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# CURATED PAST EXAM ITEMS

## - Solutions -

### GI 101 – Ratemaking and Reserving

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#### **Important Information:**

- These curated past exam items are intended to allow candidates to focus on past SOA fellowship assessments. These items are organized by topic and learning objective with relevant learning outcomes, source materials, and candidate commentary identified. We have included items that are relevant in the new course structure, and where feasible we have made updates to questions to make them relevant.
- Where an item applies to multiple learning objectives, it has been placed under each applicable learning objective.
- Candidate solutions other than those presented in this material, if appropriate for the context, could receive full marks. For interpretation items, solutions presented in these documents are not necessarily the only valid solutions.
- Learning Outcome Statements and supporting syllabus materials may have changed since each exam was administered. New assessment items are developed from the current Learning Outcome Statements and syllabus materials. The inclusion in these curated past exam questions of material that is no longer current does not bring such material into scope for current assessments.
- Thus, while we have made our best effort and conducted multiple reviews, alignment with the current system or choice of classification may not be perfect. Candidates with questions or ideas for improvement may reach out to [education@soa.org](mailto:education@soa.org). We expect to make updates annually.

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## GI 101 – LEARNING OBJECTIVE 1

### 1. Topic: Introduction and Key Considerations

The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.

**GIRR Fall 2020 Question 15 (LOs 1d, 1i, 4b, and 4c)****Learning Outcomes:**

- (1d) Understand the components of ultimate values.
- (1i) Describe how and why data are segregated and aggregate.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 4, and 23.

**Commentary on Question:**

*This question tests the candidate's understanding of unpaid ALAE and unpaid ULAE.*

**Solution:**

- (a) Describe one way a reinsurer might assess the reasonableness of an estimate of unpaid ULAE.

Either one of the following is acceptable:

- Consider the reinsurer from a run-off perspective.
- Estimate the number of years to run-off the claim liabilities and the estimated cost per year.

- (b) Recommend one of the two approaches from the table above to use in estimating unpaid ULAE. Justify your recommendation.

Either one of the following is acceptable:

- Kittel refinement because it incorporates reported claims which reduces distortion from exposure growth.
- Kittel refinement because the classical paid-to-paid overstates the ULAE ratio (numerator) when exposure is growing.

- (c) Estimate unpaid ULAE as of December 31, 2019 using the approach you selected in part (b).

Ratio of ULAE to claims (Kittel refinement): average of 2018 and 2019 = 7.20%

For the ULAE ratio selection, use the average of the most recent 2 years to reflect the growing exposure base.

$$\begin{aligned}\text{Unpaid ULAE} &= (\text{ULAE ratio} \times \text{pure IBNR}) + [\text{ULAE ratio} \times \text{multiplier} \times (\text{case estimates} + \\ &\quad \text{development on case estimates})] \\ &= (0.072 \times 1,600,000 \times 0.2) + 0.072 \times 0.75 \times (3,510,000 + 0.8 \times 1,600,000) \\ &= 281,700.\end{aligned}$$



- (d) Determine calendar year 2019 incurred ULAE.

$$\begin{aligned}\text{CY 2019 incurred ULAE} &= \text{2019 paid ULAE} + \text{Change in outstanding in 2019} \\ &= 880,000 + 281,700 - 270,000 = 891,700.\end{aligned}$$

- (e) Critique your colleague's recommendation.

Any two of the following are acceptable:

- ALAE shouldn't be evaluated on a calendar year basis because ALAE reflect development over time.
- ALAE is more directly related to the size of a claim and should be evaluated like claim experience.
- ALAE are directly attributable to claims and should be analyzed similar to claims while ULAE are general and not assigned to claims.
- Accident year detail is recorded for ALAE which allows a deeper analysis.
- ALAE reporting requires accident year detail.

## GIRR Fall 2021 Question 6 (LOs 1d, 1f, 3g, and 3j)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (1f) Demonstrate the importance of understanding key terminology and interrelationships.
- (3g) Estimate ultimate values using the methods cited in (3e).
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 16 and 22.

### Commentary on Question:

*This question tests the candidate's understanding of the evaluation and selection of estimated IBNR under various circumstances.*

### Solution:

- (a) Describe what an *actuarial central estimate* represents according to U.S. ASOPs.

An actuarial central estimate represents an expected value over the range of reasonably possible outcomes.

- (b) Assess the validity of the following statement:

“Credibility is not utilized in projecting unpaid claims for reserving.”

Invalid; credibility is often reflected implicitly when projecting ultimate claims.

- (c) Calculate the indicated IBNR as of December 31, 2020 for each of the frequency-severity method projections above.

	(1)	(2)	(3)
Accident Year	Reported Claims	<u>Indicated IBNR</u>	
		Development Based	Claim Closure
2015	5,051,008	2,154	2,479
2016	5,453,150	55,306	53,536
2017	5,764,966	136,626	102,293
2018	5,967,139	275,802	337,862
2019	6,294,143	531,932	761,852
2020	5,980,004	1,173,792	1,398,061

Notes: (1) = (Earned Premium)(Reported Claim Ratio Triangle Latest Diagonal)

e.g., 2017:  $5,764,966 = 8,669,122 \times 66.5\%$

(2) = Ultimate Claims – (1)

e.g., 2017:  $136,626 = 5,901,592 - 5,764,966$

$$(3) = \text{Ultimate Claims} - (1)$$

e.g., 2017:  $102,293 = 5,867,259 - 5,764,966$

- (d) Critique the appropriateness of each method as a potential IBNR selection for accident year 2018.
- (i) Paid development method
  - (ii) Reported development method
  - (iii) Paid Bornhuetter Ferguson method
  - (iv) Reported Bornhuetter Ferguson method
- (i) Paid development is not appropriate because it is under-responsive to large claim.
- (ii) Reported development is not appropriate because it is over-responsive to large claim.
- (iii) Paid Bornhuetter Ferguson is not appropriate because it is under-responsive to large claim.
- (iv) Reported Bornhuetter Ferguson is an appropriate method because it is not distorted by large claim and also recognizes relative immaturity of a liability coverage.

## GIRR Fall 2021 Question 18 (LOs 1d, 3f, 3g, 4a, and 4b)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).
- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 15, and 23.

### Commentary on Question:

*This question tests the candidate's understanding of selecting development factors and estimating a tail factor using Boor's algebraic method. It also tests the calculation of unpaid ULAE using the classical paid-to-paid method, as well as an understanding of the Kittel refinement to the classical paid-to-paid method and the Mango and Allen smoothing adjustment.*

### Solution:

- (a) Select age-to-age development factors for all periods excluding the tail factor. Justify your selections.

Adjusted Age-to-Age Development Factors Excluding the Large Claim							
Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96
2013	3.191	1.675	1.352	1.197	1.122	1.091	1.063
2014	3.058	1.673	1.305	1.201	1.141	1.094	
2015	2.846	1.691	1.334	1.218	1.131		
2016	2.858	1.700	1.321	1.198			
2017	2.727	1.726	1.332				
2018	2.732	1.729					
2019	2.716						
All Years Avg.	2.876	1.699	1.329	1.204	1.131	1.092	1.063
Avg. excl. high&low	2.844	1.698	1.329				
Volume Wtd. Avg.	2.861	1.699	1.329	1.204	1.131	1.092	
5 Year Avg.	2.776	1.704					
3 Year Avg.	2.725	1.718	1.329	1.206			
Selected	2.725	1.718	1.329	1.206	1.131	1.093	1.063

Justification for selection: Selected 3 years average to recognize trend down the columns.

Notes: Adjusted factors for large claim:

$$AY_{2017}, 24-36 = 1.726 = (1,082 - 150)/540$$

$$AY2017, 36-48 = 1.332 = (1,391 - 150)/(1,082 - 150)$$

$$Volume\ Wtd.\ Avg.,\ 24-36: 1.699 = (866 + 875 + 876 + 923 + 1,082 + 968 - 150)/(517 + 523 + 518 + 543 + 540 + 560)$$

$$Volume\ Wtd.\ Avg.,\ 36-48: 1.329 = (1,171 + 1,142 + 1,169 + 1,219 + 1,391 - 150)/(866 + 875 + 876 + 923 + 1,082 - 150)$$

- (b) Derive a paid tail factor using Boor's algebraic method.

		(1)	(2)	(3) = (1)(2)	(4)	(5) = (4)/(3)
				Estimated Claims		
Accident Year	Actual Paid	Paid Development Factors		96	Ultimate Claims from Reported Development Method	Implied Tail Factor
		72-84	84-96			
2013	1,824			1,824	1,975	1.083
2014	1,712		1.063	1,820	1,974	1.085
2015	1,610	1.093	1.063	1,870	2,032	1.087
Selected:						1.085

- (c) Calculate ultimate claims using the paid development method and the tail factor of 1.072.

		(1)	(2)	(3)	(4) = (2)(3)
				Age-to-Ultimate Development Factors	Ultimate Claims
Accident Year	Actual Paid	Paid Claims Excluding Large Claim			
2013	1,824	1,824		1.072	1,955
2014	1,712	1,712		1.140	1,951
2015	1,610	1,610		1.245	2,004
2016	1,460	1,460		1.408	2,056
2017	1,391	1,241		1.698	2,257
2018	968	968		2.257	2,184
2019	573	573		3.877	2,222
2020	224	224		10.566	2,367
Total	9,762	9,612			16,997

e.g.,  $1,241 = 1,391 - 150$   
 $1.698 = 1.206 \times 1.131 \times 1.093 \times 1.063 \times 1.072$

- (d) Calculate the unpaid ULAE as of December 31, 2020 using the classical paid-to-paid method and a multiplier of 50%.

$$\text{Case outstanding} = 14,660 - 9,762 = 4,898$$

$$\text{IBNR} = 17,065 - 14,660 = 2,405$$

$$\text{Unpaid ULAE} = 0.08 \times 2,405 + 0.8 \times 0.5 \times 4,898 = 388.$$

- (e) Describe the Kittel refinement to the classical paid-to-paid method and the weakness it is designed to address.

Kittel method derives ULAE ratio by comparing paid ULAE to average of paid and reported claims (rather than paid to paid ratio used in Classical method).

Kittel's change addresses some of the distortion that can arise with increasing (changing) exposures because reported claims react quicker to exposure changes.

- (f) Describe the Mango and Allen smoothing adjustment.

The Mango and Allen Smoothing Adjustment uses expected claim in place of actual claims.

## GIRR Spring 2022 Question 2 (LOs 1d, 2a, 3c, 3d)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11 and 14.

### Commentary on Question:

*This question tests the candidate's understanding of creating a development triangle from detailed claims transaction data, and diagnostic tests that can be used on data triangles.*

### Solution:

- (a) Update both development triangles shown above to include the claim transactions not captured due to the system error.

Accident Year	Incremental Reported Claims - Missing (000)			
	12	24	36	48
2018		15		20
2019	75		-10	
2020		65		
2021				

Accident Year	Cumulative Reported Claims - Missing (000)			
	12	24	36	48
2018	0	15	15	35
2019	75	75	65	
2020	0	65		
2021	0			

Accident Year	Reported Claims (000)			
	12	24	36	48
2018	1,196	1,540	1,653	1,758
2019	1,344	1,682	1,973	
2020	1,294	1,772		
2021	1,451			

e.g.,  $1,344 = 1,269 + 75$

Accident Year	Incremental Reported Counts - Missing			
	12	24	36	48
2018		1		1
2019	1			
2020				
2021				

Accident Year	Cumulative Reported Counts - Missing			
	12	24	36	48
2018	0	1	1	2
2019	1	1	1	
2020	0	0		
2021	0			

Accident Year	Reported Counts			
	12	24	36	48
2018	230	251	261	267
2019	236	256	266	
2020	231	251		
2021	234			

- (b) Determine calendar year 2021 reported claims.

$$\begin{aligned} \text{Calendar year 2021 reported claims (000)} \\ = (1,451 + 1,772 + 1,973 + 1,758) - (1,294 + 1,682 + 1,653) = 2,325 \end{aligned}$$

- (c) Determine case reserves as of December 31, 2021, for accident year 2021 only.

$$\text{Accident Year 2021 case reserves (000)} = 1,451 - 800 = 651$$

- (d) Describe the investigative tests you would recommend using for the following independent situations:



- (i) The claim department implemented a new definition of claims to distinguish between reported incidents that are valid claims and incidents not covered under the insurance policy.
- (ii) The claim department implemented a new initiative to increase their use of partial settlements.
- (i) Ratios of closed no pay counts to closed counts
- (ii) Any of the following is acceptable:
  - Ratios of paid claims to reported claims
  - Average paid claims (paid claims divided by closed counts)
  - Average paid claims on closed with payment counts (paid claims divided by counts closed with payment)
- (e) Provide two examples of company operational changes that could cause an increase in average reported claims without affecting reported counts.

**Commentary on Question:**

*Other answers are possible.*

Any two of the following are acceptable:

- Case reserve strengthening
- Increase in policy limits
- Expanded coverage
- Increase in defense costs, e.g., increased use of outside counsel

**GIRR Fall 2022 Question 7 (LOs 1j, 3c, and 3d)****Learning Outcomes:**

- (1j) Describe qualitative information required for actuarial work.
- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 5 and 14.

**Commentary on Question:**

*This question tests investigative analysis of various development triangles.*

**Solution:**

- (a) Describe two operational changes that could have caused this decrease.
  - A change in systems or process for reporting counts could cause a decrease in frequency.
  - A change in the definition of claim counts could cause a decrease in frequency.
- (b) Describe one external environmental change that could have caused this decrease.

Any one of the following is acceptable:

  - Legislative change implementing tort reform which reduces claims filed.
  - Court interpretation clarifying (confirming) a coverage exclusion.
- (c) Identify a change in pattern in this triangle.

There is a significant decrease along the latest diagonal.
- (d) Describe two possible operational changes that could have caused the pattern change identified in part (b).

**Commentary on Question:**

*This part of the question incorrectly referenced part (b) instead of part (c). The following solution relates to reference to the pattern change identified in part (c). Candidates who answered based on part (b) were graded on that basis.*

- This could be the result of a decrease from slowing down of the payment of claims (claim settlement).
  - Alternatively, it could be a result of increasing from a significant change in case estimates.
- (e) Describe an additional test to further investigate the change in pattern identified in part (b).

**Commentary on Question:**

*This part of the question incorrectly referenced part (b) instead of part (c). The following solution relates to reference to the pattern identified in part (c). Candidates who answered based on part (b) were graded on that basis.*

*Candidates can choose to either refer to the change in claim settlement or the change in case adequacy.*

Change in claim settlement could be confirmed by evaluating the ratios of closed to reported counts to see if a similar pattern is evident (i.e., significant decrease along the latest diagonal).

Change in case adequacy could be confirmed by evaluating average case estimates to see if there is a significant increase along the most recent diagonal.

## GIRR Fall 2022 Question 18 (LOs 1d, 2a)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3 and 11.

### Commentary on Question:

*This question tests the candidate's understanding of the construction of claims data triangles.*

### Solution:

- (a) Construct a cumulative reported claim development triangle by report year.

Report Year	Cumulative Paid Claims (000)						
	2015	2016	2017	2018	2019	2020	2021
2015	330	1,710	3,025	3,602	3,720	3,741	3,746
2016	0	351	2,206	3,685	4,113	4,204	4,212
2017	0	0	436	1,925	3,177	4,110	4,278
2018	0	0	0	423	2,015	3,197	3,867
2019	0	0	0	0	449	2,124	3,664
2020	0	0	0	0	0	354	2,063
2021	0	0	0	0	0	0	584

Report Year	Reported Claims (000) = Cumulative Paid Claims + Case Estimates						
	2015	2016	2017	2018	2019	2020	2021
2015	1,499	3,078	3,387	3,718	3,741	3,746	3,746
2016	0	1,672	3,554	3,907	4,207	4,227	4,228
2017	0	0	1,892	3,303	3,866	4,287	4,296
2018	0	0	0	1,827	3,364	3,717	4,070
2019	0	0	0	0	1,696	3,825	4,217
2020	0	0	0	0	0	1,897	3,774
2021	0	0	0	0	0	0	1,934

Left justify the reported claims triangle by evaluation age:

Report Year	Reported Claims (000)						
	12	24	36	48	60	72	84
2015	1,499	3,078	3,387	3,718	3,741	3,746	3,746
2016	1,672	3,554	3,907	4,207	4,227	4,228	
2017	1,892	3,303	3,866	4,287	4,296		
2018	1,827	3,364	3,717	4,070			
2019	1,696	3,825	4,217				
2020	1,897	3,774					
2021	1,934						

- (b) Calculate the calendar year 2020 reported claims for the coverage above.

$$\begin{aligned}\text{CY 2020 Reported Claims} &= \text{CY 2020 (Paid Claims + Change in Case Reserves)} \\ &= 4,256 + 569 = 4,825\end{aligned}$$

- (c) Update the reported claim development triangle from part (a) to include the missing claim transactions.

Triangle of missing information:

Report Year	Reported Claims (000)							Transaction
	12	24	36	48	60	72	84	
2015								
2016					-5	-5		4
2017								2*
2018		-15	-15	-15				1
2019	10	10	10					3
2020		5						6
2021	30							5

Corrected reported claims triangle:

Report Year	Reported Claims (000)							Transaction
	12	24	36	48	60	72	84	
2015	1,499	3,078	3,387	3,718	3,741	3,746	3,746	
2016	1,672	3,554	3,907	4,207	4,222	4,223		4
2017	1,892	3,303	3,866	4,287	4,296			2*
2018	1,827	3,349	3,702	4,055				1
2019	1,706	3,835	4,227					3
2020	1,897	3,779						6
2021	1,964							5

Note: \* Transaction 2 does not change reported triangle

- (d) Calculate the calendar year 2021 incurred claims.

$$\begin{aligned}\text{CY 2021 incurred claims} &= \text{CY 2021 (reported claims + change in IBNR)} \\ &= 4,601 + 200 = 4,801\end{aligned}$$

## **GIRR Spring 2023 Question 2 (LOs 1g, 2a)**

### **Learning Outcomes:**

- (1g) Identify different types of data used for actuarial work.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.

### **Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 4 and 11.

### **Commentary on Question:**

*This question tests the candidate's understanding of the different types of data used for actuarial work, as well as adjusting development triangles of claims and counts from changes in transactions.*

### **Solution:**

- (a) Provide one advantage and one disadvantage to aggregating claims data by policy year.

Advantage: There is a precise matching of the premiums and the claims arising from those premiums.

Disadvantage: There is a time lag associated with this type of aggregation.

- (b) Provide one disadvantage to aggregating claims data by report year.

Disadvantage: It does not capture claims that have been incurred but not yet reported (pure IBNR).

(c) Construct new data triangles with corrections for this claim file.

