspect of the potential error. An additional source is the estimation error (parameter variance). Hence, the 95% confidence interval should be wider than the one indicated.

(e) Critique Part 5.

Part 5: *Clark recommends several graphs that can be used to verify if the assumptions of the model hold. Two such graphs are on the worksheet. One graph plots normalized residuals against increment age and the other plots them against the expected increments. The second graph shows a curved, rather than horizontal pattern. Hence the underlying assumptions may not hold.*

This part of the report is correct.

4. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Commentary on Question:

This question tested a candidate's understanding of modeling the chain ladder method. It required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the following amounts using the chain ladder (CL) method:

(i) Age-to-age factors

(ii) Expected values for each DY (starting with DY 2), based on the previous DY, for which there are observed values

Part (i): The average development for each interval between development years. They are calculated as follows: For the period 1-2 it is the (sum of DY 2) divided by the (sum of DY1 excluding AY10), for the period 2-3 it is the (sum of DY 3) divided by the (sum of DY2 excluding AY9), and so on.

Part (ii): Each amount is the prior observed amount multiplied by the appropriate age-to-age factor. They are calculated as follows: the DY2 column is the 1-2 factor multiplied by the DY1 observed column from AY1 to AY9, the DY3 column is the 2-3 factor multiplied by the DY2 observed column from AY1 to AY8, and so on.

4. Continued

(b) Construct the following scatterplots for each of DYs 1 to 3:

(i) Development along with a fitted regression line

(ii) Weighted residuals versus values from the previous DY

For part (i):
DY1: the x-axis is DY1 and the y-axis is DY2, for AY1 to AY9
DY2: the x-axis is DY2 and the y-axis is DY3, for AY1 to AY8
DY3: the x-axis is DY3 and the y-axis is DY4, for AY1 to AY7

For part (ii):
The same x-axis is used as in part (i). The y-axis residuals are calculated as the estimated values from part (i) for DYb less the data for DYb divided by the square root of the data from the DYa. In which Yb =Ya + 1.

The scatterplots are completed automatically when these amounts are input in the appropriate cells.

(c) Determine the validity of the assumptions underlying the CL estimates based upon part (b). Justify your determination.

Commentary on Question:
The model solution is an example of a full credit solution.

The first set of charts from part (b)(i) looks good, but that is mostly due to scale. However, the line should be through the origin which is not the case here.

The second set from part (b)(ii) shows a pattern of residuals that are negative, then positive. Threfore, the assumptions are not valid.

(d) Assume that the scatterplots indicate that the assumptions are not valid. Mack provides two other formulas for calculating age-to-age factors that could be tested.

State one of these alternatives. Do not do any calculations.

Commentary on Question:
The model solution is an example of a full credit solution.

f_{k0} is the C(i,k) squared weighted average of the individual development factors

4. Continued

(e) Input the appropriate values to perform this regression analysis.

AY1: the x1 column is DY1, the increment column is DY2 minus DY1, for AY1 to AY9
AY2: the x2 column is DY2, the increment column is DY3 minus DY2, for AY1 to AY8
AY3: the x3 column is DY3, the increment column is DY4 minus DY2, for AY1 to AY7

The regressions are completed automatically when these amounts are input in the appropriate cells.

(f) Determine the validity of the assumptions underlying the CL estimates based upon the results of the regression analysis in part (e). Justify your determination.

Commentary on Question:
The model solution is an example of a full credit solution.

The intercept is not zero for DY2 because it is more than double the standard error. This violates Mack's assumption because it is significant. For DY1, the intercept is nearly significant at 1.6 standard deviations. This is a borderline violation of Mack's assumption.

The slopes are clearly not zero for D1 to D3, which supports Mack's assumption.

(g) Propose an alternative model that is more consistent with the results of the regression analysis in part (e).

Commentary on Question:
The model solution is an example of a full credit solution.

The "linear model plus constant" model is more appropriate when both terms are significant.