What's in the current data:

AY		12	24	36	48
2019	Cumulative paid claims	0	1,500	2,500	60,000
2019	Case estimate	0	900,000	900,000	400,000
2019	Reported Claims	0	901,500	902,500	460,000
2019	Reported Counts	0	1	1	1

What should have been in the data:

AY		12	24	36	48
2019	Cumulative paid claims	0	1,500	2,500	60,000
2019	Case estimate	90,000	90,000	90,000	40,000
2019	Reported Claims	90,000	91,500	92,500	100,000
2019	Reported Counts	1	1	1	1

Accident	Revised Reported Claims					
Year	12	24	36	48	60	72
2017	2,147,785	3,025,674	3,620,901	4,136,684	4,362,359	4,382,594
2018	2,219,814	3,071,925	3,876,926	4,331,668	4,596,920	
2019	2,432,602	3,344,013	4,112,135	4,714,225		
2020	2,591,328	3,398,123	4,339,405			
2021	2,582,962	3,768,518				
2022	2,735,738					

Accident	Revised Reported Counts					
Year	12	24	36	48	60	72
2017	729	895	998	1,082	1,119	1,122
2018	727	900	1,019	1,089	1,130	
2019	744	911	1,022	1,102		
2020	765	902	1,042			
2021	763	939				
2022	767					

There is no change to paid claims and no change to closed counts.



- (d) Calculate calendar year 2022 reported claims, based on corrected data.

Change in reported for accident years 2016 and prior: 7,200

Sum of latest diagonal of adjusted reported claims triangle: 24,537,400

Sum of previous diagonal of adjusted reported claims triangle: 18,787,247

Calendar year 2022 reported claims:  
 $= 24,537,400 - 18,787,247 + 7,200 =$  5,757,353

## GIRR Fall 2023 Question 1 (LOs 1d, 2a, 3d)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3d) Analyze development triangles for investigative testing.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11, and 14.

### Commentary on Question:

*This question tests the candidate's understanding of claim triangles and identifying anomalies in the data.*

### Solution:

- (a) Update both triangles to include the missing transactions.

Claim ID 100 – Changes to accident year (AY) 2019 row of each triangle:

Paid Claims				
AY	12	24	36	48
2019		0	6	6

Case Estimates				
AY	12	24	36	48
2019		5	0	0

Reported Claims				
AY	12	24	36	48
2019	0	5	6	6

Claim ID 200 – Changes to accident year (AY) 2020 row of each triangle:

Paid Claims			
AY	12	24	36
2020	0	6	6

Case Estimates			
AY	12	24	36
2020	17	4	4

Reported Claims			
AY	12	24	36
2020	17	10	10

Claim ID 300 – Changes to accident year (AY) 2021 row of each triangle:

Paid Claims		
AY	12	24
2021	0	11

Case Estimates		
AY	12	24
2021	29	29

Reported Claims		
AY	12	24
2021	29	40

Revised triangles:

Accident	Reported Claims (000)			
Year	12	24	36	48
2019	1,148	1,788	2,532	3,416
2020	3,444	4,903	6,857	
2021	5,739	12,210		
2022	8,035			

Accident	Paid Claims (000)			
Year	12	24	36	48
2019	138	466	888	1,431
2020	413	1,275	3,154	
2021	689	4,151		
2022	1,286			

- (b) Identify an anomaly in the triangle of ratios of paid claims to reported claims based on the corrected triangles from part (a).

Accident Year	Ratios of Paid Claims to Reported Claims			
	12	24	36	48
2019	0.12	0.26	0.35	0.42
2020	0.12	0.26	0.46	
2021	0.12	0.34		
2022	0.16			

For calendar year 2022 (i.e., the latest diagonal), the ratios have increased significantly.

- (c) Describe two operational changes that could have caused the anomaly you identified in part (b).

**Commentary on Question:**

*Only operational changes were given credit. Noting a decrease in the adequacy of case estimates is not sufficient without the explanation of what operational change could lead to a decrease in the adequacy of case estimates.*

- The insurer implemented new processes to speed-up the settlement of claims.
- A change to the approval process that decreased case estimates.

- (d) Calculate incurred claims for calendar year 2021.

Reported claims as of Dec. 31, 2021:  $5,739 + 4,903 + 2,532 = 13,174$

(i.e., the 2021 calendar year diagonal in the revised reported claims triangle)

Reported claims as of Dec. 31, 2020:  $3,444 + 1,788 = 5,232$

Ultimate claims as of Dec. 31, 2021:  $13,174 + 38,476 = 51,650$

Ultimate claims as of Dec. 31, 2020:  $5,232 + 17,722 = 22,954$

CY2021 incurred claims:

= Ultimate claims as of Dec. 31, 2021 – Ultimate claims as of Dec. 31, 2020

=  $51,650 - 22,954 = 28,696$

## GIRR Spring 2024 Question 7 (LOs 1d, 3e, 3f, 3g)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 15, 17, 18.

### Commentary on Question:

*This question tests the candidate's understanding of the development method, the Bornhuetter Ferguson method, and the Benktander method of estimating IBNR.*

### Solution:

- (a) Calculate the IBNR for each AY as of December 31, 2023 using:

- (i) the Development method,
  - (ii) the Bornhuetter Ferguson method, and
  - (iii) two iterations of the Benktander method.
- (i)

AY	Reported Claims	CDF	Development Method Ultimate Claims	Development Method IBNR
2021	5,613,235	1.2556	7,047,851	1,434,616
2022	4,682,692	1.5958	7,472,822	2,790,130
2023	3,554,432	2.3060	8,196,475	4,642,043

- (ii)

AY	Historical Earned Premiums	Claim Trend Factor @6.1%	Premium On-Level Factor	Claim Ratio at Each AY Cost Level	Expected Claims Based on Claim Ratio
2021	10,119,409	1.1257	1.034	69.81%	7,064,127

2022	10,552,425	1.0610	1.020	73.06%	7,709,934
2023	10,850,455	1.0000	1.000	76.00%	8,246,346

AY	Ultimate Claims BF Method	BF Method IBNR
2021	7,051,164	1,437,929
2022	7,561,352	2,878,660
2023	8,224,719	4,670,287

(iii)

AY	BK Method (Ultimate Claims)		BK Method (IBNR)	
	Iteration 1	Iteration 2	Iteration 1	Iteration 2
2021	7,048,525	7,047,988	1,435,290	1,434,753
2022	7,505,877	7,485,164	2,823,185	2,802,472
2023	8,212,471	8,205,534	4,658,039	4,651,102

- (b) Explain if this business is performing better or worse than expected for AY 2023 using the methods above.

2023 claim ratio for each method:

Development method	75.5%
BF method	75.8%
BK 2 <sup>nd</sup> iteration	75.6%

Expected claim ratio: 76.0%

Since all claim ratios are lower than the expected claim ratio, all are performing better than expected.

- (c) Identify one other weakness of the Benktander method.

There is not a clear sense as to the improvement in the estimation of ultimate claims from additional iterations.

### GIRR Fall 2024 Question 3 (LOs 1l, 6d, 6e)

#### Learning Outcomes:

- (1l) Understand credibility as used for actuarial work.
- (6d) Calculate loadings for catastrophes and large claims.
- (6e) Apply loadings for catastrophes and large claims in ratemaking.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 6 and 31.

#### Commentary on Question:

*This question tests the candidate's understanding of loadings for large claims by analyzing claims at various limits.*

#### Solution:

- (a) Identify two other considerations in assigning credibility to an experience set of data.

#### Commentary on Question:

*This question is about assigning the credibility and not about what is considered for the complement of credibility.*

Any two of the following are acceptable:

- The number of years of claim data underlying the experience
- The stability or variability observed in claims from year to year
- The presence or absence of large or unusual claims
- Changes in the internal or external environment
- The age, relevance, and reliability of the experience
- The age, relevance, and reliability of other data to which the complement of credibility would be applied

- (b) Calculate the loadings for 500,000 to total limits for each accident year.

Severity trend for 1,000,000 limit =  $7.0\% \times 0.70 + 6.0\% \times 0.30 = 6.7\%$

Severity trend for total limit =  $8.6\% \times 0.50 + 7.0\% \times 0.50 = 7.8\%$

Accident Year	Trend Period	<u>Severity Trend at:</u>		Trended Claims at 1,000,000 Limit	Total Limit
		6.7%	7.8%		
2021	4.667	1.353	1.420	5,817,559	6,365,155
2022	3.667	1.268	1.317	5,541,683	6,068,833
2023	2.667	1.189	1.222	5,813,421	6,228,374

Accident Year	Loading for 1,000,000 to Total Limit	Loadings for 500,000 to 1,000,000 Limit	Loadings for 500,000 to Total Limit
2021	1.094	1.196	1.309
2022	1.095	1.165	1.276
2023	1.071	1.185	1.270

- (c) Recommend a loading for 500,000 to total limits for ratemaking purposes. Justify your recommendation.

Average of 2022 and 2023 = 1.273

Justification:

- Accident year 2021 loading is much higher than 2022 & 2023
- Therefore, use most recent 2 years as it is more stable, and it uses the most recent data.



## GIRR Fall 2024 Question 4 (LOs 1d, 2a, 2c)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11, and 12.

### Commentary on Question:

*This question tests the candidate's understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies. The candidate also needs to calculate reported claim ratios and IBNR.*

### Solution:

- (a) Verify the earned premiums for calendar years 2021, 2022, and 2023.

Written Date	Written Premium	Months Earned in Calendar Year			Earned Premium in Calendar Year		
		2021	2022	2023	2021	2022	2023
1-Apr-21	120	9	12	3	45	60	15
1-May-21	120	8	12	4	40	60	20
1-Jun-21	120	7	12	5	35	60	25
1-Jul-21	120	6	12	6	30	60	30
1-Aug-21	120	5	12	7	25	60	35
1-Sep-21	120	4	12	8	20	60	40
1-Oct-21	120	3	12	9	15	60	45
1-Nov-21	120	2	12	10	10	60	50
1-Dec-21	120	1	12	11	5	60	55
1-Jan-22	120	0	12	12	0	60	60
1-Feb-22	120	0	11	12	0	55	60
1-Mar-22	120	0	10	12	0	50	60
1-Apr-22	120	0	9	12	0	45	60
1-May-22	120	0	8	12	0	40	60
1-Jun-22	120	0	7	12	0	35	60
1-Jul-22	120	0	6	12	0	30	60
1-Aug-22	120	0	5	12	0	25	60
1-Sep-22	120	0	4	12	0	20	60

Written Date	Written Premium	Months Earned in Calendar Year			Earned Premium in Calendar Year		
		2021	2022	2023	2021	2022	2023
1-Oct-22	120	0	3	12	0	15	60
1-Nov-22	120	0	2	12	0	10	60
1-Dec-22	120	0	1	12	0	5	60
1-Jan-23	120	0	0	12	0	0	60
1-Feb-23	120	0	0	11	0	0	55
1-Mar-23	120	0	0	10	0	0	50
1-Apr-23	120	0	0	9	0	0	45
1-May-23	120	0	0	8	0	0	40
1-Jun-23	120	0	0	7	0	0	35
1-Jul-23	120	0	0	6	0	0	30
1-Aug-23	120	0	0	5	0	0	25
1-Sep-23	120	0	0	4	0	0	20
1-Oct-23	120	0	0	3	0	0	15
1-Nov-23	120	0	0	2	0	0	10
1-Dec-23	120	0	0	1	0	0	5
1-Jan-24	120	0	0	0	0	0	0
1-Feb-24	120	0	0	0	0	0	0
1-Mar-24	120	0	0	0	0	0	0
Total					225	930	1,425

- (b) Calculate the unearned premiums as of each year-end for 2021, 2022, and 2023.

	31-Dec-2021	31-Dec-2022	31-Dec-2023
Earned Premiums:	225	930	1,425
Written premiums:	1,080	1,440	1,440
Unearned premiums	855	1,365	1,380

e.g.,  $1,365 = 1,440 - 930 + 855$

- (c) Calculate in-force premiums as of December 31, 2023.

There are 24 policies in-force as of December 31, 2023. (#10 through 33)

In-force premiums =  $24 \times 120 = 2,880$

- (d) Describe a scenario where the market analyst's conclusion would be incorrect.

Either of the following is acceptable:

- DEF writes the same volume of written premiums as ABC but annual term policies.

- DEF writes the same volume of written premiums as ABC with 2-year term policies but books annually.

(e) Calculate the reported claim ratios for each of calendar years 2022 and 2023.

Reported claims for CY2022: 319

Reported claims for CY2023: 622

Claim ratio for CY2022: 34.3%

Claim ratio for CY2023: 43.6%

(f) Calculate IBNR for accident years 2022 and 2023.

Ultimate claim ratio for AY2021 =  $135/225 = 60\%$

Accident Year	Earned Premiums	Ultimate Claims	IBNR
2022	930	558	112
2022	1,425	855	427

e.g.,  $558 = 0.6 \times 930$ ;  $112 = 558 - 446$

## GI 101 – LEARNING OBJECTIVE 2

### 2. Topic: Preparing Claims and Exposure Data for Actuarial Work

The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

## GIRR Fall 2020 Question 1 (LOs 2a)

### Learning Outcomes:

(2a) Create development triangles of claims and counts from detailed claim transaction data.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 11.

### Solution:

#### Commentary on Question:

*This question tests the constructions of claims data triangles as well as the candidate's ability to recognize inconsistencies with claims data triangles.*

### Solution:

(a) Identify the inconsistencies in the data triangles.

Reported Claims = Cumulative Paid + Case Estimates

Accident Year	Reconciled Reported Claims (000)			
	12	24	36	48
2016	12,800	16,380	18,350	<b>18,680</b>
2017	13,700	17,810	<b>19,550</b>	
2018	15,200	<b>19,070</b>		
2019	<b>14,720</b>			

The most recent diagonal does not reconcile, as Reported claims > Paid claims + Case estimates.

(b) Provide one potential cause for the data issue identified in part (a).

There are either paid claims or case estimates missing from the latest calendar year.

(c) Construct revised paid claims and case estimates triangles incorporating this additional information.

Changes to cumulative paid:

Increase in AY2017 @ 12 months by 11,000 {both indemnity + ALAE}

Increase in AY2017 @ 24 months by 25,000 {both indemnity + ALAE}

Increase in AY2017 @ 36 months by 25,000 {both indemnity + ALAE}

*{Note: no payments and no case adjustment in CY2019, so still the same values by the end of 2019}*

Changes in case estimates:

Increase in AY2017 @ 12 months by 30,000 {latest case estimate}

Increase in AY2017 @ 24 months by 20,000 {latest case estimate}

Increase in AY2017 @ 36 months by 20,000 {latest case estimate}

*{Note: no payments and no case adjustments in CY2019, so still the same values by the end of 2019}*

Accident Year	Restated Paid Claims (000)			
	12	24	36	48
2016	9,730	14,580	17,430	18,300
2017	9,461	15,345	18,435	
2018	10,940	16,090		
2019	11,100			

Accident Year	Restated Case Estimates (000)			
	12	24	36	48
2016	3,070	1,800	920	380
2017	4,280	2,510	1,160	
2018	4,260	2,980		
2019	3,620			

e.g., Restated paid claims for AY2017 @ 12 months = 9,450 + 11,000 / 1,000

- (d) Calculate the calendar year 2018 reported claims using the revised triangles from part (c).

CY reported = (Case estimate at end of year) – (Case estimate at beginning of year) + (Paid claims during the year)

Case estimate at the end of 2018 = 4,260 + 2,510 + 920 = 7,690

Case estimate at the end of 2017 = 4,280 + 1,800 = 6,080

Paid during 2018 = 10,940 + 15,345 – 9,461 + 17,430 – 14,580 = 19,674

CY 2018 reported claims = 21,284

## GIRR Fall 2020 Question 9 (LOs 2d, 3e, 3f, 3g)

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 15 and 19.

### Commentary on Question:

*This question tests the estimating of ultimate claims using the Cape Cod method.*

### Solution:

- (a) Describe why premium on-level factors are typically used in the Cape Cod method but not in the Bornhuetter Ferguson method.

The Bornhuetter Ferguson method uses an external a priori estimate as an expected claim ratio. (This should implicitly be at appropriate rate level).

The Cape Cod method derives one adjusted expected claim ratio from all historical data, so all years need to reflect the same rate level.

- (b) Describe a situation in which an actuary may choose to derive an adjusted expected pure premium instead of an adjusted expected claim ratio when using the Cape Cod method.

Any one of the following is acceptable:

- If rate change history is not available/reliable
- If on-level premium adjustment factors are not available/reliable
- If exposure base is not inflation-sensitive, then using exposures simplifies the calculation

- (c) Explain why confidence in the development method is a consideration in selecting the decay factor.

As the decay factor approaches 0, projected ultimate claims in the Generalized Cape Cod method approach results from the development method. So, an actuary who has significant confidence in the development method can choose a smaller decay factor.

- (d) Calculate premium on-level factors for each accident year, to use in the Cape Cod method as of December 31, 2019.

**Commentary on Question:**

*The diagram is helpful to solve the question but not required for credit.*

2015	2016	2017	2018	2019
A		B		C

Level	Rate Level	<u>Percent Premium Earned in Each CY at Rate Level</u>				
	Index	2015	2016	2017	2018	2019
A	1.00000	100.0%	50.0%			
B	1.06000		50.0%	100.0%	87.5%	12.5%
C	1.11300				12.5%	87.5%
Total		100.0%	100.0%	100.0%	100.0%	100.0%

Average rate level in each CY:      1.00000   1.03000   1.06000   1.06663   1.10638

On-level factors for reserving:      1.1064   1.0742   1.0438   1.0373   1.0000

e.g.,  $1.0373 = 1.10638 / 1.06663$

- (e) Calculate the projected ultimate claims for each accident year using the Cape Cod method.

Accident Year	(1) Earned Premiums (EP)	(2) Premium On-Level Factors from part (d)	(3) = (1)(2) On-Level Earned Premiums	(4) Reported CDFs	(5) = 1/(4) Expected % Reported	(6) = (3)(5) Used-Up On-Level EP
2015	16,100	1.1064	17,813	1.030	97.1%	17,294
2016	17,600	1.0742	18,905	1.055	94.8%	17,919
2017	18,300	1.0438	19,101	1.100	90.9%	17,364
2018	19,800	1.0373	20,538	1.300	76.9%	15,798
2019	21,600	1.0000	21,600	1.700	58.8%	12,706
	93,400		97,956			81,082



	(7)	(8)	(9)	(10) = (7)(8)(9)	(11)
Accident Year	Actual Reported Claims excluding Large Claim	<u>Claim Adjustment Factors</u>		Adjusted Claims	Expected Claims
		Trend at 5%	Tort Reform		
2015	11,150	1.2155	0.90	12,198	11,548
2016	11,380	1.1576	0.95	12,515	12,191
2017	11,190	1.1025	1.00	12,337	12,287
2018	10,870	1.0500	1.00	11,414	13,872
2019	9,040	1.0000	1.00	9,040	15,319
	53,630			57,503	65,217

Adjusted Expected Claim Ratio: 70.92% = 57,503 / 81,082

Notes: (7) for AY2018: 10,870 = 11,470 – 800

(11) = [(3) × 0.7092] / [(8)(9)]

	(12)	(13) = 1 – (5)	(14) = (11)(13)	(15) = (12) + (14)
Accident Year	Actual Reported Claims	Expected % Unreported	Expected Unreported	Projected Ultimate
2015	11,150	2.9%	336	11,486
2016	11,380	5.2%	636	12,016
2017	11,190	9.1%	1,117	12,307
2018	11,470	23.1%	3,201	14,671
2019	9,040	41.2%	6,308	15,348
	54,230		11,598	65,828

### GIRR Fall 2020 Question 16 (LOs 2d, 5b, 5e, 6g)

#### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 26, 27, and 32.

#### Commentary on Question:

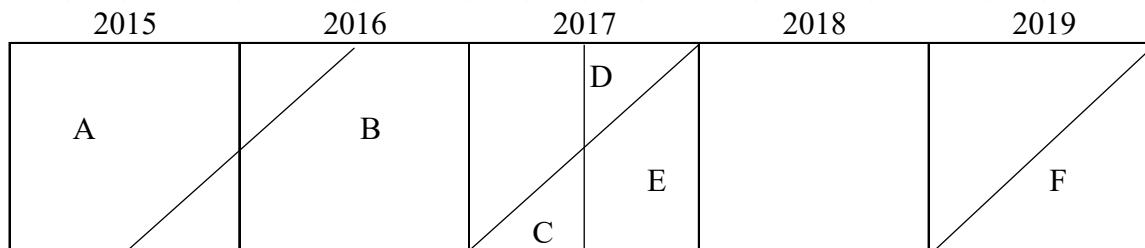
*This question tests basic ratemaking using a claim ratio approach. The candidate also needs to understand earned premiums adjusted to current rate level for ratemaking purposes.*

#### Solution:

- (a) Calculate premium on-level factors for accident years 2015-2019 to use for ratemaking purposes.

#### Commentary on Question:

*The diagram is helpful to solve the question but not required for credit.*



		<u>Percent Premium Earned in Each CY at Rate</u>				
	<u>Rate Level</u>	<u>Level</u>				
Level	Index	2015	2016	2017	2018	2019
A	1.00000	87.5%	12.5%			
B	1.08000	12.5%	87.5%	37.5%		
C	1.18800			12.5%		
D	0.86400			12.5%		
E	0.95040			37.5%	100.0%	50.0%
F	0.99792					50.0%
Total		100.0%	100.0%	100.0%	100.0%	100.0%

Average rate level in each CY:      1.01000   1.07000   1.01790   0.95040   0.97416

On-level factors for ratemaking:      0.9880   0.9326   0.9804   1.0500   1.0244

e.g.,     $0.97416 = 0.5 \times 0.95040 + 0.5 \times 0.99792$   
 $1.2044 = 0.99792 / 0.97416$

- (b) Calculate the trended on-level claim ratios for each accident year.

Trend from the average accident date in each AY (i.e., July 1) to the average accident date in future rating period.

Average accident date in future rating period: November 1, 2021

Accident Year	Trending Period in Years	Earned Premiums	<u>Premium Adj. Factors</u>		Trended Earned Prem. at Current Rate Level
			Trend at 1.00%	On-Level	
2015	6.333	11,755,570	1.0650	0.9880	12,370,486
2016	5.333	11,864,520	1.0545	0.9326	11,668,350
2017	4.333	12,406,530	1.0441	0.9804	12,698,923
2018	3.333	12,492,860	1.0337	1.0500	13,559,877
2019	2.333	12,394,530	1.0235	1.0244	12,995,072

e.g., for AY2019:     $1.0235 = 1.01^{2.333}$   
 $12,995,072 = 12,394,530 \times 1.0235 \times 1.0244$

Accident Year	Ultimate Claims	Pure Premium Trend Factor at 4.00%	Regulation Adjustment to Claims	Trended Claims	Trended Claim Ratio
2015	8,130,150	1.2820	0.80	8,338,086	67.40%
2016	7,970,110	1.2327	0.80	7,859,570	67.36%
2017	7,781,380	1.1853	0.90	8,300,615	65.36%
2018	8,001,680	1.1397	1.00	9,119,247	67.25%
2019	7,995,960	1.0958	1.00	8,762,239	67.43%

e.g., for AY 2019:  $1.0958 = 1.04^{2.333}$   
 $8,762,239 = 7,995,960 \times 1.0958 \times 1.00$   
 $67.43\% = 8,762,239 / 12,995,072$

- (c) Recommend a trended claim ratio to use for ratemaking. Justify your recommendation.

Accident Year	Trended Claim Ratio	Accident Year Weights
2015	67.40%	10%
2016	67.36%	15%
2017	65.36%	20%
2018	67.25%	25%
2019	67.43%	30%

Weighted average trended claim ratio = 66.96%

Justification: No significant outliers, so average of all years with more weight to more recent experience.

- (d) Calculate the indicated rate change.

Weighted average trended claim ratio	66.96%
Ratio of ULAE to claims	10.00%
Weighted average trended claim ratio including ULAE = $0.6696 \times (1 + 0.10) =$	73.65%
Fixed expenses as ratio to premiums at current rate level	6.00%
Variable expenses (ratio to premiums)	19.00%
Profit and contingencies ratio to premiums	5.00%
Permissible claim ratio = $(1 - 0.19 - 0.05) / (1 + 0.06/0.7365) =$	70.28%
Indicated rate change = $0.7365 / 0.7028 - 1 =$	4.81%

- (e) Explain why an indicated rate increase of 5% is not necessarily indicative of deteriorating experience.

We are told that rates were adequate at the time of the rate change. Therefore, if experience does not get better or worse after the change, then experience should change with expected net trend.

Net trend = (claim trend)/(premium trend) – 1 =  $(1 + 0.04) / (1 + 0.01) - 1 = 2.97\%$

Time from the change to the effective date of the new rates = 1.5 years

Therefore, experience should change with respect to net trend =  $(1 + 0.0297)^{1.5} - 1 = 4.5\%$

Since this is close to the rate change implemented at that time, this is as expected and does not indicate deteriorating experience.

## GIRR Spring 2021 Question 1 (LOs 2c, 2d)

### Learning Outcomes:

- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
- (2d) Adjust historical earned premiums to current rate levels.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 12 & 13.

### Commentary on Question:

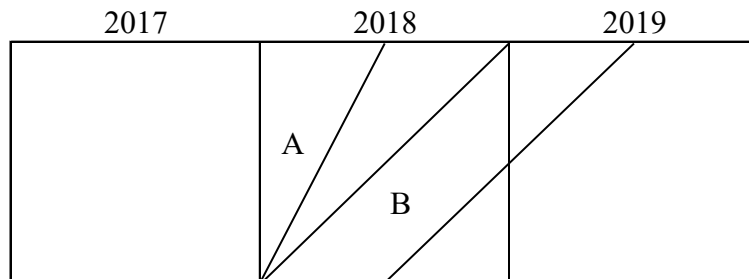
*This question tests the candidate's understanding of earned premiums and adjusting earned premiums to current rate levels for ratemaking purposes.*

### Solution:

- (a) Calculate the 2018 earned premium.

### Commentary on Question:

*Candidates who made use of the diagram did better on this question.*



1. Policies in force as of Dec. 31, 2017:

A: These are the policies that are in force as of Dec. 31, 2017 and expire in 2018 Area =  $\frac{1}{2} \times 1 \times \frac{1}{2} = 25\%$

$$\text{Earned premium} = 2,500 \times 750 \times 0.25 = 468,750$$

B: These are the policies from 2017 that expired in 2018 and then renewed Area =  $\frac{1}{2} - \frac{1}{8} = 37.5\%$

$$\text{Earned premium} = 2,500 \times 750 \times 0.375 \times 0.80 \times (1 + 0.04) = 585,000$$

2. Policies written new from July 1, 2018:

$$\text{Earned premium} = 2,750 \times 780 \times 0.50 = 1,072,500$$

$$\text{Total 2018 earned premium} = 468,750 + 585,000 + 1,072,500 = 2,126,250.$$

- (b) Calculate the 2018 on-level earned premium to use for ratemaking.

**Commentary on Question:**

*The parallelogram approximation approach is not accurate for this question due to the different terms of the policies during the year.*

- 1 A: This area needs to reflect both rate changes to be on-level:

$$\text{On-level earned premium (OLEP)} = 468,750 \times (1 + 0.04) \times (1 + 0.05) = 511,875$$

- 1 B: This needs only needs to reflect the 2020 rate change:

$$\text{OLEP} = 585,000 \times (1 + 0.05) = 614,250$$

2. This needs only needs to reflect the 2020 rate change:

$$\text{OLEP} = 1,072,500 \times (1 + 0.05) = 1,072,500$$

$$\text{Total 2018 on-level earned premium} = 511,875 + 614,250 + 1,072,500 = 2,198,625.$$

## GIRR Fall 2021 Question 1 (LOs 2c, 2d)

### Learning Outcomes:

- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
- (2d) Adjust historical earned premiums to current rate levels.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 12 and 13.

### Commentary on Question:

*This question tests the candidate's understanding of written premiums, earned premiums and unearned premiums. In addition, this question tests the candidate's understanding of adjusting premiums to current rate levels.*

### Solution:

- (a) Calculate the 2020 calendar year total written premiums.

Policy#	2020 Written Premium	
1	2,205	written in 2020 (gets the 5% renewal increase)
2	1,440	written in 2020 (Feb 1 & Aug 1 renewal)
3	1,800	written in 2020
3	-600	Cancellation (4 months remaining at the time of cancellation)
Total	4,845	

- (b) Calculate the 2020 calendar year total earned premiums.

Policy#	Period	Original Written Premium	# of Months Earned in 2020	2020 Earned Premium
1	Jan 1, 2020 to Oct 31, 2020	2,100	10	1,750.00
1	Nov 1, 2020 to Dec 31, 2020	2,205	2	367.50
2	Feb 1, 2020 to July 31, 2020	720	6	720.00
2	Aug 1, 2020 to Dec 31, 2020	720	5	600.00
3	April 1, 2020 to Nov 30, 2020	1,800	8	1,200.00
Total 2020 Earned Premium				4,637.50

Notes:  $2,205 = 2,100 \times 1.05$



$$368 = 2,205 \times 2/12$$

- (c) Calculate the total unearned premiums as of December 31, 2020.

Policy#	Unearned Premium
1	1,837.50
2	120.00
3	0.00
Total	1,957.50

Notes:  $1,837.50 = 2,205 - 367.50$

Policy 3 not in force at the end of 2020, therefore unearned premium = 0

- (d) Explain why the parallelogram approach would be inaccurate for this calculation.

**Commentary on Question:**

*Policy duration is not relevant to the parallelogram approach being inaccurate.*

Any two of the following are acceptable:

- The policies are not written evenly throughout the period.
- The parallelogram approach is an approximation method and with so few policies the actual calculation is more accurate.
- The parallelogram approach is more appropriate for an entire book of business and not few individual policies.

- (e) Calculate the 2020 total earned premiums adjusted to the current rate level.

Policy#	Period	2020 Earned Premium	Earned Premium at Current Rate Level
1	Jan 1, 2020 to Oct 31, 2020	1,750.00	1,837.50
1	Nov 1, 2020 to Dec 31, 2020	367.50	367.50
2	Feb 1, 2020 to July 31, 2020	720.00	756.00
2	Aug 1, 2020 to Dec 31, 2020	600.00	630.00
3	April 1, 2020 to Dec 1, 2020	1,200.00	1,260.00
Total 2020 Earned Premium at Current Rate Level			4,851.00

## GIRR Fall 2021 Question 6 (LOs 2d, 3g)

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.  
 (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 17, and 19.

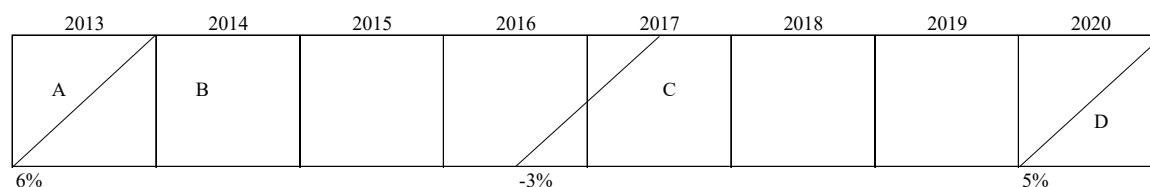
### Question:

### Commentary on Question:

*This question tests the candidate's understanding of adjusting earned premiums to current rate levels as well as estimating ultimate claims using the expected method and the Cape Cod method.*

### Solution:

- (a) Calculate premium on-level factors for all accident years for projecting claim ratios as of December 31, 2020.



Rate Change History							
Effective Date of Rate Change	Rate Change %	Rate Level Index	Percent Premium Earned in Each CY at Rate Level				
			2011	2012	2013	2014	2015
Initial		1.00000	100.00%	100.00%	50.00%	-	-
Jan. 1, 2013	6.0%	1.06000	-	-	50.00%	100.00%	100.00%
Jul. 1, 2016	-3.0%	1.02820	-	-	-	-	-
Jan. 1, 2020	5.0%	1.07961	-	-	-	-	-
Average Rate Level in each CY:			1.00000	1.00000	1.03000	1.06000	1.06000
On-Level Factors for reserving:			1.05391	1.05391	1.02321	0.99425	0.99425

Rate Change History			Percent Premium Earned in Each CY at Rate Level				
Effective Date of Rate Change	Rate Change %	Rate Level Index	2016	2017	2018	2019	2020
Initial		1.00000	-	-	-	-	-
Jan. 1, 2013	6.0%	1.06000	87.50%	12.50%	-	-	-
Jul. 1, 2016	-3.0%	1.02820	12.50%	87.50%	100.00%	100.00%	50.00%
Jan. 1, 2020	5.0%	1.07961	-	-	-	-	50.00%

Average Rate Level in each CY:	1.05603	1.03218	1.02820	1.02820	1.05391
On-Level Factors for reserving:	0.99799	1.02105	1.02500	1.02500	1.00000

e.g., 2016       $1.05603 = (1.06 \times 0.875) + (1.0282 \times 0.125)$   
 $0.99799 = 1.05391 / 1.05603$

(b) Calculate projected ultimate claims for all accident years using the expected method.

	(1)	(2)	(3)	(4)	(5)
Accident Year (AY)	On-Level Factors	Tort Reform	Trended On- Level Claim Ratio	Claim Ratio at Cost Level of Each AY	Projected Ultimate Claims
2011	1.05391	0.80	67.0%	84.5%	4,889,698
2012	1.05391	0.80	66.5%	84.5%	4,456,640
2013	1.02321	0.80	53.7%	82.0%	3,999,255
2014	0.99425	0.90	68.8%	70.8%	3,417,196
2015	0.99425	1.00	68.3%	63.8%	3,270,117
2016	0.99799	1.00	59.6%	64.0%	3,455,112
2017	1.02105	1.00	66.8%	65.5%	3,388,744
2018	1.02500	1.00	64.5%	65.7%	3,136,239
2019	1.02500	1.00	62.1%	65.7%	2,999,591
2020	1.00000	1.00		64.1%	3,154,776
Total			64.1%		36,167,367

Notes: (3) = [(Projected ultimate claims from development method)(2) / [(Earned premiums)(1)]  
(3)<sub>Total</sub> = Average of AY2011 through AY2019  
(4) = 64.1% × (1) / (2)  
(5) = (4)(Earned premiums)

(c) Calculate projected ultimate claims for all accident years using the Cape Cod method.

	(6)	(7)	(8) = (6)(7)	(9)	(10)
Accident Year (AY)	On-Level Earned Premium	Expected % Paid	Used-Up On- Level Earned Premium	Adjusted Paid Claims at Dec. 31, 2020	Expected Claims
2011	6,099,959	96.5%	5,887,991	3,944,320	4,918,179
2012	5,559,714	92.5%	5,143,121	3,418,400	4,482,598
2013	4,989,120	86.5%	4,315,848	2,316,800	4,022,549
2014	4,795,868	78.2%	3,749,702	2,578,140	3,437,100
2015	5,099,389	70.2%	3,581,032	2,447,000	3,289,164
2016	5,387,869	55.5%	2,988,280	1,780,460	3,475,237
2017	5,284,375	39.5%	2,088,686	1,395,000	3,408,482
2018	4,890,621	26.3%	1,286,667	829,600	3,154,507
2019	4,677,534	13.7%	639,357	396,900	3,017,063
2020	4,919,527	4.5%	221,920	180,900	3,173,151
Total			29,902,603	19,287,520	36,378,030
Adjusted Expected Claim Ratio:				64.5%	

Notes: (6) = (1)(Earned Premiums)  
 (7) = 1 / (Cumulative Development Factors)  
 (9) = (2)(Paid Claims as of December 31, 2020)  
 Adjusted Expected Claim Ratio = 19,287,520 / 29,902,603  
 (10) = 64.5%×(6)/(2)

	(11) = 1 – (7)	(12) = (10)(11)	(13)
Accident Year (AY)	Expected % Unpaid	Expected Unpaid Claims	Projected Ultimate Claims
2011	3.5%	170,902	5,101,302
2012	7.5%	335,884	4,608,884
2013	13.5%	542,835	3,438,835
2014	21.8%	749,766	3,614,366
2015	29.8%	979,358	3,426,358
2016	44.5%	1,547,762	3,328,222
2017	60.5%	2,061,256	3,456,256
2018	73.7%	2,324,592	3,154,192
2019	86.3%	2,604,671	3,001,571
2020	95.5%	3,030,010	3,210,910
Total		14,347,036	36,340,896

Notes: (13) = (12) + (Paid Claims as of December 31, 2020)

## GIRR Fall 2021 Question 16 (LOs 2a, 3c, 3d)

### Learning Outcomes:

- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 11 and 14.

### Commentary on Question:

*This question tests the candidate's understanding of creating a development triangle from detailed claims transaction data, identifying potential issues with data triangles, and diagnostic tests that can be used on data triangles.*

### Solution:

- (a) Define “maturity age” in the context of a claim development triangle.

The maturity age refers to the time interval from the beginning of the experience period to the valuation date of the claims.

- (b) Construct a development triangle of cumulative reported claims, by accident year, with maturity ages 6, 12, 18, 24, 30 and 36 months.

Accident Year (AY)	<u>Incremental Paid Claims at Maturity Age (in Months)</u>					
	6	12	18	24	30	36
2018	50	100	250	0	55	75
2019	265	0	30	185		
2020	0	275				

e.g., AY2019 at 6 months:  $265 = 190 + 75$

AY	<u>Cumulative Paid Claims at Maturity Age (in Months)</u>					
	6	12	18	24	30	36
2018	50	150	400	400	455	530
2019	265	265	295	480		
2020	0	275				

e.g., AY2019 at 18 months:  $295 = 265 + 0 + 30$

AY	<u>Case Estimates at Maturity Age (in Months)</u>					
	6	12	18	24	30	36
2018	150	200	75	390	410	350
2019	35	260	225	0		
2020	550	65				

e.g., AY2019 at 12 months:  $260 = 35 + 225 + 0$

AY	<u>Reported Claims at Maturity Age (in Months)</u>					
	6	12	18	24	30	36
2018	200	350	475	790	865	880
2019	300	525	520	480		
2020	550	340				

Reported claims = Cumulative paid claims + Case estimates

e.g., AY2019 at 12 months:  $525 = 265 + 260$

- (c) Select which line of business was the likely source for each of the following claims, providing a justification for each selection:
- (i) Claim 2 is likely Automobile physical damage as it has a short reporting delay and was settled within 6 months of claim occurrence.
  - (ii) Claim 3 is likely Medical malpractice claim as it has a long reporting delay and has not closed within 36 months of its occurrence.
  - (iii) Claim 7 is likely Workers' compensation claim as it was reopened after its initial settlement.
- (d) Identify two anomalies relating to this triangle.