5. **Learning Objectives:**

7. The candidate will understand the application of game theory to the allocation of risk loads.

Learning Outcomes:

(7a) Allocate a risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question tested a candidate's ability to calculate property catastrophe risk loads based upon Mango's approach. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the renewal risk load for each account using the following methods:

(i) Marginal Variance

(ii) Shapley

Commentary on Question:

Note that the events are independent when calculating variance for the accounts.

(i) Marginal Variance

For each account j, the variance is equal to the sum over the events of $\text{Loss}_j(i)^2 \times p(i) \times [1 - p(i)]$.

For the three accounts combined, the variance is calculated in a similar manner.

For each account, the marginal variance is the variance for the three accounts combined minus the sum of the variance for the other two accounts.

For each account, the risk load is the marginal variance for the account times λ.

(ii) Shapley

Create a variance-covariance matrix for the three accounts as follows:

For each cell (j, k) in the 3×3 matrix representing account j and account k the value is the sum over events i of $\text{Loss}_j(i) \times \text{Loss}_k(i)^2 \times p(i) \times [1 - p(i)]$.

For each account, the Shapley value is the sum of its variance and the covariances with the other two accounts (i.e., sum of the account column)

For each account, the risk load is the Shapley value for the account times λ.

(b) Demonstrate that the Shapley method is renewal additive.

5. Continued

Calculate the variance for the three accounts combined as follows:
For each event *i*, the loss, L(*i*) is the sum of the losses for each account.
The variance for the three accounts combined is equal to the sum over the events *i* of $\text{Loss}(i)^2 \times p(i) \times [1 - p(i)]$.

The risk load for the three accounts combined is the variance for the three accounts combined times λ.

Add the risk load for the three accounts from the Shapley method.

These two risk load amounts should be equal which demonstrates that the Shapley method is renewal additive.

6. **Learning Objectives:**

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4a) Explain the mathematics of excess of loss coverages in graphical terms.

Sources:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Commentary on Question:

This question tested a candidate's knowledge of Lee's approach to excess of loss coverage.

Solution:

(a) Show the formula for each of the following amounts from this policy:

(i) Expected payment per loss using the layer method

(ii) Expected payment per loss using the size method

(iii) Expected total payments in a policy period

(iv) Increased limits factor (ILF) for limit k with basic limit b

(v) Derivative of the ILF for limit k with basic limit b

(vi) Expected basic limits loss after constant inflation of factor a (i.e., $x \rightarrow x'$ in which $x' = ax$)

(i) C[0, L]
(ii) A[0, L] + $L \times G(L)$
(iii) E{N}× C[0, L]
(iv) C[0, k] / C[0, b]
(v) $G(k)$ / C[0, b]
(vi) a C[0, b / a]

(b) Show the formula for each of the following amounts from this policy:

(i) Expected payment per loss using the layer method

(ii) Expected payment per loss using the size method

6. **Continued**

Commentary on Question:
There are several versions of correct formulas that could be shown for part (ii). Only one correct formula was required for full credit. The model solution is an example of a full credit solution.

(i) C[R, $R+L$]
(ii) A[R, $R+L$] + $(R+L) \times G(R+L) - R \times G(R)$

(c) State what is required of ILFs to pass the consistency test.

Commentary on Question:
The model solution is an example of a full credit solution.

ILFs must increase at a decreasing rate as the limit increases.

7. Learning Objectives:

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4g) Estimate the premium asset for retrospectively rated polices for financial reporting.

Sources:

Estimating the Premium Asset on Retrospectively Rated Policies, Teng and Perkins

Discussion of Estimating the Premium Asset on Retrospectively Rated Policies, Feldblum

Commentary on Question:

This question tested a candidate's ability to calculate the premium asset on retrospectively rated policies. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

BP = Basic Premium, SP = Standard Premium, LCF = Loss Conversion Factor,
TM = Tax Multiplier, EPLE = Cumulative Expected Percentage of Loss Emerged,
IEPLE = Incremental Expected Percentage of Loss Emerged,
ELR = Expected Loss Ratio, EMLR = Emerged Loss Ratio, LCR = Loss Capping Ratio,
ILCR = Incremental LCR, CLCR = Cumulative LCR,
LEPA = Loss elimination ratio from per accident limit,
LEMM = Loss elimination ratio from retro formula maximum and minimum

Solution:

(a) Calculate the cumulative premium development to loss development (CPDLD) ratio for each retrospective adjustment period using the formula approach.