Any two of the following are acceptable:

- Reported pure premiums decreased in AYs 2015-2016, then increased again in AYs 2017 and subsequent accident years.
- Reported pure premium for AY2014 increased significantly at 72 months, then decreased again at 84 months.
- Reported pure premium development is increasing over time (i.e., development factors increase down each column).

- (e) Describe a business, operational, or environmental change that could cause each of the anomalies identified in part (d).

**Commentary on Question:**

*Only one change is needed for each anomaly identified in part (d).*

- Reported pure premiums decreased in AYs 2015-2016, then increased again in AYs 2017 and subsequent accident years:
  - Changes in policy terms (e.g., limits, deductibles) could cause PP to change over time.
  - Changes in the type of insureds (exposures) could cause PP to change over time.
- Reported pure premium for AY2014 increased significantly at 72 months, then decreased again at 84 months:
  - The reporting of a large claim (or case estimate) which then decreased/normalized could cause an increase, then decrease in reported pure premiums.
  - The reporting of a large claim, which was subsequently covered by reinsurance (or subrogation) could cause an increase, then decrease in reported pure premiums.
- Reported pure premium development is increasing over time (i.e., development factors increase down each column).
  - Change in policy terms (e.g., limits, deductibles) could cause development to change over time.
  - Change in the type of insureds (exposures) could cause development to change over time.
  - Change in case reserve adequacy (or claim settlement patterns) could cause development to change over time.

### GIRR Spring 2022 Question 1 (LOs 2c, 2d)

#### Learning Outcomes:

- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
- (2d) Adjust historical earned premiums to current rate levels.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 12 and 13.

#### Commentary on Question:

*This question tests the candidate's understanding of written premiums and adjusting premiums to current rate levels.*

#### Solution:

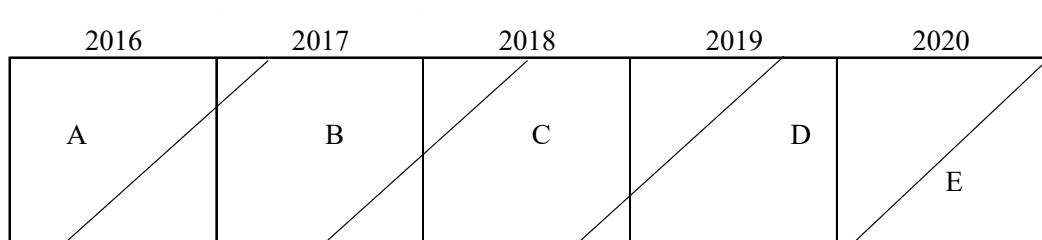
- (a) Calculate the calendar year 2018 written premiums.

$$789,520 = 782,020 - 785,000 + 792,500$$

- (b) Calculate the 2017, 2018, and 2019 on-level earned premiums, applicable for ratemaking, using the parallelogram method.

#### Commentary on Question:

*The diagram is helpful to solve the question but not required for credit.*





Level	Rate Level Index	Percent Premium Earned in Each Calendar Year at Rate Level		
		2017	2018	2019
A	1.00000	3.13%	-	-
B	1.02000	84.38%	12.50%	-
C	1.06080	12.50%	84.38%	28.13%
D	1.13506	-	3.13%	71.88%
E	1.16911	-	-	-
Total		100.0%	100.0%	100.0%

Average rate level in each CY:            1.02448      1.05802      1.11417

On-level factors for ratemaking:            1.1412      1.1050      1.0493

e.g.,     $1.02448 = 0.0313 \times 1.0 + 0.8438 \times 1.02 + 0.125 \times 1.608$   
 $1.1412 = 1.16911 / 1.02448$

Calendar Year	(1) On-Level Factor	(2) Earned Premiums	(3) = (1)(2) On-Level Earned Premiums
2017	1.1412	778,650	888,578
2018	1.1050	782,020	864,128
2019	1.0493	789,880	828,826

- (c) Calculate the 2018 earned premium adjusted to current rate levels for ratemaking purposes for these two policies using the extension of exposures approach.

Policy	Months earned in 2018	% Earned in 2018	2018 Earned Premium	Future Rate Changes		2018 On- Level Earned Premium
				Oct. 1, 2018	Feb. 1, 2020	
1	8	66.7%	3,333.33	7.0%	3.0%	3,673.67
2	2	16.7%	1,166.67	n/a	3.0%	1,201.67
			4,500.00			4,875.33

e.g.,     $3,333.33 = 5,000 \times 0.667$   
 $3,673.67 = 3,333.33 \times 1.07 \times 1.03$

- (d) Explain why the answer in part (c) results in a different answer from multiplying the 2018 earned premiums for these two policies by the 2018 on-level factor calculated in part (b).

The parallelogram approach is an approximation method that assumes policies are written evenly throughout the year. These 2 policies do not represent policies that are written evenly (i.e., they are individual policies and not representative of the average). The extension of exposures approach is more accurate for individual policies.

- (e) Critique this recommendation.

Recommend consistency, so adding the earned premiums from the 2 policies to the total earned premiums and then multiplying by the factor is recommended.

## GIRR Spring 2022 Question 2 (LOs 1d, 2a, 3c, 3d)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11 and 14.

### Commentary on Question:

*This question tests the candidate's understanding of creating a development triangle from detailed claims transaction data, and diagnostic tests that can be used on data triangles.*

### Solution:

- (a) Update both development triangles shown above to include the claim transactions not captured due to the system error.

Accident Year	Incremental Reported Claims - Missing (000)			
	12	24	36	48
2018		15		20
2019	75		-10	
2020		65		
2021				

Accident Year	Cumulative Reported Claims - Missing (000)			
	12	24	36	48
2018	0	15	15	35
2019	75	75	65	
2020	0	65		
2021	0			

Accident Year	Reported Claims (000)			
	12	24	36	48
2018	1,196	1,540	1,653	1,758
2019	1,344	1,682	1,973	
2020	1,294	1,772		
2021	1,451			

e.g.,  $1,344 = 1,269 + 75$

Accident Year	Incremental Reported Counts - Missing			
	12	24	36	48
2018		1		1
2019	1			
2020				
2021				

Accident Year	Cumulative Reported Counts - Missing			
	12	24	36	48
2018	0	1	1	2
2019	1	1	1	
2020	0	0		
2021	0			

Accident Year	Reported Counts			
	12	24	36	48
2018	230	251	261	267
2019	236	256	266	
2020	231	251		
2021	234			

- (b) Determine calendar year 2021 reported claims.

$$\begin{aligned} \text{Calendar year 2021 reported claims (000)} \\ = (1,451 + 1,772 + 1,973 + 1,758) - (1,294 + 1,682 + 1,653) = 2,325 \end{aligned}$$

- (c) Determine case reserves as of December 31, 2021, for accident year 2021 only.

$$\text{Accident Year 2021 case reserves (000)} = 1,451 - 800 = 651$$

- (d) Describe the investigative tests you would recommend using for the following independent situations:

- (i) The claim department implemented a new definition of claims to distinguish between reported incidents that are valid claims and incidents not covered under the insurance policy.
- (ii) The claim department implemented a new initiative to increase their use of partial settlements.
- (i) Ratios of closed no pay counts to closed counts
- (ii) Any of the following is acceptable:
  - Ratios of paid claims to reported claims
  - Average paid claims (paid claims divided by closed counts)
  - Average paid claims on closed with payment counts (paid claims divided by counts closed with payment)
- (e) Provide two examples of company operational changes that could cause an increase in average reported claims without affecting reported counts.

**Commentary on Question:**

*Other answers are possible.*

Any two of the following are acceptable:

- Case reserve strengthening
- Increase in policy limits
- Expanded coverage
- Increase in defense costs, e.g., increased use of outside counsel

## GIRR Fall 2022 Question 2 (LOs 2d)

### Learning Outcomes:

(2d) Adjust historical earned premiums to current rate levels.

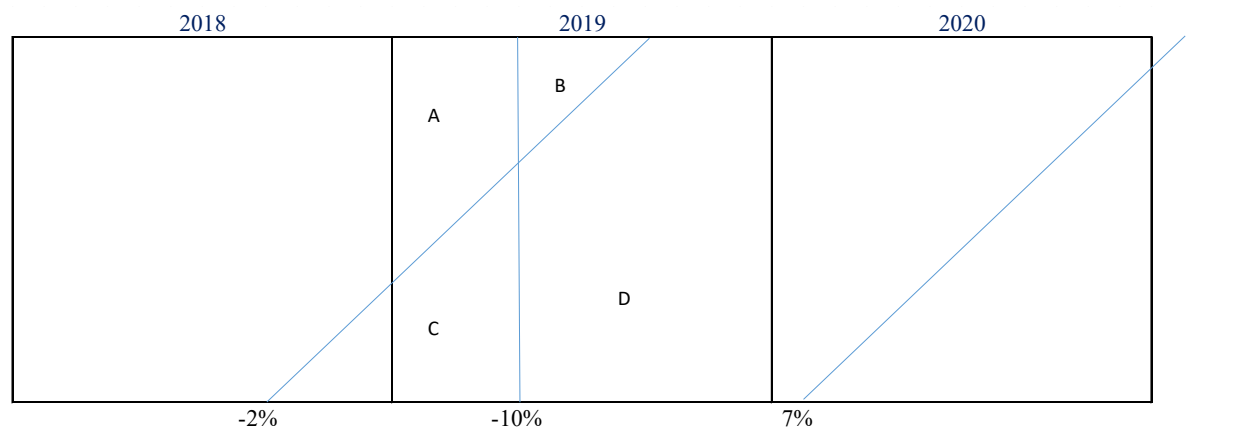
### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13.

*This question tests the candidate's ability to adjust premium to current rate levels for ratemaking purposes. Candidates generally did well with the calculations.*

### Solution:

(a) Calculate the 2019 earned premium adjusted to current rate levels for ratemaking purposes.



Area	Rate Level Index	% Earned in 2019
A	1.05	$1/2 \times 4/12 = 16.67\%$
B	$1.05 \times 0.9 = 0.945$	$1/2 \times 4/12 \times 4/12 = 5.56\%$
C	$1.05 \times 0.98 = 1.029$	$1/2 \times 4/12 = 16.67\%$
D	$1.05 \times 0.98 \times 0.9 = 0.9261$	$100\% - 16.67\% - 5.56\% - 16.67\% = 61.11\%$

Weighted average rate level in CY 2019 =  $1.05 \times 16.67\% + \dots + 0.9261 \times 61.11\%$   
 $= 0.9650$

Current rate level =  $1.05 \times 0.98 \times 1.07 \times 1.03 \times 0.9 = 1.0207$

On-level factor =  $1.0207 / 0.9650 = 1.0577$

CY 2019 earned premium at current rate level for ratemaking purposes:  
 $= 1,400,000 \times 1.0577 = 1,480,819$

- (b) Explain why the answer to part (a) would be higher if all policies were six-month policies instead of twelve-month policies.

**Commentary on Question:**

*Candidates need to provide an explanation for credit. Candidates struggled to fully explain the impacts from the given changes.*

- With all policies being 6-month policies, more of the area of 2019 would be at lower rates (lower % at rate level 1.05, higher % at rate level 0.9261).
  - Therefore, the average rate level in 2019 should be lower.
  - The current rate level remains unchanged.
  - Therefore, the on-level factor would be higher than the value from part (a).
- (c) Explain what effect this change would have on the on-level calculation from part (a).
- The average premium would increase to reflect such a change but would expect claims would increase as policyholders would receive more coverage. Therefore, expect no change to the on-level calculation.

## GIRR Fall 2022 Question 11 (LOs 2b, 2c)

### Learning Outcomes:

- (2b) Describe the different types of exposures used for conducting actuarial work.
- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 12.

### Commentary on Question:

*This question tests the candidate's understanding of earned and unearned exposures. Candidates generally did well on the calculations but struggled to describe how the concepts apply to different types of policies and coverage.*

### Solution:

- (a) Describe the option(s) for recognizing written exposures on each policy.

Policy number 101 should record written exposure at the initial effective date.

Policy number 102 can record written exposure based on:

- (i) at the initial effective date
  - (ii) annual basis only, thus, the total written exposure is divided into equivalent annual values and recorded on the anniversary of the effective date.
- (b) Calculate the percentage premium *earned* on December 31, 2021 for policy number 101.

2 months earned by end of year, therefore % earned =  $2/6 = 33.3\%$

- (c) Calculate the percentage premium *unearned* on December 31, 2021 for policy number 102.

Date written = July 15, 2021, therefore 5.5 months earned by Dec 31, 2021

Therefore,  $24 - 5.5 = 18.5$  months unearned as of Dec. 31, 2021. % unearned =  $18.5/24 = 77.1\%$

- (d) Explain why a warranty policy is not likely to have exposures earned evenly throughout the policy term.
  - A warranty policy is typically a multi-year policy.
  - In warranty coverage, the exposure to claims is often significantly greater in the later years of the policy term than in the early years.



- As a result, a pro rata earning of the premium is not appropriate given that the financial reporting objective is to earn revenue (i.e., premium) in accordance with the delivery of service (i.e., protection for the policyholder from loss).
- (e) Describe three types of coverages or policies, other than a warranty policy, where it may not be appropriate to assume premiums are earned evenly throughout the policy term.

Any three of the following are acceptable:

- (i) property catastrophe coverage for hurricanes or hail coverage are examples of GI where exposure to claims is concentrated over specific months
- (ii) aggregate stop loss coverage has much greater exposure near the end of the policy term rather than during the initial months of coverage
- (iii) policies covering seasonal risks like snowmobile coverage have loss concentrated in the winter months
- (iv) ocean marine insurance may have cessation of shipping operations for three months
- (v) new home warranty policies and policies for product warranties that provide protection for mechanical breakdown or manufacturer defects are typically longer than one year and the exposure to claims is often significantly greater in the later years
- (vi) financial and performance guarantee
- (vii) retrospectively-rated policies have final premiums determined after the policy expiration, which should be written and earned when it enters the insurer's system
- (viii) reinstatement premium may be included within the original premium or may require additional premiums to be paid and can have a distorting effect on earned premium

## GIRR Fall 2022 Question 18 (LOs 1d, 2a)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3 and 11.

### Commentary on Question:

*This question tests the candidate's understanding of the construction of claims data triangles.*

### Solution:

- (a) Construct a cumulative reported claim development triangle by report year.

Report Year	Cumulative Paid Claims (000)						
	2015	2016	2017	2018	2019	2020	2021
2015	330	1,710	3,025	3,602	3,720	3,741	3,746
2016	0	351	2,206	3,685	4,113	4,204	4,212
2017	0	0	436	1,925	3,177	4,110	4,278
2018	0	0	0	423	2,015	3,197	3,867
2019	0	0	0	0	449	2,124	3,664
2020	0	0	0	0	0	354	2,063
2021	0	0	0	0	0	0	584

Report Year	Reported Claims (000) = Cumulative Paid Claims + Case Estimates						
	2015	2016	2017	2018	2019	2020	2021
2015	1,499	3,078	3,387	3,718	3,741	3,746	3,746
2016	0	1,672	3,554	3,907	4,207	4,227	4,228
2017	0	0	1,892	3,303	3,866	4,287	4,296
2018	0	0	0	1,827	3,364	3,717	4,070
2019	0	0	0	0	1,696	3,825	4,217
2020	0	0	0	0	0	1,897	3,774
2021	0	0	0	0	0	0	1,934

Left justify the reported claims triangle by evaluation age:

Report Year	Reported Claims (000)						
	12	24	36	48	60	72	84
2015	1,499	3,078	3,387	3,718	3,741	3,746	3,746
2016	1,672	3,554	3,907	4,207	4,227	4,228	
2017	1,892	3,303	3,866	4,287	4,296		
2018	1,827	3,364	3,717	4,070			
2019	1,696	3,825	4,217				
2020	1,897	3,774					
2021	1,934						

- (b) Calculate the calendar year 2020 reported claims for the coverage above.

$$\begin{aligned}\text{CY 2020 Reported Claims} &= \text{CY 2020 (Paid Claims + Change in Case Reserves)} \\ &= 4,256 + 569 = 4,825\end{aligned}$$

- (c) Update the reported claim development triangle from part (a) to include the missing claim transactions.

Triangle of missing information:

Report Year	Reported Claims (000)							Transaction
	12	24	36	48	60	72	84	
2015								
2016					-5	-5		4
2017								2*
2018		-15	-15	-15				1
2019	10	10	10					3
2020		5						6
2021	30							5

Corrected reported claims triangle:

Report Year	Reported Claims (000)							Transaction
	12	24	36	48	60	72	84	
2015	1,499	3,078	3,387	3,718	3,741	3,746	3,746	
2016	1,672	3,554	3,907	4,207	4,222	4,223		4
2017	1,892	3,303	3,866	4,287	4,296			2*
2018	1,827	3,349	3,702	4,055				1
2019	1,706	3,835	4,227					3
2020	1,897	3,779						6
2021	1,964							5

Note: \* Transaction 2 does not change reported triangle

- (d) Calculate the calendar year 2021 incurred claims.

$$\begin{aligned}\text{CY 2021 incurred claims} &= \text{CY 2021 (reported claims + change in IBNR)} \\ &= 4,601 + 200 = 4,801\end{aligned}$$

## GIRR Spring 2023 Question 1 (LOs 2b, 2c, 2d)

### Learning Outcomes:

- (2b) Describe the different types of exposures used for conducting actuarial work.
- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
- (2d) Adjust historical earned premiums to current rate levels.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 12 and 13.

### Commentary on Question:

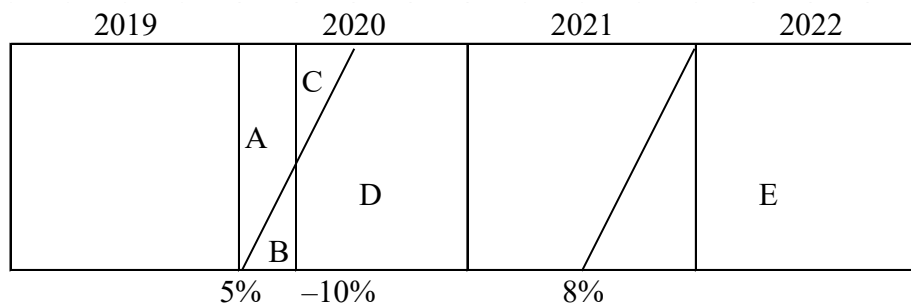
*This question tests the candidate's understanding of the earnings of policies with different policy terms, as well as adjusting earned premiums to current rate level for ratemaking purposes.*

### Solution:

- (a) State the two key assumptions of the parallelogram method.
  - Policies are written evenly over the experience period
  - Exposures are earned evenly over the policy term
- (b) Calculate the calendar year 2020 on-level premium to be used for a ratemaking analysis.