$IEPLE_i = EPLE_i - EPLE_{i-1}$
$EMLR_i = ELR \times EPLE_i$
$CLCR_i = 100\% - LEPA_i - LEMM_i$
$ILCR_1 = CLCR_1$
For i = 2 to 4,
$ILCR_i = (EMLR_i \times CLCR_i - EMLR_{i-1} \times CLCR_{i-1}) / (EMLR_i - EMLR_{i-1})$

$PDLD_1 = (BPF \times TM / (EMLR_1)) + (TM \times LCF \times ILCR_1)$
For i = 2 to 4, $PDLD_i = TM \times LCF \times ILCR_i$

For j = 1 to 4,

$$CPDLD_j = \sum_{i=j}^{4} PDLD_i \times IEPLE_i \div \sum_{i=j}^{4} IEPLE_i$$

7. Continued

(b) State the formula to estimate the premium asset that includes the CPLD ratio as one of the elements in the formula.

As of the valuation date, for each policy year,
Premium Asset =
Expected Future Loss Emergence × CPDLD + Premium Booked from Prior Adjustment – Premium Booked as of the valuation date.

(c) Identify two situations where an empirical approach to estimating PDLD ratios would be preferred to the formula approach.

Commentary on Question:
There are several ways that this could be answered correctly. The model solution is an example of a full credit solution.

- Different retrospective parameters apply to many different regions.
- Historical patterns of PDLD show stability.

(d) Provide a reason the PDLD method might be preferred to Fitzgibbon's method.

Commentary on Question:
There are several ways that this could be answered correctly. The model solution is an example of a full credit solution.

It follows the actual retrospective premium adjustment formula, so it's easier to explain and justify to underwriters.

8. **Learning Objectives:**

3. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(3a) Describe a risk margin analysis framework.

(3b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(3c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question tested a candidate's understanding of risk margins as set out in Marshall. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Describe two considerations why correlation effects exist within internal systemic risk.

Commentary on Question:

The model solution is an example of a full credit solution.

The correlation between valuation classes may be caused by the "same actuary effect" and the use of the same template or valuation models across different valuation classes.

There may be linkages between the premium liability methodology and outcomes from the outstanding claim valuation, which contributes to the correlation effects between outstanding claim and premium liabilities.

(b) Calculate the following for the company:

(i) Total internal systemic risk CoV

(ii) Total external systemic risk CoV

(iii) Total consolidated CoV for all sources of risk

(iv) Risk margin at the 80% adequacy level

8. Continued

Commentary on Question:
ISR = Internal Systemic Risk, ESR = External Systemic Risk,
ISRρ(A, B)= ISR Correlation between A and B, CE = Central Estimate

(i)

Calculate the ISR variances for both OSC and PL, for both Auto and Home as $(\text{CE} \times \text{ISR CoV})^2$

ISR covariance for (Auto OSC, Auto PL) is calculated as follows
ISRρ(Auto OSC, Auto PL) × ISR CoV(Auto OSC) × ISR CoV(Auto PL) × CE(Auto OSC) × CE(Auto PL)

ISR covariances for (Auto OSC, Home OSC) and (Auto PL, Home OSC) are calculated similarly. Note that Home PL has no correlation with the other Line-Liability combinations, so it may be excluded from the ISR covariance calculations.

$$\text{Total ISR CoV} = [\sum \text{ISR variances} + 2 \times \sum \text{ISR covariances}]^{1/2} / \sum \text{CE}$$

(ii)

$$\text{Total ESR CoV} = [\sum[\text{CE(i)ESR CoV(i)}]^{1/2} / \sum \text{CE}$$

(iii)

Total Consolidated CoV
$$= [\text{Independent risk CoV}^2 + \text{Total ISR CoV}^2 + \text{Total ESR CoV}^2]^{1/2}$$

(iv)

$$\text{Risk Margin 80\% adequacy} = \text{Total Consolidated CoV} \times z\ value_{0.8} \times \sum \text{CE}$$

9. Learning Objectives:

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5k) Test for risk transfer in reinsurance contracts.

Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm

Insurance Risk Transfer and Categorization of Reinsurance Contracts, Gurenko, Itigin and Wiechert

Commentary on Question:

This question tested a candidate's understanding of methods to measure the existence of risk transfer. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Describe two examples of contracts where the risk transfer is "reasonably self-evident."

Commentary on Question:

The model solution is an example of a full credit solution.

- Straight quota share with fixed terms
- Fixed premium per-risk excess of loss

(b) Describe two advantages of using ERD and RCR versus using VaR and TVaR.

Commentary on Question:

The model solution is an example of a full credit solution.

- ERD and RCR do not rely on a fixed (arbitrary) selection of a percentile. TVaR and VaR rely on a selection of a percentile.
- ERD and RCR capture all capital-destroying loss events, while TVaR and VaR generally do not.

(c) Determine whether risk transfer exists in this contract using the Max QP test with α equal to 4.

9. Continued

Commentary on Question:
P(x) is the probability of loss amount x.

Calculate $F(x)$ as the cumulative probability at loss amount, x.
$F^*(x) = 1 - [1 - F(x)]^{1/2}$
$P^*(x)$ values are the incremental values of $F^*(x)$
$E(x) = \sum xP(x)$
$E^*(x) = \sum xP^*(x)$
$RTD = E^*(x) - E(x)$

For this contract, 4 times the RTD of 30.7 is 122.7. This is greater than the premium of 48 so it passes the risk transfer test based on the Max QP test methodology.

10. **Learning Objectives:**

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

Learning Outcomes:

(2a) Estimate ultimate claims for excess limits and layers.

(2b) understand the difference in development patterns and trends for excess limits and layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2^{nd} Ed. (2022), Friedland

- Appendix G

Commentary on Question:

This question tested a candidate's knowledge regarding the development of excess limits and layers. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the total IBNR for the layer of 150,000 excess of 50,000 using volume-weighted average loss development factors.

Commentary on Question:

DY = development year, AY = Accident Year

Step 1: Calculate IBNR for the 50,000 limit.
Using the data triangle at the 50,000 limit: For each stage of development, DYx to DYx+1, the age-to-age factor is the sum of the DYx+1 column divided by [the sum of the DYx column excluding the amount from the current valuation date (i.e., the last data point in the column)]

The cumulative development factors (CDFs) are calculated as follows: 72 months to ultimate is set to 1.0. The 60 months to ultimate factor is the 60-72 months age-to-age factor times the 72 months to ultimate factor. The 48 months to ultimate factor is the 48-60 months age-to-age factor times the 72 to ultimate factor. This relation continues for all development ages.

IBNR by AY is calculated as follows: For AY2023 it the DY1 amount times the (12 months to ultimate factor minus 1.0), for AY 2022 it is the DY2 amount times the (24 months to ultimate factor minus 1.0), and so on.

Step 2: Calculate total IBNR for the 200,000 limit.
Subtract the total reported amount at 200,000 limit (sum of the diagonal from the 200,000 limit triangle) from the provided total ultimate at 200,000 limit.

10. **Continued**

Step 3: Calculate IBNR for the layer 150,000 excess 50,000.
This is the total IBNR at 200,000 limit minus the total IBNR at 50,000 limit.

(b) Explain why the expected method would be a viable alternative to the development method.

Commentary on Question:
There are several possible acceptable responses to this question. The model solution is an example of a full credit solution.

The expected method is often used for immature experience periods when excess claims data may be too sparse, which is the case here. Furthermore, there is no history to calculate the development factors.

(c) Describe a limitation of using the expected method.

For more mature experience periods, the lack of sensitivity to change in reported claims makes it less accurate.

(d) Calculate the AY 2023 IBNR for the layer using the expected method.

Step 1: For each of 50,000 and 200,000 limit, calculate trended earned premium. All the claims have been trended to December 31, 2023. The premium is provided for calendar year 2023 – it is reasonable to assume that the average earned date is the middle of the year. Therefore, we trend the premiums by one-half year given an annual premium inflation trend of 3%.