6-month policies:

- in force premium on Jan 1, 2020: 3,000,000
- annualized premium: 6,000,000



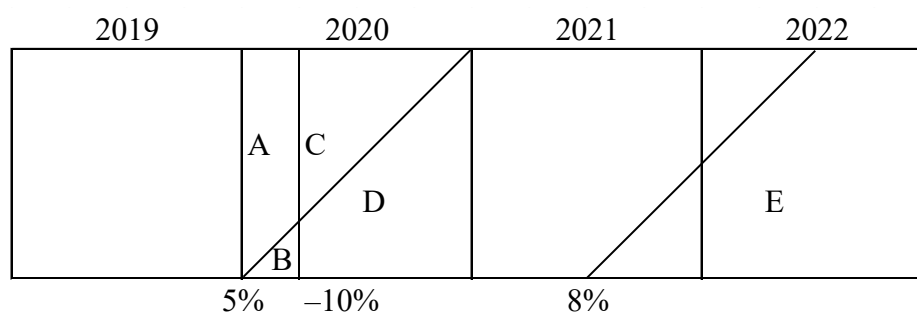
Section	Rate Level Index	Percent Premium Earned in CY2020 at Rate Level
A	1.00000	18.75%
B	1.05000	6.25%
C	0.90000	6.25%
D	0.94500	68.75%
E	1.02060	

Average rate level: 0.9591  
On-level factor: 1.0642  
On-level earned premium: 6,384,985

e.g.,  $1.0642 = 1.0206 / 0.9591$

12-month policies:

- in force premium on Jan 1, 2020: 9,000,000



Section	Rate Level Index	Percent Premium Earned in CY2020 at Rate Level
A	1.00000	21.88%
B	1.05000	3.13%
C	0.90000	28.13%
D	0.94500	46.88%
E	1.02060	

Average rate level: 0.9477  
On-level factor: 1.0770  
On-level earned premium: 9,692,755

Total CY2020 earned premium at current rate level for ratemaking:  
 $= 6,384,985 + 9,692,755 = 16,077,740$

- (c) Provide two examples of general insurance policies where exposures are not usually earned evenly throughout the policy term.

Any two of the following are acceptable:

- Policies covering seasonal risks
- Warranty
- Financial guarantee
- Property catastrophe and aggregate stop-loss reinsurance
- Retrospectively-rated policies
- Policies with reinstatement premium

## GIRR Spring 2023 Question 2 (LOs 1g, 2a)

### Learning Outcomes:

- (1g) Identify different types of data used for actuarial work.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 4 and 11.

### Commentary on Question:

*This question tests the candidate's understanding of the different types of data used for actuarial work, as well as adjusting development triangles of claims and counts from changes in transactions.*

### Solution:

- (a) Provide one advantage and one disadvantage to aggregating claims data by policy year.

Advantage: There is a precise matching of the premiums and the claims arising from those premiums.

Disadvantage: There is a time lag associated with this type of aggregation.

- (b) Provide one disadvantage to aggregating claims data by report year.

Disadvantage: It does not capture claims that have been incurred but not yet reported (pure IBNR).



(c) Construct new data triangles with corrections for this claim file.

What's in the current data:

AY		12	24	36	48
2019	Cumulative paid claims	0	1,500	2,500	60,000
2019	Case estimate	0	900,000	900,000	400,000
2019	Reported Claims	0	901,500	902,500	460,000
2019	Reported Counts	0	1	1	1

What should have been in the data:

AY		12	24	36	48
2019	Cumulative paid claims	0	1,500	2,500	60,000
2019	Case estimate	90,000	90,000	90,000	40,000
2019	Reported Claims	90,000	91,500	92,500	100,000
2019	Reported Counts	1	1	1	1

Accident	Revised Reported Claims					
Year	12	24	36	48	60	72
2017	2,147,785	3,025,674	3,620,901	4,136,684	4,362,359	4,382,594
2018	2,219,814	3,071,925	3,876,926	4,331,668	4,596,920	
2019	2,432,602	3,344,013	4,112,135	4,714,225		
2020	2,591,328	3,398,123	4,339,405			
2021	2,582,962	3,768,518				
2022	2,735,738					

Accident	Revised Reported Counts					
Year	12	24	36	48	60	72
2017	729	895	998	1,082	1,119	1,122
2018	727	900	1,019	1,089	1,130	
2019	744	911	1,022	1,102		
2020	765	902	1,042			
2021	763	939				
2022	767					

There is no change to paid claims and no change to closed counts.

- (d) Calculate calendar year 2022 reported claims, based on corrected data.

Change in reported for accident years 2016 and prior: 7,200

Sum of latest diagonal of adjusted reported claims triangle: 24,537,400

Sum of previous diagonal of adjusted reported claims triangle: 18,787,247

Calendar year 2022 reported claims:  
 $= 24,537,400 - 18,787,247 + 7,200 =$  5,757,353

## GIRR Fall 2023 Question 1 (LOs 1d, 2a, 3d)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3d) Analyze development triangles for investigative testing.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11, and 14.

### Commentary on Question:

*This question tests the candidate's understanding of claim triangles and identifying anomalies in the data.*

### Solution:

- (a) Update both triangles to include the missing transactions.

Claim ID 100 – Changes to accident year (AY) 2019 row of each triangle:

Paid Claims				
AY	12	24	36	48
2019		0	6	6

Case Estimates				
AY	12	24	36	48
2019		5	0	0

Reported Claims				
AY	12	24	36	48
2019	0	5	6	6

Claim ID 200 – Changes to accident year (AY) 2020 row of each triangle:

Paid Claims			
AY	12	24	36
2020	0	6	6

Case Estimates			
AY	12	24	36
2020	17	4	4

Reported Claims			
AY	12	24	36
2020	17	10	10

Claim ID 300 – Changes to accident year (AY) 2021 row of each triangle:

Paid Claims		
AY	12	24
2021	0	11

Case Estimates		
AY	12	24
2021	29	29

Reported Claims		
AY	12	24
2021	29	40

Revised triangles:

Accident	Reported Claims (000)			
Year	12	24	36	48
2019	1,148	1,788	2,532	3,416
2020	3,444	4,903	6,857	
2021	5,739	12,210		
2022	8,035			

Accident	Paid Claims (000)			
Year	12	24	36	48
2019	138	466	888	1,431
2020	413	1,275	3,154	
2021	689	4,151		
2022	1,286			

- (b) Identify an anomaly in the triangle of ratios of paid claims to reported claims based on the corrected triangles from part (a).

Accident Year	Ratios of Paid Claims to Reported Claims			
	12	24	36	48
2019	0.12	0.26	0.35	0.42
2020	0.12	0.26	0.46	
2021	0.12	0.34		
2022	0.16			

For calendar year 2022 (i.e., the latest diagonal), the ratios have increased significantly.

- (c) Describe two operational changes that could have caused the anomaly you identified in part (b).

**Commentary on Question:**

*Only operational changes were given credit. Noting a decrease in the adequacy of case estimates is not sufficient without the explanation of what operational change could lead to a decrease in the adequacy of case estimates.*

- The insurer implemented new processes to speed-up the settlement of claims.
- A change to the approval process that decreased case estimates.

- (d) Calculate incurred claims for calendar year 2021.

Reported claims as of Dec. 31, 2021:  $5,739 + 4,903 + 2,532 = 13,174$

(i.e., the 2021 calendar year diagonal in the revised reported claims triangle)

Reported claims as of Dec. 31, 2020:  $3,444 + 1,788 = 5,232$

Ultimate claims as of Dec. 31, 2021:  $13,174 + 38,476 = 51,650$

Ultimate claims as of Dec. 31, 2020:  $5,232 + 17,722 = 22,954$

CY2021 incurred claims:

= Ultimate claims as of Dec. 31, 2021 – Ultimate claims as of Dec. 31, 2020

=  $51,650 - 22,954 = 28,696$

## GIRR Fall 2023 Question 9 (LOs 2d)

### Learning Outcomes:

(2d) Adjust historical earned premiums to current rate levels.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13.

### Commentary on Question:

*This question tests the candidate's ability to adjust premium to current rate levels for ratemaking purposes.*

### Solution:

- (a) Provide one reason why the company would want to write more 6-month policies in this situation.

Rates have been increasing since the change, so the higher premiums will be earned faster to help keep up with the needed rate changes.

- (b) Calculate the premium on-level factors for calendar years 2019 through 2022 to use in estimating expected claim ratios for the ratemaking analysis.

On-level factors for all 12-month policies:

2019	2020	2021	2022	2023
1	1.0300	1.1021		1.16823
3%	7%		6%	

### Rate Change History

Effective Date of Rate Change	Rate Change %	Rate Level Index	Percent Premium Earned in Each CY at Rate Level			
			2019	2020	2021	2022
	0.0%	1.00000	87.50%	12.50%	-	-
1-Jul-19	3%	1.03000	12.50%	75.00%	12.50%	-
1-Jul-20	7%	1.10210	-	12.50%	87.50%	71.88%
1-Apr-22	6%	1.16823	-	-	-	28.13%
Total			100.00%	100.00%	100.00%	100.00%
Average Rate Level in each CY:			1.00375	1.03526	1.09309	1.12070
On-Level Factors:			1.1639	1.1284	1.0687	1.0424

On-level factors for all 6-month policies:

2019	2020	2021	2022	2023
1	1.03	1.03×1.07		1.03×1.07×1.06
3%	7%		6%	

Rate Change History

Effective Date of Rate Change	Rate Change %	Rate Level Index	<u>Percent Premium Earned in Each CY at Rate Level</u>			
			2019	2020	2021	2022
	0.0%	1.00000	87.50%	12.50%	-	-
1-Jul-19	3%	1.03000	12.50%	62.50%	-	-
1-Jul-20	7%	1.10210	-	25.00%	100.00%	50.00%
1-Apr-22	6%	1.16823	-	-		50.00%
Total			100.00%	100.00%	100.00%	100.00%
Average Rate Level in each CY:			1.00375	1.04428	1.10210	1.13516
On-Level Factors:			1.1639	1.1187	1.0600	1.0291
Combined On-Level factors: (50% weights)			1.1639	1.1236	1.0644	1.0358

(c) Explain why the on-level factors needed for reserving would be lower than the on-level factors calculated in part (b).

- On-level factors for reserving are adjusted to the 2022 average rate level.
- This level is a lower value than the current rate level for ratemaking, leading to lower on-level factors.

(d) Provide one situation where actuaries would need to determine an estimate of ultimate premiums.

Any one of the following is acceptable:

- adjustments to ultimate are required when analyzing policy year data that is not yet completed
- when conducting actuarial work for lines of business where the premiums are subject to audit of exposures following the completion of the policy year
- for lines of business that are subject to retrospective experience rating adjustments

## GIRR Fall 2023 Question 15 (LOs 2c, 2d)

### Learning Outcomes:

- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
- (2d) Adjust historical earned premiums to current rate levels.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 12 and 13.

### Commentary on Question:

*This question tests the candidate's understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies. The candidate also needs to understand earned premiums adjusted to current rate level.*

### Solution:

- (a) Calculate the written premiums for 2022.

(1) Policy Number	(2) Policy Premium	(3) Policy Effective Date	(4) Policy Expiration Date	(5) % Written in 2022	(6) = (2)(5) 2022 Written Premiums
501	5,000	July 1, 2020	June 30, 2022	0%	0
502	3,600	April 1, 2021	March 31, 2024	33%	1,200
503	2,400	January 1, 2022	December 31, 2024	33%	800
504	4,800	September 1, 2022	August 31, 2024	50%	2,400
Total					4,400

- (b) Calculate the earned premiums for 2022.

(1) Policy Number	(7) % Earned in 2022	(8) = (2)(7) 2022 Earned Premiums
501	6/24 = 25.0%	1,250
502	12/36 = 33.3%	1,200
503	12/36 = 33.3%	800
504	4/24 = 16.7%	800
Total		4,050



- (c) Calculate the unearned premiums as of December 31, 2022.

(1)	(9)	(10)	(11)	(12) = (9)(11)/12
Policy Number	Equivalent Annual Premium	Written Date in 2022	# Months Unearned as of Dec 31, 2022	UEP as of Dec. 31, 2022
501	2,500	Jul. 1, 2022	0	0
502	1,200	Apr. 1, 2022	3	300
503	800	Jan. 1, 2022	0	0
504	2,400	Sep. 1, 2022	8	1,600
Total				1,900

- (d) Recalculate the 2022 earned premium for policy 504.

Annual premium written on Sep. 1, 2022:	2,400
Number of months of earned premium during 1st year: (i.e., Sep. 1, 2022 to Sep. 1, 2023)	6
Monthly earned premium:	400
# of months in 2022 vehicle was operated:	1
2022 earned premium:	400

- (e) Recalculate the unearned premium as of December 31, 2022 for policy 504.

# of months unearned as of Dec. 31, 2022 (excluding months in 2023 vehicle was not operated):	5
Unearned premium as of Dec. 31, 2022:	2,000

- (f) Describe why the parallelogram approximation would not be appropriate when adjusting historical premiums to current rate levels for policies such as policy 504.

It would not be appropriate because premiums are not earned evenly throughout the policy term.

## GIRR Spring 2024 Question 1 (LOs 2c, 2d)

### Learning Outcomes:

- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
- (2d) Adjust historical earned premiums to current rate levels.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 12 and 13.

### Commentary on Question:

*This question tests the candidate's understanding of certain details of individual insurance policies and ability to make correct calculations of earned premium, unearned premium and written premium for various policies. The candidate also needs to understand earned premiums adjusted to current rate level.*

### Solution:

- (a) Calculate the total earned premium for calendar year 2022.

Policy	Period in CY2022	Monthly Premium	# policies	# of months earned in 2022	CY2022 Earned Premium
Block	Jan-March 2022	175.00	1,000	3	525,000
Block	April-Dec 2022	183.75	800	9	1,323,000
100	Mar 1-Dec 31, 2022	250.00	1	10	2,500
200	May 1-Dec 31, 2022	175.00	1	8	1,400
300	July 1-Dec 31, 2022	116.67	1	6	700
400	n/a		1	0	0
Total					1,852,600

- (b) Calculate the total unearned premium as of December 31, 2023.

Policy	Monthly Premium	# policies	# of months outstanding on Dec. 31, 2023	Total Unearned Premium
Block	198.45	560	3	333,396
100	262.50	1	2	525
200	175.00	1	4	700
300	116.67	1	0	0
400	200.00	1	2	400

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Total	335,021
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- (c) Calculate the calendar year 2022 earned premium at current rate levels using the extension of exposures method.

Policy	Period	Monthly Premium from Part (a)	Rate change to Current Rate Level	Monthly Premium at Current Rates	# policies	# of months earned in 2022	CY2022 Earned Premium at Current Rate Levels
Block	Jan-March 2022	175.00	13.40%	198.45	1,000	3	595,350
Block	April-Dec 2022	183.75	8.00%	198.45	800	9	1,428,840
100	Mar 1-Dec 31, 2022	250.00	13.40%	283.50	1	10	2,835
200	May 1-Dec 31, 2022	175.00	8.00%	189.00	1	8	1,512
300	July 1-Dec 31, 2022	116.67	8.00%	126.00	1	6	756
400	n/a				1	0	0
Total							2,029,293

- (d) State why the parallelogram approach is not as accurate as the extension of exposures method used in part (c).

The exposures are not evenly distributed over time.

## GIRR Spring 2024 Question 5 (LOs 2d, 5b, 5e, 6f, 6g, 6h, 6j, 6k)

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6f) Explain the requirements for loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate loadings for catastrophes and large claims.
- (6h) Apply loadings for catastrophes and large claims in ratemaking.
- (6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6k) Demonstrate the use of credibility in ratemaking.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 26, 31, and 32.

### Commentary on Question:

*This question tests the candidate's ability to calculate the indicated average rate, while considering adjustments to earned premium and a loading for non-hurricane weather claims.*

### Solution:

- (a) Calculate the trended ultimate non-hurricane weather excluding hail pure premium per 100 EHY for all years.

Average accident date in future rating period: June 1, 2025 (9 months after start date).

# months from 2023 average accident date to June 1, 2025: 23

Accident Year	Ultimate		Trending Period (months)	Trend Factors	
	Frequency per 100 EHY	Severity		Frequency @ -1.0%	Severity @ 5.0%
2014	2.02	4,100	131	0.8961	1.7034
2015	0.39	3,500	119	0.9051	1.6223
2016	1.99	2,900	107	0.9143	1.5450
2017	0.1	4,400	95	0.9235	1.4715
2018	1.99	2,800	83	0.9328	1.4014
2019	0.8	4,200	71	0.9423	1.3347
2020	0.63	2,600	59	0.9518	1.2711
2021	2.73	3,600	47	0.9614	1.2106
2022	0.56	2,100	35	0.9711	1.1529

<u>Ultimate</u>			<u>Trend Factors</u>		
Accident Year	Frequency per 100 EHY	Severity	Trending Period (months)	Frequency @ -1.0%	Severity @5.0%
2023	1.69	3,100	23	0.9809	1.0980

<u>Trended Ultimate</u>			
Accident Year	Frequency per 100 EHY	Severity	Pure Premium per 100 EHY
2014	1.810	6,983.94	12,642
2015	0.353	5,678.00	2,004
2016	1.819	4,480.60	8,152
2017	0.092	6,474.43	598
2018	1.856	3,923.89	7,284
2019	0.754	5,605.56	4,226
2020	0.600	3,304.87	1,982
2021	2.625	4,358.07	11,438
2022	0.544	2,421.15	1,317
2023	1.658	3,403.88	5,643
Average:			
-all years	1.211	4,663.44	5,529

- (b) Recommend the trended ultimate non-hurricane weather excluding hail pure premium per 100 EHY to use in determining a weather loading. Justify your recommendation.

Recommend all years average: 5,529

Justification: should use more years to smooth out fluctuations; no significant trend.

- (c) Calculate the non-hurricane weather excluding hail loading percentage to use for ratemaking.

Selected state S PP per 100 EHY 5,529

Credibility-Weighted Pure Premium per 100 EHY 5,069.96

Expected Non-Hurricane Weather Claims 909,095.18

Weather loading as a claim ratio =  $909,095/13,089,711 = 6.95\%$

- (d) Identify two considerations when choosing the number of years and/or the weights to assign to each of the years.

Any 2 of the following are acceptable:

- professional judgment
- assessment of the relevance and reliability of the insurer's historical experience
- whether there are regulation requirements
- balance between stability and responsiveness
- management input
- credibility consideration - want enough years for full credibility, if possible
- also acceptable to note that give more weight to recent experience to account for recent changes

- (e) Recommend the number of years to include when estimating the weighted average trended claim ratio for the indicated rate change. Justify your recommendation.

AY	Ultimate Counts	Running Total
		Ultimate Counts
2019	1,070	5,447
2020	1,075	<b>4,377</b>
2021	1,074	3,302
2022	1,141	2,228
2023	1,087	1,087

Recommend 4 years.

Justification: Full credibility (3,654) is met by including at least the most recent 4 years.

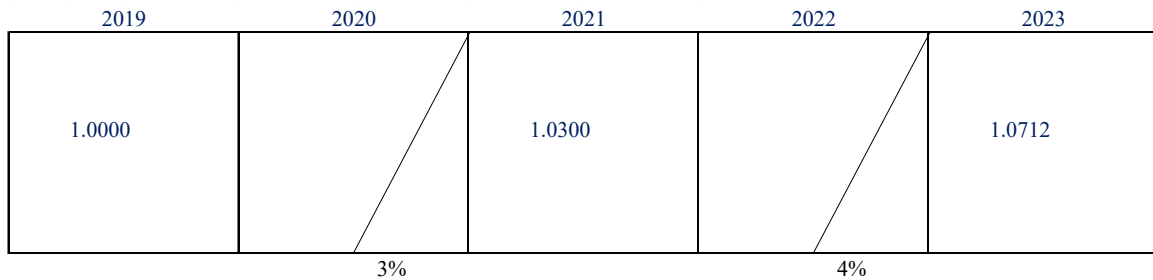
- (f) Recommend the weights to assign to each year when estimating the weighted average trended claim ratio for the indicated rate change. Justify your recommendation.