Step 2: For each of 50,000 and 200,000 limit, calculate the ultimate claims using the appropriate expected loss ratios for the corresponding limit.

Step 3: Calculate the AY 2023 IBNR for the layer 150,000 excess 50,000.
The AY 2023 IBNR for the layer is the ultimate for the layer minus the reported for the layer in which:

- The layer ultimate is the ultimate for the 200,000 limit from step 2 minus the ultimate for the 50,000 limit from step 2.
- The reported for the layer is the 200,000 limit reported amount for AY 2023 at 12 development months minus the 50,000 limit reported amount for AY 2023 at 12 development months.

10. Continued

(e) Calculate the AY 2023 IBNR for the layer using the ILF method.

Step 1: Convert the premium ILF to a claim ILF.
This is the premium ILF time the expected loss ratio for the 200,000 limit divided by the expected loss ratio for the 50,000 limit.

Step 2: Calculate the AY 2023 layer ultimate claims at the 200,000 limit.
This is the ultimate claims from the 50,000 limit from part (a) times the premium ILF from step 1.

Step 3: Calculate the AY 2023 IBNR for the layer 150,000 excess 50,000.
The AY 2023 IBNR for the layer is the ultimate for the layer minus the reported for the layer in which:

- The layer ultimate is the ultimate for the 200,000 limit from step 2 minus the ultimate for the 50,000 limit from part (a).
- The reported for the layer is the 200,000 limit reported amount for AY 2023 at 12 development months minus the 50,000 limit reported amount for AY 2023 at 12 development months.

11. Learning Objectives:

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6b) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 35: Claims-Made Ratemaking

Commentary on Question:

This question tested a candidate's understanding of the reasons for claims-made policies and the principles of claims-made ratemaking.

Solution:

(a) Describe these circumstances.

Commentary on Question:

This question tested a candidate's understanding of the fundamentals of claims-made ratemaking. There are a number of circumstances that may be referenced for this. Full credit was given for providing a description with at least two appropriate circumstances. The model solution is an example of a full credit solution.

The industry was faced with a basic inability to accurately set the price for the occurrence policy form, because most of the claims arising out of any given year's professional services would not be reported until well after the insurer had accepted a fixed price for an open-ended promise to indemnify. Many insurers ceased writing this kind of insurance, and others decided to charge prices they deemed high enough creating an availability and affordability problem.

(b) Identify two reasons that this shift to claims-made coverage was not as prevalent outside of the United States.

Commentary on Question:

There are more than two reasons. The model solution is an example of a full credit solution.

- Greater role of socialized health care in other countries.
- Greater use of mechanisms for dispute resolution in other countries.

(c) Define the claims-made coverage retroactive date.

The earliest accident date for which coverage is provided under a claims-made policy.

11. **Continued**

(d) Marker and Mohl identified five principles of claims-made ratemaking.

State four of these principles.

Commentary on Question:
The model solution includes all five. Only four were required for full credit.

- A claims-made policy should always cost less than an occurrence policy, as long as claim costs are increasing.
- Whenever there is a sudden, unpredictable change in the underlying trend, claims-made policies priced on the basis of the prior trend will be closer to the correct price than occurrence policies priced in the same way. [
- Whenever there is a sudden unexpected shift in the reporting pattern, the cost of mature claims-made coverage will be affected very little if at all relative to occurrence coverage.
- Claims-made policies incur no liability for pure IBNR claims so the risk of reserve inadequacy is greatly reduced.
- The investment income earned from claims-made policies is substantially less than under occurrence policies

12. **Learning Objectives:**

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6a) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

Commentary on Question:

This question tested a candidate's understanding of deductibles and self-insured retentions. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Describe three ways that a self-insured retention (SIR) differs from the use of a deductible.

Commentary on Question:

There are more than three ways. The model solution is an example of a full credit solution.

- In most traditional GI policies with relatively small deductibles, defense costs are outside of the deductible and outside of the policy limits, and insurers are responsible to pay for all defense costs. In contrast, with an SIR program the insured would typically take the lead on the defense of all claims until the SIR is breached.
- In most traditional GI policies with a deductible, the policy limit is eroded by the deductible. However, under an SIR, there is no erosion of the limit.
- In most traditional GI policies with a large deductible, the insurer is typically required to hold collateral to ensure that the insurer can pay the entire claim regardless of whether or not the insured will reimburse the insurer for the deductible. Collateral is not generally required for an SIR program.

(b) Provide the following with respect to an insurer's application of this approach.

(i) Definition of elimination ratio

(ii) Formula for elimination ratio

Commentary on Question:

The model solution is an example of a full credit solution.

12. **Continued**

(i) Definition of elimination ratio
The proportion of claims eliminated by the deductible relative to the claims underlying the estimate of the base rate, which is associated with an insurer's base deductible.

(ii) Formula for elimination ratio
(claims eliminated by the deductible − claims eliminated by the base deductible)
divided by
(total ground up claims − claims eliminated by the base deductible)

(c) The reduction in premium is not proportional to the size of the deductible for many lines of general insurance, particularly automobile physical damage coverages and personal property insurance.

Explain why this should be expected.

Commentary on Question:
The model solution is an example of a full credit solution.

This is because lines like automobile physical damage have a claims distribution with many smaller claims. As such, a change in the deductible will tend to have a disproportionate effect on premium as it will have a much larger effect on smaller claims.

(d) Determine the amount the insurer would pay to the insured for this loss under the following scenarios. State any assumptions required.

(i) The policy has no deductible.

(ii) The policy has a deductible of 2,500.

Commentary on Question:
The model solution is an example of a full credit solution.

(i) The policy has no deductible.
Coinsurance penalty % = 1-100,000/(200,000*60%) = 16.7%
Claim payment = 40,000 × (1-0.167) = 33,333

12. **Continued**

(ii) The policy has a deductible of 2,500.

The policy deductible may come before or after the coinsurance penalty, depending on the policy wording.

- If before the coinsurance penalty, the claim payment is:
  (40,000 – 2,500) × (1 – 0.167) = 31,250

- If before the coinsurance penalty, the claim payment is:
  40,000 × (1 – 0.167) – 2,500 = 30,833

13. **Learning Objectives:**

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5c) Analyze and describe the various types of reinsurance.

(5f) Calculate the price for a property per risk excess treaty.

(5i) Describe considerations involved in pricing property catastrophe covers.

Sources:
Basics of Reinsurance Pricing, Clark

Commentary on Question:
This question tested a candidate's ability to analyze per risk excess treaties. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the nominal rate on line.

This is the annual premium divided by the occurrence limit
= 20 million / 150 million
= 13.3%

(b) Calculate the underwriting loss (excluding expenses) to ABC Re if a loss fully exhausts the limit.

This is the underwriting result (times −1 because we want the loss) produced with a claim amount of 150 million.

Underwriting loss
= −1×[annual premium – claim amount + additional premium]

Additional premium
= 50% × (150 million + 10%×150 million – 150 million)
= 66 million

Underwriting loss
= −1×[20 million −150 million + 66 million]
= 64 million

13. Continued

(c) Calculate the premium for an equivalent traditional risk cover.

Premium
= 20 million – (1 – 10%) × 80% × 20 million
= 5.6 million

(d) Calculate the rate on line for an equivalent traditional risk cover.

Rate on line
= 5.6 million / (5.6 million + 64 million)
= 8.0%

(e) Construct a counterproposal that should be acceptable to both ABC Re and JKL. Justify your answer.

Commentary on Question:
There are many different counterproposals that should be acceptable to both parties. These counterproposals should do one or more of the following: increase premium, decrease profit commission, increase margin, increase additional premium. This should be done to ensure that the rate on line for the equivalent traditional risk cover is greater than 10% (because there is a full loss once every 10 years) . The model solution is an example of a full credit solution increasing premium and the margin so that the rate on line for the equivalent traditional risk cover is just above 10%. A full credit solution should calculate this amount to show that their counterproposal upholds the target of being just above 10%.

Increasing the premium to 23 million, and the margin to 15%, should be acceptable to both parties with a rate on line of just over 10%.

The rate on line for the equivalent traditional risk cover is 10.6%