**Commentary on Question:**

*Candidates can also select weights judgmentally, as long as the number of years used matches the number of years recommended in part (e).*

AY	Earned Exposures	AY Weights		
		Initial	Limited	Balanced
2020	19,937	27.3%	23.4%	24.4%
2021	17,061	23.4%	23.4%	24.4%
2022	17,992	24.7%	24.6%	25.6%
2023	17,931	24.6%	24.6%	25.6%
Total	72,921		96.0%	

(g) Calculate the indicated rate change for this line of business.



Rate Index	Area in CY				
	2019	2020	2021	2022	2023
1.0000	100%	75%	0%	0%	0%
1.0300	0%	25%	100%	75%	0%
1.0712	0%	0%	0%	25%	100%
Average rate level:	1.0000	1.0075	1.0300	1.0403	1.0712
On-level factor:	1.0712	1.0632	1.0400	1.0297	1.0000

Claim Ratio Trend:  $(1 + -1.0\%)(1 + 5\%) - 1 = 3.95\%$

AY	Earned Premiums	On-Level Factor	On-Level Earned Premiums	Ultimate Claims
2019	13,510,549	1.07120	14,472,500	8,709,600
2020	13,268,660	1.06323	14,107,582	8,673,608
2021	11,739,370	1.04000	12,208,945	7,919,295
2022	12,638,750	1.02970	13,014,158	8,605,528
2023	13,089,711	1.00000	13,089,711	9,489,317

AY	Claim Trend Period (yrs)	Claim Trend Factor	Trended Ult. Claims	Claim Ratio	Weights
2019	5.9167	1.25761	10,953,253	75.68%	0.0%
2020	4.9167	1.20982	10,493,496	74.38%	24.4%
2021	3.9167	1.16385	9,216,849	75.49%	24.4%
2022	2.9167	1.11962	9,634,939	74.03%	25.6%
2023	1.9167	1.07708	10,220,730	78.08%	25.6%
Weighted:				75.51%	

Weighted Average Trended Claim Ratio (including non-hurricane weather loading): 82.46%

Ratio of ULAE to Claims 5.00%

Weighted Average Trended Claim Ratio including ULAE =  $0.8245 \times (1 + 6.7598) =$  86.58%

Fixed Expenses as Ratio to Premiums at Current Rate Level 3.00%

Variable Expenses - Ratio to Premiums	12.00%
Profit and Contingencies Ratio to Premiums	4.00%
Permissible Claim Ratio = $(1 - 0.12 - 0.04) / (1 + 0.03/0.8658) =$	81.19%
Indicated Rate Change = $0.8658 / 0.8119 - 1 =$	6.64%



### GIRR Spring 2024 Question 13 (LOs 2a)

#### Learning Outcomes:

(2a) Create development triangles of claims and counts from detailed claim transaction data.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 11.

#### Commentary on Question:

*This question tests the constructions of claims data triangles as well as the candidate's ability to recognize inconsistencies with claims data triangles.*

#### Solution:

(a) Verify that the change in case estimates during calendar year 2023 from the industry summary should be 223,240.

Accident Year	Case Estimates					
	12	24	36	48	60	72
2015	786,844	564,811	308,931	160,024	48,442	0
2016	795,613	613,589	329,380	140,620	45,963	0
2017	865,750	653,990	358,166	158,396	55,255	0
2018	971,601	688,324	387,347	163,712	48,728	0
2019	985,138	757,423	408,513	205,511	86,907	
2020	1,069,993	795,296	445,648	300,044		
2021	1,110,968	873,229	457,851			
2022	1,252,106	896,859				
2023	1,306,801					

Calendar Year (CY)	Case Estimates at End of Year	Case Change in CY
2022	2,825,222	
2023	3,048,462	223,240

(b) Identify the value that was reported in error to the industry bureau.

CY	Paid at End of Year	Paid in CY
2022	26,688,847	
2023	28,641,623	1,952,776

The claims paid in CY 2023 was incorrect, which likely caused the error in the change in case in CY 2023.

- (c) Construct a *reported* count triangle that reflects the development on these two claim files over time. Make sure to correctly label your triangles.

Claim #4400:

- AY 2021
- Reported in 2021 (12 months)
- Stays a reported count at 24 and 36 months

Accident Year	Reported Counts		
	12	24	36
2021	1	1	1

Claim #5500:

- AY 2021
- Reported in 2022 (24 months), so zero at 12 and 1 at 24 & 36 months

Accident Year	Reported Counts		
	12	24	36
2021	0	1	1

- (d) Construct a *closed* count triangle that reflects the development on these two claim files over time. Make sure to correctly label your triangles.

Claim #4400:

- closed in 2022, so closed counts should be a 1 at 24 months, reopened in 2023 so remove the 1 at 36 months

Accident Year	Closed Counts		
	12	24	36
2021	0	1	0

Claim #5500:

- deemed invalid claim in 2023, so closed count is 1 at 36 months

Accident Year	Closed Counts		
	12	24	36
2021	0	0	1

## GIRR Fall 2024 Question 4 (LOs 1d, 2a, 2c)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11, and 12.

### Commentary on Question:

*This question tests the candidate's understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies. The candidate also needs to calculate reported claim ratios and IBNR.*

### Solution:

- (a) Verify the earned premiums for calendar years 2021, 2022, and 2023.

Written Date	Written Premium	Months Earned in Calendar Year			Earned Premium in Calendar Year		
		2021	2022	2023	2021	2022	2023
1-Apr-21	120	9	12	3	45	60	15
1-May-21	120	8	12	4	40	60	20
1-Jun-21	120	7	12	5	35	60	25
1-Jul-21	120	6	12	6	30	60	30
1-Aug-21	120	5	12	7	25	60	35
1-Sep-21	120	4	12	8	20	60	40
1-Oct-21	120	3	12	9	15	60	45
1-Nov-21	120	2	12	10	10	60	50
1-Dec-21	120	1	12	11	5	60	55
1-Jan-22	120	0	12	12	0	60	60
1-Feb-22	120	0	11	12	0	55	60
1-Mar-22	120	0	10	12	0	50	60
1-Apr-22	120	0	9	12	0	45	60
1-May-22	120	0	8	12	0	40	60
1-Jun-22	120	0	7	12	0	35	60
1-Jul-22	120	0	6	12	0	30	60
1-Aug-22	120	0	5	12	0	25	60
1-Sep-22	120	0	4	12	0	20	60

Written Date	Written Premium	Months Earned in Calendar Year			Earned Premium in Calendar Year		
		2021	2022	2023	2021	2022	2023
1-Oct-22	120	0	3	12	0	15	60
1-Nov-22	120	0	2	12	0	10	60
1-Dec-22	120	0	1	12	0	5	60
1-Jan-23	120	0	0	12	0	0	60
1-Feb-23	120	0	0	11	0	0	55
1-Mar-23	120	0	0	10	0	0	50
1-Apr-23	120	0	0	9	0	0	45
1-May-23	120	0	0	8	0	0	40
1-Jun-23	120	0	0	7	0	0	35
1-Jul-23	120	0	0	6	0	0	30
1-Aug-23	120	0	0	5	0	0	25
1-Sep-23	120	0	0	4	0	0	20
1-Oct-23	120	0	0	3	0	0	15
1-Nov-23	120	0	0	2	0	0	10
1-Dec-23	120	0	0	1	0	0	5
1-Jan-24	120	0	0	0	0	0	0
1-Feb-24	120	0	0	0	0	0	0
1-Mar-24	120	0	0	0	0	0	0
Total					225	930	1,425

- (b) Calculate the unearned premiums as of each year-end for 2021, 2022, and 2023.

	31-Dec-2021	31-Dec-2022	31-Dec-2023
Earned Premiums:	225	930	1,425
Written premiums:	1,080	1,440	1,440
Unearned premiums	855	1,365	1,380

e.g.,  $1,365 = 1,440 - 930 + 855$

- (c) Calculate in-force premiums as of December 31, 2023.

There are 24 policies in-force as of December 31, 2023. (#10 through 33)

In-force premiums =  $24 \times 120 = 2,880$

- (d) Describe a scenario where the market analyst's conclusion would be incorrect.

Either of the following is acceptable:

- DEF writes the same volume of written premiums as ABC but annual term policies.

- DEF writes the same volume of written premiums as ABC with 2-year term policies but books annually.

(e) Calculate the reported claim ratios for each of calendar years 2022 and 2023.

Reported claims for CY2022: 319

Reported claims for CY2023: 622

Claim ratio for CY2022: 34.3%

Claim ratio for CY2023: 43.6%

(f) Calculate IBNR for accident years 2022 and 2023.

Ultimate claim ratio for AY2021 =  $135/225 = 60\%$

Accident Year	Earned Premiums	Ultimate Claims	IBNR
2022	930	558	112
2022	1,425	855	427

e.g.,  $558 = 0.6 \times 930$ ;  $112 = 558 - 446$

**GIRR Fall 2024 Question 10 (LOs 2a, 3e, 3g)****Learning Outcomes:**

- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 16 and 22.

**Commentary on Question:**

*This question tests the candidate's understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies. The candidate also needs to understand earned premiums adjusted to current rate level.*

**Solution:**

- (a) Provide an example of another line of business that often has a long lag between the occurrence date and the report date.

Any of the following are acceptable:

- Errors & Omissions
- Medical malpractice
- Any type of bodily injury liability only coverage

- (b) Provide an example of a line of business where claim files are commonly reopened.

Any of the following are acceptable:

- Workers compensation
- Any type of bodily injury liability only coverage

- (c) Construct a revised cumulative paid claims triangle adjusted for the legislative change.

**Commentary on Question:**

*It is necessary to start with incremental paid claims, as the reform affects claims paid after a certain date and not the cumulative of all claims paid to that date.*

Accident Year	Incremental Paid Claims					
	12	24	36	48	60	72
2018	1,518,006	1,766,528	1,553,804	1,308,213	798,483	204,638
2019	1,582,770	1,969,314	1,523,378	1,064,621	903,118	
2020	1,573,601	2,034,384	1,315,593	1,284,989		
2021	1,608,502	1,795,820	1,492,737			
2022	1,448,977	1,890,519				
2023	1,791,306					

Accident Year	Adjustment Factors for Tort Reform					
	12	24	36	48	60	72
2018	0.80	0.80	0.80	0.90	1.00	1.00
2019	0.80	0.80	0.90	1.00	1.00	
2020	0.80	0.90	1.00	1.00		
2021	0.90	1.00	1.00			
2022	1.00	1.00				
2023	1.00					

Accident Year	Adjusted Incremental Paid Claims = Incremental Paid Claims × Adjustment Factors					
	12	24	36	48	60	72
2018	1,214,405	1,413,222	1,243,043	1,177,392	798,483	204,638
2019	1,266,216	1,575,451	1,371,040	1,064,621	903,118	
2020	1,258,881	1,830,946	1,315,593	1,284,989		
2021	1,447,652	1,795,820	1,492,737			
2022	1,448,977	1,890,519				
2023	1,791,306					

Accident Year	Adjusted Cumulative Paid Claims					
	12	24	36	48	60	72
2018	1,214,405	2,627,627	3,870,670	5,048,062	5,846,545	6,051,183
2019	1,266,216	2,841,667	4,212,707	5,277,328	6,180,446	
2020	1,258,881	3,089,826	4,405,419	5,690,408		
2021	1,447,652	3,243,472	4,736,209			
2022	1,448,977	3,339,496				
2023	1,791,306					

- (d) Verify the projected ultimate claims for accident years 2024 and 2025.

Accident Year	Projected Frequency	Projected Counts	Projected Severity	Projected Ultimate Claims
2024	10.57%	1,120.23	6,342.50	7,105,054
2025	10.54%	1,128.46	6,818.19	7,694,043

e.g.,  $10.57\% = 10.6\% \times (1 - 0.3\%)$   
 $1,120.23 = 10.57\% \times 10,600$   
 $6,342.50 = 5,900 \times (1 + 7.5\%)$   
 $7,105,054 = 1,120.23 \times 6,342.50$

- (e) Calculate the claims expected to be paid in calendar years 2024 and 2025, using the results from part (c).

**Commentary on Question:**

*Age-to-ultimate factors are calculated by dividing the given ultimate claims by cumulative paid claims to date (i.e., the latest diagonal).*

Accident Year	Cumulative Paid Claims						Projected Ultimate Claims
	12	24	36	48	60	72	
2018	1,518,006	3,284,534	4,838,338	6,146,551	6,945,034	7,149,672	7,149,672
2019	1,582,770	3,552,084	5,075,462	6,140,083	7,043,201	7,289,724	7,289,724
2020	1,573,601	3,607,985	4,923,578	6,208,567	7,231,724	7,484,846	7,484,846
2021	1,608,502	3,404,322	4,897,059	6,280,054	7,314,992	7,571,028	7,571,028
2022	1,448,977	3,339,496	4,873,746	6,250,157	7,280,168	7,534,985	7,534,985
2023	1,791,306	4,087,339	5,965,167	7,649,810	8,910,480	9,222,361	9,222,361
2024	1,380,051	3,148,951	4,595,660	5,893,535	6,864,776	7,105,054	7,105,054
2025	1,494,453	3,409,990	4,976,627	6,382,093	7,433,847	7,694,043	7,694,043

	12-24	24-36	36-48	48-60	60-72	72-84
Age-to-age:	2.282	1.459	1.282	1.165	1.035	1.000
Age-to-ult:	5.148	2.256	1.546	1.206	1.035	1.000

e.g.,  $5.148 = 9,222,361 / 1,791,306$   
 $2.282 = 5.148 / 2.256$



Accident Year	Incremental Paid Claims					
	12	24	36	48	60	72
2018	1,518,006	1,766,528	1,553,804	1,308,213	798,483	204,638
2019	1,582,770	1,969,314	1,523,378	1,064,621	903,118	246,523
2020	1,573,601	2,034,384	1,315,593	1,284,989	1,023,157	253,122
2021	1,608,502	1,795,820	1,492,737	1,382,995	1,034,938	256,036
2022	1,448,977	1,890,519	1,534,250	1,376,411	1,030,011	254,817
2023	1,791,306	2,296,033	1,877,828	1,684,643	1,260,671	311,881
2024	1,380,051	1,768,900	1,446,709	1,297,876	971,241	240,278
2025	1,494,453	1,915,537	1,566,637	1,405,466	1,051,754	260,196

CY2024 paid claims: 7,863,009

CY2025 paid claims: 7,805,652

## GIRR Fall 2024 Question 11 (LOs 2d, 5b, 5e)

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13 and 27.

### Commentary on Question:

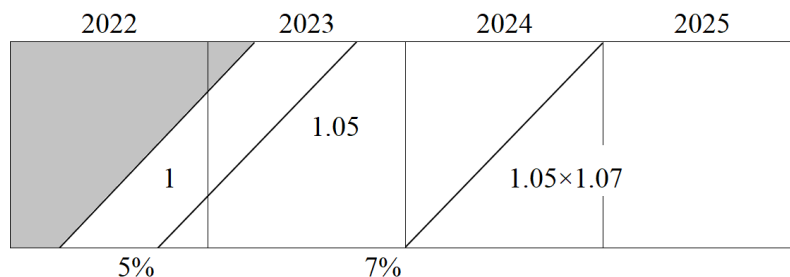
*This question tests the candidate's ability to adjust premium to current rate levels and adjust premiums for trend for ratemaking purposes.*

### Solution:

- (a) Calculate the on-level premium factors for calendar year 2022 and 2023.

### Commentary on Question:

*Since the company started writing a new line of business on March 1, 2022, the shaded area in the diagram below has no earned premiums and should not be included in estimating the percent of premiums earned in each calendar year.*

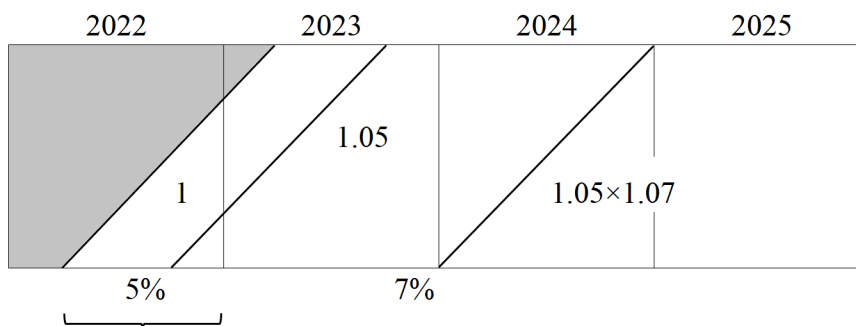


Rate Change History			Percent Premium Earned in Each Calendar Year (CY) at Rate Level	
Effective Date of Rate Change	Rate Change %	Rate Level Index	2022	2023
Prior to Mar 1/22			65.28%	1.39%
		1.00000	29.17%	20.83%
1-Sep-22	5%	1.05000	5.56%	77.78%
1-Jan-24	7%	1.12350	-	-
Total			34.72%	98.61%
Average Rate Level in each CY:			1.00800	1.03944
On-Level Factors:			1.1146	1.0809

Notes:

- $5.56\% = 0.5(4/12)^2$
- $29.17\% = 0.5(10/12)^2 - 5.56\%$
- Avg rate level in CY 2022 =  $(1.0000 \times 29.17\% + 1.0500 \times 5.56\%) / 34.72\% = 1.0080$
- $20.83\% = 0.5(8/12)^2 - 0.5(2/12)^2$
- $77.78\% = 1 - 0.5(8/12)^2$
- Avg rate level in CY 2023 =  $(1.0000 \times 20.83\% + 1.0500 \times 77.78\%) / 98.61\% = 1.0394$

(b) Calculate premium trend factors for calendar year 2022 and 2023.



for 2022: all policies written between Mar 1, 2022 & Dec 31, 2022 contribute toward 2022 EP

for 2023: all policies written between Mar 1, 2022 & Dec 31, 2023 contribute toward 2023 EP

Trend from average written date in experience period to average written date in future rating period.

Calendar Year	Average Written Date in Experience Period	Average Written Date Rating Period	Trending Period in Months	Trending Period in Years	Trend Factor
2022	1-Aug-22	1-Oct-25	38	3.167	0.98425
2023	1-Feb-23	1-Oct-25	32	2.667	0.98672

e.g.,  $0.98425 = (1 - 0.005)^{3.167}$

## GI 101 – LEARNING OBJECTIVE 3

### 3. Topic: Projecting Ultimate Claims

The candidate will know how to calculate and evaluate projected ultimate values.

## GIRR Fall 2020 Question 2 (LOs 3e, 3f, 3g)

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 18.

### Commentary on Question:

*This question tests the candidate's understanding of the Bornhuetter Ferguson and Benktander methods for estimating ultimate claims.*

### Solution:

- (a) Calculate the total ultimate claims using the Bornhuetter Ferguson method applied to the following two claim amounts:
  - (i) Paid claims
  - (ii) Reported claims

Accident Year	(1) Actual Claims as of Dec. 31, 2019	(2) Reported	(3) Ultimate Claims from Development Method on		(4) Expected Claims
	Paid	Reported	Paid	Reported	
2016	889,190	898,170	916,755	916,133	889,488
2017	916,340	964,570	1,014,895	1,003,537	998,479
2018	824,940	959,230	1,065,872	1,077,820	1,113,814
2019	586,850	838,362	1,140,237	1,139,829	1,142,919

	(6) = (3)/(1)    (7) = (4)/(2)		(8)	(9)
	Age-to-Ultimate Dev.		BF Estimate	
Accident	<u>Factors Based on</u>		<u>Ultimate Claims</u>	
Year	Paid	Reported	Paid	Reported
2016	1.0310	1.0200	915,935	915,611
2017	1.1076	1.0404	1,013,301	1,003,341
2018	1.2921	1.1236	1,076,709	1,081,780
2019	1.9430	1.3596	1,141,539	1,140,646
Total			4,147,484	4,141,378

Notes:      (8) = (1) + (5)[1 – 1/(6)]  
                  (9) = (2) + (5)[1 – 1/(7)]

- (b) Evaluate the reasonableness of the inputs for the Bornhuetter Ferguson method in part (a) by comparing the following two amounts:
- (i) Actual paid claims to expected paid claims
- (ii) Actual reported claims to expected reported claims

	(10) = 1/(6)    (11) = 1/(7)		(12) =	(13) =	(14) = (1) –	(15) =
	Expected % Dev. At		(5)(10)	(5)(11)	(12)	(2) –
Accident	<u>Dec. 31, 2019</u>		<u>Expected Claims</u>		<u>Difference</u>	
Year	Paid	Reported	Paid	Reported	Paid	Reported
2016	0.9699	0.9804	862,743	872,047	26,447	26,123
2017	0.9029	0.9612	901,518	959,708	14,822	4,862
2018	0.7740	0.8900	862,045	991,264	–37,105	–32,034
2019	0.5147	0.7355	588,230	840,634	–1,380	–2,273
Total			3,214,536	3,663,654	2,784	–3,322

Overall, it appears reasonable, but there are some AY's that are not reasonable (e.g., 2016 & 2018).

- (c) Identify two reasons that might cause the differences shown in part (b).

Any two of the following are acceptable:

- development pattern
- trend rate
- selected values for expected claims
- existence of unusually large claims

- (d) Describe a reason why the Benktander method might be preferred to estimate ultimate claims.

A situation where you would want to put more weight (confidence) on the development method but still give consideration to the Bornhuetter Ferguson method.

- (e) Calculate the total ultimate claims applied to paid claims using one iteration of the Benktander method.

$$(16) = (1) + (8)[1 - 1/(6)]$$

Accident Year	Benktander Estimate
2016	916,730
2017	1,014,740
2018	1,068,322
2019	1,140,869
Total	4,140,661

## GIRR Fall 2020 Question 7 (LOs 3j)

### Learning Outcomes:

(3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 22.

### Commentary on Question:

*This question tests the candidate's understanding of the evaluation and selection of estimated ultimate claims under various circumstances.*

### Solution:

Recommend two methods for projecting ultimate claims that are appropriate for each line of business without repeating any methods. Justify your recommendations for all four methods

#### Line of Business A:

- Frequency-Severity method because it allows separate analysis of claim frequency from claim severity. This method should show increasing claim frequency trend and potential increases in severity from policy limits.
- Cape Cod method because it uses actual claim experience to determine expected claims. The use of expected claims will also moderate some of the volatility and claim trends can be explicitly reflected in expected claims.

#### Line of Business B:

- Development method applied to reported claims because experience is relatively stable and mature enough for development patterns.
- Bornhuetter Ferguson method applied to reported claims because this is a new line of business which means using the a priori expected claim ratio is appropriate. The Bornhuetter Ferguson method will not over-project large losses to date. A priori expectations can reflect industry data if available.



## GIRR Fall 2020 Question 9 (LOs 2d, 3e, 3f, 3g)

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 15 and 19.

### Commentary on Question:

*This question tests the estimating of ultimate claims using the Cape Cod method.*

### Solution:

- (a) Describe why premium on-level factors are typically used in the Cape Cod method but not in the Bornhuetter Ferguson method.

The Bornhuetter Ferguson method uses an external a priori estimate as an expected claim ratio. (This should implicitly be at appropriate rate level).

The Cape Cod method derives one adjusted expected claim ratio from all historical data, so all years need to reflect the same rate level.

- (b) Describe a situation in which an actuary may choose to derive an adjusted expected pure premium instead of an adjusted expected claim ratio when using the Cape Cod method.

Any one of the following is acceptable:

- If rate change history is not available/reliable
- If on-level premium adjustment factors are not available/reliable
- If exposure base is not inflation-sensitive, then using exposures simplifies the calculation

- (c) Explain why confidence in the development method is a consideration in selecting the decay factor.

As the decay factor approaches 0, projected ultimate claims in the Generalized Cape Cod method approach results from the development method. So, an actuary who has significant confidence in the development method can choose a smaller decay factor.

- (d) Calculate premium on-level factors for each accident year, to use in the Cape Cod method as of December 31, 2019.

**Commentary on Question:**

*The diagram is helpful to solve the question but not required for credit.*

2015	2016	2017	2018	2019
A		B		C

Level	Rate Level	<u>Percent Premium Earned in Each CY at Rate Level</u>				
	Index	2015	2016	2017	2018	2019
A	1.00000	100.0%	50.0%			
B	1.06000		50.0%	100.0%	87.5%	12.5%
C	1.11300				12.5%	87.5%
Total		100.0%	100.0%	100.0%	100.0%	100.0%

Average rate level in each CY:      1.00000   1.03000   1.06000   1.06663   1.10638

On-level factors for reserving:      1.1064   1.0742   1.0438   1.0373   1.0000

e.g.,  $1.0373 = 1.10638 / 1.06663$

- (e) Calculate the projected ultimate claims for each accident year using the Cape Cod method.

Accident Year	(1) Earned Premiums (EP)	(2) Premium On-Level Factors from part (d)	(3) = (1)(2) On-Level Earned Premiums	(4) Reported CDFs	(5) = 1/(4) Expected % Reported	(6) = (3)(5) Used-Up On-Level EP
2015	16,100	1.1064	17,813	1.030	97.1%	17,294
2016	17,600	1.0742	18,905	1.055	94.8%	17,919
2017	18,300	1.0438	19,101	1.100	90.9%	17,364
2018	19,800	1.0373	20,538	1.300	76.9%	15,798
2019	21,600	1.0000	21,600	1.700	58.8%	12,706
	93,400		97,956			81,082

	(7)	(8)	(9)	(10) = (7)(8)(9)	(11)
Accident Year	Actual Reported Claims excluding Large Claim	<u>Claim Adjustment Factors</u>		Adjusted Claims	Expected Claims
		Trend at 5%	Tort Reform		
2015	11,150	1.2155	0.90	12,198	11,548
2016	11,380	1.1576	0.95	12,515	12,191
2017	11,190	1.1025	1.00	12,337	12,287
2018	10,870	1.0500	1.00	11,414	13,872
2019	9,040	1.0000	1.00	9,040	15,319
	53,630			57,503	65,217

Adjusted Expected Claim Ratio: 70.92% = 57,503 / 81,082

Notes: (7) for AY2018: 10,870 = 11,470 – 800

(11) = [(3) × 0.7092] / [(8)(9)]

	(12)	(13) = 1 – (5)	(14) = (11)(13)	(15) = (12) + (14)
Accident Year	Actual Reported Claims	Expected % Unreported	Expected Unreported	Projected Ultimate
2015	11,150	2.9%	336	11,486
2016	11,380	5.2%	636	12,016
2017	11,190	9.1%	1,117	12,307
2018	11,470	23.1%	3,201	14,671
2019	9,040	41.2%	6,308	15,348
	54,230		11,598	65,828

**GIRR Fall 2020 Question 17 (LOs 3h, 3i)****Learning Outcomes:**

- (3h) Explain the effect of changing conditions on the projection methods cited in (3e).  
(3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 21.

**Commentary on Question:**

*This question tests the candidate's understanding of how claims data is affected by various changing conditions and the appropriateness of various methods of estimating ultimate claims under changing conditions.*

**Solution:**

- (a) Provide two different examples of changing conditions that are likely to decrease the latest diagonal of a reported claim triangle.

Any two of the following are acceptable:

- Decrease in case reserve adequacy
- Slowdown in settlement pattern
- Tort reform reducing open claims

- (b) Describe how an increase in attachment point for an excess of loss reinsurer could affect a reported claim triangle.

Any two of the following are acceptable:

- Increase in severity
- Increase in severity trend
- Longer pattern/tail
- Could affect row (or multiple rows) depending on implementation period

- (c) Explain what affect the claim ratio deterioration is likely to have on reported claim development factors.

If all other assumptions are steady-state, then deterioration in the claim ratio will not impact the patterns in the claim development triangle.

- (d) Explain which of the following two methods is likely to produce a more accurate estimate of ultimate claims in recent accident years in this scenario:

- (i) the development method applied to reported claims, or

- (ii) the Bornhuetter Ferguson method applied to reported claims.

Since the development factors are not impacted by the claim deterioration, the development method will produce a better estimate than the Bornhuetter Ferguson method which will be understated because the expected claim ratio is not picking up the unexpected deterioration.

## GIRR Fall 2020 Question 19 (LOs 3e, 3g)

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 15 and 20.

### Commentary on Question:

*This question tests the calculation of ultimate claims using the development method applied to claims the Berquist-Sherman adjustment for change in claims settlement.*

### Solution:

- (a) Calculate the triangle of adjusted closed counts.

Accident Year	Adjusted Closed Counts Excluding Large Claim Counts					Ultimate Counts
	12	24	36	48	60	
2015	564	864	1,060	1,187	1,256	1,256
2016	678	1,038	1,274	1,426		1,509
2017	576	882	1,082			1,282
2018	606	929				1,350
2019	699					1,557

e.g., AY2018 @ 12 months:  $606 = 1,350 \times 0.449$

- (b) Calculate total unpaid claims using the development method applied to paid claims, adjusted for changes in settlement rates.

Accident Year	Adjusted Paid Claims Excluding Large Claims				
	12	24	36	48	60
2015	600,585	2,136,841	4,787,346	6,664,813	7,213,000
2016	525,026	2,264,528	5,219,251	8,044,000	
2017	564,056	1,969,044	4,601,000		
2018	698,435	2,145,000			
2019	832,000				

e.g., AY2018 @ 12 months:  $698,435 = 85,287e^{(0.00347 \times 606)}$

Development factors (3-year volume weighted average):

	12-24	24-36	36-48	48-60	60-
Age-to-age	3.568	2.293	1.470	1.082	1.000
Age-to-ult	13.017	3.648	1.591	1.082	1.000

Accident Year	Paid Claims	Age-to- ultimate Dev. Factors	Large Claims Reported	Ult. Claims Incl. Large	Large Claims Paid	Unpaid Claims
2015	7,213,000	1.000		7,213,000		0
2016	8,044,000	1.082	801,000	9,506,627	615,000	847,627
2017	4,601,000	1.591		7,319,331		2,718,331
2018	2,145,000	3.648	923,000	8,747,519	297,000	6,305,519
2019	832,000	13.017		10,829,959		9,997,959
	22,835,000			43,616,436		19,869,436

e.g., AY 2018:

$$8,747,519 = 2,145,000 \times 3.648 + 923,000$$

$$6,305,519 = 8,747,519 - (2,145,000 + 297,000)$$

- (c) Assess the appropriateness of relying on the accident year 2019 ultimate claims from part (b) when selecting ultimate claims.

The AY2019 cumulative paid development factor is highly leveraged (13.017). Therefore, we should likely seek other methods for selecting ultimate claims.

## **GIRR Spring 2021 Question 2 (LOs 3c, 3d)**

### **Learning Outcomes:**

- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.

### **Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 14.

### **Commentary on Question:**

*This question tests investigative analysis of various development triangles.*

### **Solution:**

- (a) Provide one possible interpretation of this anomaly. Justify your interpretation.

A large claim may have been reported between 24 and 36 months (which remains unpaid).  
Justification:

- A large reported claim would explain the increase in average reported claims for accident year 2014 only, with no change in average paid claims.
- A large reported but unpaid claim would explain the decrease in the ratios for paid to reported claims for accident year 2014 only beginning at 36 months.
- A single large claim would not have a material effect on counts.

- (b) Identify another anomaly from the diagnostics.

- Latest 2 diagonals (i.e., calendar years 2019-2020) for ratios of paid to reported claims is low

OR

- Latest 2 diagonals (i.e., calendar years 2019-2020) for ratios of closed to reported counts is low

- (c) Provide one possible interpretation of the anomaly you identified in part (b). Justify your interpretation.

This appears to be a slow-down in settlement patterns. Justification:

- Changes on the diagonal often relate to settlement changes or case adequacy changes.
- Either paid claims have decreased or reported claims have increased.
- Closed counts and paid claims have both decreased.
- Since average reported claims didn't change, this does not appear to be a change in case adequacy.



**GIRR Spring 2021 Question 3 (LOs 3g, 4a, 4b, 4c, 5b, 5c, 5d, 5e)****Learning Outcomes:**

- (3g) Estimate ultimate values using the methods cited in (3e).
- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.
- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16, 23, and 26.

**Commentary on Question:**

*This question tests the candidate's understanding of claims trend analysis and selection as well as estimating ultimate claims using the development-based frequency-severity method. This question also tests the candidate's understanding of estimating unpaid ULAE using the classical paid-to-paid method with the Mango-Allen smoothing adjustment.*

**Solution:**

- (a) Explain why this may happen when using the development-based frequency-severity method.  
  
For the development-based frequency-severity method, the severity would be developed to an ultimate value separately, which might not equal the developed ultimate claims divided by the developed ultimate counts.
- (b) Recommend a claim frequency at the accident year 2020 cost level. Justify your recommendation.

Accident Year	(1) Earned Exposures	(2) Projected Ultimate Counts	(3) Based on Development Method Claims	(4) Severity
2015	25,200	2,088	9,028,629	4,324
2016	26,700	2,194	9,779,132	4,458
2017	25,300	2,063	9,477,056	4,594
2018	24,500	1,983	9,375,491	4,733
2019	23,900	1,933	8,987,726	4,724
2020	24,200	1,709	7,810,473	4,749
Total	149,800	11,970	54,458,507	

Accident Year	(5) = (2)/(1) Indicated Frequency	(6) <sub>t</sub> = (5) <sub>t</sub> / (5) <sub>t-1</sub> Year-to-year Change	(7) Frequency Trend @ -0.78%	(8) = (5)(7) Trended Frequency
2015	0.082857		0.961604	0.079676
2016	0.082172	-0.008266	0.969163	0.079638
2017	0.081542	-0.007676	0.976782	0.079648
2018	0.080939	-0.007392	0.984461	0.079681
2019	0.080879	-0.000743	0.992200	0.080248
2020	0.070620	-0.126842	1.000000	0.070620

Frequency trend selection: (column 6): Average of 2016-2018 = -0.78%  
(2019 & 2020 are outliers)

Recommended 2020 cost level frequency (column 8): average excluding 2020  
= 0.0798  
(all other years are stable and 2020 is an outlier)

- (c) Calculate ultimate claims using the development-based frequency-severity method and the recommended claim frequency from part (b).

Accident Year	(4) Indicated Severity	(9) <sub>t</sub> = (4) <sub>t</sub> / (4) <sub>t-1</sub> Year-to-year Change	(10) Severity Trend @ 3.06%	(11) = (4)(10) Trended Severity
2015	4,324		1.162655	5,027.32
2016	4,458	0.030990	1.128134	5,029.22
2017	4,594	0.030507	1.094638	5,028.77
2018	4,733	0.030257	1.062136	5,027.09
2019	4,724	-0.001902	1.030600	4,868.55
2020	4,749	0.005292	1.000000	4,749.00

Severity trend selection: (column 9): Average of 2016-2018 = 3.06%  
(2019 & 2020 are outliers)

Recommended 2020 cost level frequency (column 11): Average of 2016-2018  
= 5,028.10  
(2019 & 2020 are outliers)

	(12) = (1)×0.0798/(7)	(13) = 5.028.10/(10)	(14) = (12)(13)
Accident Year	Ultimate Counts	Ultimate Severity	Ultimate Claims
2015	2,090.83	4,324.67	9,042,137
2016	2,198.00	4,457.01	9,796,505
2017	2,066.50	4,593.39	9,492,263
2018	1,985.55	4,733.95	9,399,499
2019	1,921.82	4,878.81	9,376,179
2020	1,930.76	5,028.10	9,708,066
Total			56,814,649

- (d) Calculate the expected claims paid for calendar years 2017 through 2020.

	12	24	36	48	60	72
Cumulative paid claims development factors by maturity age (CDF)	11.245	2.017	1.228	1.063	1.010	1.000
% Cumulative Paid (1/CDF)	8.9%	49.6%	81.4%	94.1%	99.0%	100.0%
% Incremental Paid	8.9%	40.7%	31.9%	12.6%	4.9%	1.0%

e.g., % incremental paid at 24 months = 40.7% = 49.6% - 8.9%

Accident Year	Ultimate Claims from Part (c)	Projected in Calendar Year			
		2017	2018	2019	2020
2015	9,042,137	2,880,340	1,142,940	446,367	89,526
2016	9,796,505	3,985,781	3,120,642	1,238,293	483,607
2017	9,492,263	844,132	<b>3,861,998</b>	3,023,727	1,199,837
2018	9,399,499		835,883	3,824,256	2,994,177
2019	9,376,179			833,809	3,814,768
2020	9,708,066				863,323
Total		7,710,253	8,961,462	9,366,451	9,445,237

e.g., Accident year 2017 expected paid claims in calendar year 2018

$$= 0.407 \times 9,492,263 = \mathbf{3,861,998}$$

- (e) Recommend a ULAE ratio using the classical paid-to-paid method with the Mango-Allen smoothing adjustment. Justify your recommendation.

Calendar Year	Paid ULAE	Expected Claims from Part (d)	Ratio ULAE to Claims
2017	738,905	7,710,253	9.58%
2018	851,350	8,961,462	9.50%
2019	883,245	9,366,451	9.43%
2020	879,224	9,445,237	9.31%
Total	3,352,724	35,483,403	9.45%

Recommended ULAE ratio = total of all years = 9.45%, as there are no significant outliers.

**Commentary on Question:**

*Candidates could also recommend a ULAE ratio that considered the downward trend.*

- (f) Calculate the unpaid ULAE.

$$\begin{aligned} \text{Calculated unpaid ULAE} &= 9.45\% \times 4,351,459 \times (1 - 0.25) + 9.45\% \times 11,117,813 \\ &= 1,358,858. \end{aligned}$$

### GIRR Spring 2021 Question 4 (LOs 3i, 4a)

#### Learning Outcomes:

- (3i) Assess the appropriateness of the projection methods cited in (e) in varying circumstances.
- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 22 and 23.

#### Commentary on Question:

*This question tests the candidate's ability to evaluate and justify selections of ultimate values based on various methods. In addition, this question tests the candidate's understanding of estimating unpaid unallocated loss adjustment expenses.*

#### Solution:

- (a) Explain why the development method may not be appropriate for estimating unpaid claims for this coverage.

#### Commentary on Question:

*Any two of the following are acceptable.*

- The development method is not appropriate for immature experience periods (i.e., the data is less than five years).
  - The development method is not appropriate when limited or no historical experience is available.
  - The development method is not appropriate when conditions are changing (i.e., tort reform will distort development).
- (b) Recommend an appropriate method for estimating unpaid claims for this coverage. Justify your recommendation.

#### Commentary on Question:

*Although the Cape Cod method is the most appropriate recommendation, other methods are acceptable if the justification is appropriate for the circumstances. Justification should include at least three explanations.*

The Cape Cod method is recommended. Justification:

- Good for immature experience periods
- Good when limited or no historical experience is available
- Good for long-tailed coverages
- Allows for explicit trend adjustment

- Allows for explicit tort reform adjustment
  - Industry development (experience) can be used to supplement company development (which is limited to five years)
  - Cape Cod method uses actual experience
  - Cape Cod method adds stability
  - Can be applied to paid and/or reported data
- (c) Explain why the classical paid-to-paid method may not be appropriate for estimating unpaid ULAE for this coverage.

**Commentary on Question:**

*Any two of the following are acceptable.*

- Tort reform may change the relationship between payments for ULAE and payments for claims.
  - Experience period has not reached a steady-state (only five years but coverage is long-tailed).
  - Classical paid-to-paid method is not appropriate if significant changes in exposure are occurring (growth in this case).
- (d) Recommend an appropriate method for estimating unpaid ULAE for this coverage. Justify your recommendation.

**Commentary on Question:**

*Although the Mango & Allen smoothing adjustment is the most appropriate recommendation, other methods are acceptable if the justification is appropriate for the circumstances. Justification should include at least two explanations.*

The Mango & Allen smoothing adjustment is recommended. Justification:

- Appropriate for long-tail coverages
- Appropriate for changing exposure volume
- Appropriate for relatively new insurer/coverage

### GIRR Spring 2021 Question 9 (LOs 3d, 3f, 3g)

#### Learning Outcomes:

- (3d) Analyze development triangles for investigative testing.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 14 and 20.

#### Commentary on Question:

*This question tests the candidate's understanding of Berquist-Sherman adjustments when there has been a change in case estimate adequacy.*

#### Solution:

- (a) Calculate the average case estimate triangle.

Accident Year	Case Estimates = Reported Claims – Paid Claims				
	12	24	36	48	60
2016	7,600	11,200	3,800	5,240	3,600
2017	8,862	12,699	4,047	4,815	
2018	7,923	12,072	6,036		
2019	8,996	16,680			
2020	13,301				

Accident Year	Open Counts = Reported Counts – Closed Counts				
	12	24	36	48	60
2016	248	228	196	148	60
2017	253	232	200	151	
2018	265	244	210		
2019	260	239			
2020	271				

Accident Year	Average Case = Case Estimates / Open Counts				
	12	24	36	48	60
2016	30.65	49.12	19.39	35.41	60.00
2017	35.03	54.74	20.24	31.89	
2018	29.90	49.48	28.74		
2019	34.60	69.79			
2020	49.08				

- (b) Evaluate whether the average case estimate triangle indicates either decreasing, increasing or stable case reserve adequacy.

Changes in Average Case Estimates	12	24	36	48
2016-2017	14.3%	11.4%	4.4%	-9.9%
2017-2018	-14.6%	-9.6%	42.0%	
2018-2019	15.7%	41.1%		
2019-2020	41.9%			

There is some instability down each column. The last diagonal shows significant increases, suggesting a significant increase in case reserve adequacy.

- (c) Calculate IBNR by accident year using the reported development method, with a Berquist-Sherman adjustment.

Adjusted Average Case = Last Diagonal from part (a), trended to each AY at 5%:

AY	12	24	36	48	60
2016	40.38	60.29	26.07	30.37	60.00
2017	42.40	63.30	27.37	31.89	
2018	44.52	66.47	28.74		
2019	46.74	69.79			
2020	49.08				

e.g.,  $46.74 = 49.08 / 1.05$

Adjusted Case Estimates = Adjusted Average Case Estimate  $\times$  Open Counts:

AY	12	24	36	48	60
2016	10,014	13,746	5,110	4,495	3,600
2017	10,727	14,686	5,475	4,815	
2018	11,797	16,218	6,036		
2019	12,153	16,680			
2020	13,301				

e.g.,  $12,153 = 46.74 \times 260$

Adjusted Reported Claims = Paid Claims + Adjusted Case Estimates

AY	12	24	36	48	60
2016	34,414	56,546	62,710	69,495	76,000
2017	36,692	60,257	66,816	74,040	
2018	39,872	65,494	72,363		
2019	40,977	67,306			
2020	44,192				

Development Factors:



AY	12 to 24	24 to 36	36 to 48	48 to 60	60 to Ult
2016	1.643	1.109	1.108	1.094	
2017	1.642	1.109	1.108		
2018	1.643	1.105			
2019	1.643				
2020					
Average	1.643	1.108	1.108	1.094	1.000
Age-to-Ultimate	2.205	1.342	1.212	1.094	1.000

	(1)	(2)	(3) = (1)(2)	(4) = (3) – (1)
		Age-to-Ultimate Development		
AY	Reported Claims	Factor	Ultimate Claims	IBNR
2016	76,000	1.000	76,000	0
2017	74,040	1.094	80,971	6,931
2018	72,363	1.212	87,696	15,333
2019	67,306	1.342	90,343	23,037
2020	44,192	2.205	97,436	53,244
Total			432,445	98,544

- (d) Explain why the reported development method without a Berquist-Sherman adjustment would have overstated the IBNR.

Case estimates without the adjustment are lower, which would yield higher development factors.

## GIRR Spring 2021 Question 14 (LOs 2d, 3g)

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.  
 (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 17, and 19.

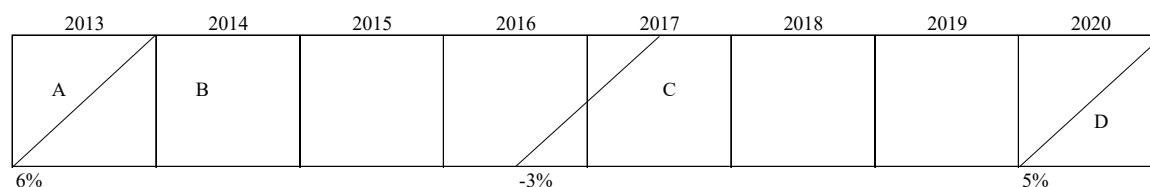
### Question:

### Commentary on Question:

*This question tests the candidate's understanding of adjusting earned premiums to current rate levels as well as estimating ultimate claims using the expected method and the Cape Cod method.*

### Solution:

- (a) Calculate premium on-level factors for all accident years for projecting claim ratios as of December 31, 2020.



Rate Change History							
Effective Date of Rate Change	Rate Change %	Rate Level Index	Percent Premium Earned in Each CY at Rate Level				
			2011	2012	2013	2014	2015
Initial		1.00000	100.00%	100.00%	50.00%	-	-
Jan. 1, 2013	6.0%	1.06000	-	-	50.00%	100.00%	100.00%
Jul. 1, 2016	-3.0%	1.02820	-	-	-	-	-
Jan. 1, 2020	5.0%	1.07961	-	-	-	-	-
Average Rate Level in each CY:			1.00000	1.00000	1.03000	1.06000	1.06000
On-Level Factors for reserving:			1.05391	1.05391	1.02321	0.99425	0.99425

Rate Change History			Percent Premium Earned in Each CY at Rate Level				
Effective Date of Rate Change	Rate Change %	Rate Level Index	2016	2017	2018	2019	2020
Initial		1.00000	-	-	-	-	-
Jan. 1, 2013	6.0%	1.06000	87.50%	12.50%	-	-	-
Jul. 1, 2016	-3.0%	1.02820	12.50%	87.50%	100.00%	100.00%	50.00%
Jan. 1, 2020	5.0%	1.07961	-	-	-	-	50.00%

Average Rate Level in each CY:	1.05603	1.03218	1.02820	1.02820	1.05391
On-Level Factors for reserving:	0.99799	1.02105	1.02500	1.02500	1.00000

e.g., 2016       $1.05603 = (1.06 \times 0.875) + (1.0282 \times 0.125)$   
 $0.99799 = 1.05391 / 1.05603$

(b) Calculate projected ultimate claims for all accident years using the expected method.

	(1)	(2)	(3)	(4)	(5)
Accident Year (AY)	On-Level Factors	Tort Reform	Trended On- Level Claim Ratio	Claim Ratio at Cost Level of Each AY	Projected Ultimate Claims
2011	1.05391	0.80	67.0%	84.5%	4,889,698
2012	1.05391	0.80	66.5%	84.5%	4,456,640
2013	1.02321	0.80	53.7%	82.0%	3,999,255
2014	0.99425	0.90	68.8%	70.8%	3,417,196
2015	0.99425	1.00	68.3%	63.8%	3,270,117
2016	0.99799	1.00	59.6%	64.0%	3,455,112
2017	1.02105	1.00	66.8%	65.5%	3,388,744
2018	1.02500	1.00	64.5%	65.7%	3,136,239
2019	1.02500	1.00	62.1%	65.7%	2,999,591
2020	1.00000	1.00		64.1%	3,154,776
Total			64.1%		36,167,367

Notes: (3) = [(Projected ultimate claims from development method)(2) / [(Earned premiums)(1)]  
(3)<sub>Total</sub> = Average of AY2011 through AY2019  
(4) = 64.1% × (1) / (2)  
(5) = (4)(Earned premiums)

(c) Calculate projected ultimate claims for all accident years using the Cape Cod method.

	(6)	(7)	(8) = (6)(7)	(9)	(10)
Accident Year (AY)	On-Level Earned Premium	Expected % Paid	Used-Up On- Level Earned Premium	Adjusted Paid Claims at Dec. 31, 2020	Expected Claims
2011	6,099,959	96.5%	5,887,991	3,944,320	4,918,179
2012	5,559,714	92.5%	5,143,121	3,418,400	4,482,598
2013	4,989,120	86.5%	4,315,848	2,316,800	4,022,549
2014	4,795,868	78.2%	3,749,702	2,578,140	3,437,100
2015	5,099,389	70.2%	3,581,032	2,447,000	3,289,164
2016	5,387,869	55.5%	2,988,280	1,780,460	3,475,237
2017	5,284,375	39.5%	2,088,686	1,395,000	3,408,482
2018	4,890,621	26.3%	1,286,667	829,600	3,154,507
2019	4,677,534	13.7%	639,357	396,900	3,017,063
2020	4,919,527	4.5%	221,920	180,900	3,173,151
Total			29,902,603	19,287,520	36,378,030
Adjusted Expected Claim Ratio:				64.5%	

Notes: (6) = (1)(Earned Premiums)  
 (7) = 1 / (Cumulative Development Factors)  
 (9) = (2)(Paid Claims as of December 31, 2020)  
 Adjusted Expected Claim Ratio = 19,287,520 / 29,902,603  
 (10) = 64.5%×(6)/(2)

	(11) = 1 – (7)	(12) = (10)(11)	(13)
Accident Year (AY)	Expected % Unpaid	Expected Unpaid Claims	Projected Ultimate Claims
2011	3.5%	170,902	5,101,302
2012	7.5%	335,884	4,608,884
2013	13.5%	542,835	3,438,835
2014	21.8%	749,766	3,614,366
2015	29.8%	979,358	3,426,358
2016	44.5%	1,547,762	3,328,222
2017	60.5%	2,061,256	3,456,256
2018	73.7%	2,324,592	3,154,192
2019	86.3%	2,604,671	3,001,571
2020	95.5%	3,030,010	3,210,910
Total		14,347,036	36,340,896

Notes: (13) = (12) + (Paid Claims as of December 31, 2020)

### GIRR Spring 2021 Question 15 (LOs 3h, 3i)

#### Learning Outcomes:

- (3h) Explain the effect of changing conditions on the projection methods cited in (3e).
- (3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 21.

#### Commentary on Question:

*This question tests the candidate's understanding of how estimates of ultimate claims are affected by various changing conditions and the appropriateness of various methods of estimating ultimate claims under changing conditions.*

#### Solution:

- (a) Explain how the changes occurring to book of business 1 might influence the estimates of ultimate claims under each of the following methods:
  - (i) The Bornhuetter Ferguson method
  - (ii) The frequency-severity method
  - (i) Historical development factors will be understated when applied to recent accident years. However, the a priori expected claim ratios will be correct if they come from pricing actuaries.
  - (ii) Historical development factors (for counts and average values) will be understated when applied to recent accident years. However, identifying the trend and possibly adjusting for it should be easier if frequency is analyzed separately from severity.
- (b) Explain how the changes occurring to book of business 2 might influence the estimates of ultimate claims under each of the following methods:
  - (i) The development method applied to reported claims
  - (ii) The Cape Cod method applied to reported claims
  - (i) The development method should not be affected by the change in claim frequency. However, this method could be over-responsive to the large claim in the recent accident year and will likely overstate the estimate in this year only.
  - (ii) Because the expected ratio is based on historical averages, this method may understate claim frequency deterioration in the recent two accident years if it is not

reflected in the trend selection. Development should not be affected by the change in claim frequency. The large claim will be appropriately reflected in the estimate without being over-responsive because the Cape Cod method uses expected unreported and does not apply development to actual claims reported.

**GIRR Spring 2021 Question 19 (LOs 3e, 3g, 3j)****Learning Outcomes:**

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 15, 18, and 22.

**Commentary on Question:**

*This question tests the calculation of ultimate claims and unpaid claims using the development method and the Bornhuetter Ferguson method.*

**Solution:**

- (a) Describe two situations when the Bornhuetter Ferguson method may be preferable to the development method.

Any two of the following situations are acceptable:

- For immature experience periods
- Following the introduction of new GI products when limited or no historical experience is available
- Following entry into a new geographical area for which limited or no historical data exists
- If there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future

- (b) Select age-to-age development factors to be used in applying the development method.

AY	12:24	24:36	36:48	48:60	60:72	Tail
2015	1.678	1.310	1.154	1.073	1.044	
2016	1.671	1.307	1.147	1.072		
2017	1.589	1.299	1.143			
2018	1.582	1.292				
2019	1.561					
Simple average:	1.616	1.302	1.148	1.073	1.044	
Latest 3 years:	1.577	1.299	1.148	1.073	1.044	
Selection:	1.577	1.299	1.148	1.073	1.044	1.100

Justification: Use the most recent 3 years to give consideration to the decreasing ratios down the columns.

- (c) Estimate ultimate claim ratios as of December 31, 2020 for all accident years using the development method and selections from part (b).

	12-Ult.	24-Ult.	36-Ult.	48-Ult.	60-Ult.	72-Ult.
Calculated CDFs	2.897	1.837	1.414	1.231	1.148	1.100

e.g.,  $1.231 = 1.073 \times 1.044 \times 1.100$

AY	Paid Claims	CDF	Ultimate Claims	Ultimate Claim Ratios
2015	14,520	1.100	15,972	68.5%
2016	14,071	1.148	16,155	71.9%
2017	12,825	1.231	15,793	70.1%
2018	11,822	1.414	16,712	77.1%
2019	7,968	1.837	14,634	70.6%
2020	3,370	2.897	9,764	54.7%
Total	64,576		89,030	

e.g.,  $16,712 = 11,822 \times 1.414$ ;  $77.1\% = 16,712 / 21,688$

- (d) Estimate ultimate claim ratios as of December 31, 2020 for all accident years using the Bornhuetter Ferguson method.



AY	Earned Premiums	Paid Claims	CDF	% Unpaid	A Priori Claim Ratio	Ultimate Claims	Ultimate Claim Ratios
2015	23,313	14,520	1.100	9%	65%	15,898	68.2%
2016	22,459	14,071	1.148	13%	65%	15,954	71.0%
2017	22,525	12,825	1.231	19%	65%	15,577	69.2%
2018	21,688	11,822	1.414	29%	65%	15,947	73.5%
2019	20,743	7,968	1.837	46%	65%	14,110	68.0%
2020	17,850	3,370	2.897	65%	60%	10,383	58.2%
Total		64,576				87,868	

$$\text{e.g., } 15,947 = 11,822 + 21,688 \times 0.65 \times (1 - 1/1.414)$$

$$73.5\% = 15,947 / 21,688$$

- (e) Recommend unpaid claims by accident year as of December 31, 2020. Justify your recommendations.

AY	Ultimate Claim Ratio from Part (c)	Ultimate Claim Ratio from Part (d)	Selected Ultimate Claim Ratio	Ultimate Claims	Unpaid Claims
2015	68.5%	68.2%	68.5%	15,972	1,452
2016	71.9%	71.0%	71.9%	16,155	2,084
2017	70.1%	69.2%	70.1%	15,793	2,968
2018	77.1%	73.5%	77.1%	16,712	4,890
2019	70.6%	68.0%	70.6%	14,634	6,666
2020	54.7%	58.2%	58.2%	10,383	7,013
Total				89,649	25,073

$$\text{e.g., } 16,712 = 0.771 \times 21,688$$

$$4,890 = 16,712 - 11,822$$

Justification: Recommend the development method for AYs 2019 and prior, and the Bornhuetter Ferguson (BF) method for AY 2020. The development method is used for older years to reflect actual experience. The BF is better for immature periods and more than half of ultimate claims for AY 2020 are unpaid. Also, BF method allows incorporation of expected change from COVID in the a priori claim ratio for AY 2020.

## GIRR Fall 2021 Question 2 (LOs 3a, 3e, 3f, 3g)

### Learning Outcomes:

- (3a) Identify considerations for selecting methods for estimating ultimate claims.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 15, 17 and 18.

### Commentary on Question:

*This question tests the candidate's understanding of the expected method. In addition, this question tests the calculation of ultimate claims using the expected and Bornhuetter Ferguson methods, as well as evaluating the reasonableness of the inputs used for the Bornhuetter Ferguson method.*

### Solution:

- (a) Describe one advantage of using the pure premium approach to the expected method, rather than the claim ratio approach.

No adjustment is required for premium rate changes.

Or

It is best to choose an exposure base that requires no adjustment.

Or

It may be possible to select a pure premium exposure base that is a leading indicator of claims experience.

- (b) Describe why reinsurers typically use the claim ratio approach to the expected method, rather than the pure premium approach.

Exposures are usually not available.

- (c) Describe why reinsurers often use the expected method rather than the development method.

Reinsurance data is often subject to significant lags in immature years, which make development-based projections less reliable.

- (d) Contrast the leveraged nature of cumulative development factors with the leveraged nature of trend factors.

Large development factors in immature periods can increase uncertainty.  
Large trend factors in older/mature periods can increase uncertainty.

- (e) Describe one approach the actuary may consider to moderate the leveraging effect of actuarial factors.

The actuary should consider excluding the oldest and most recent time periods when selecting averages based on historical experience.

Or

The actuary should consider excluding highly leveraged years (recent or old) from its selected experience period (either older or recent years).

- (f) Calculate ultimate claims using the pure premium approach to the expected method.

	(1)	(2)	(3)
Report Year (RY)	Earned Dentists	Actual Reported Claims	Cumulative Development Factors
2013	12,603	12,974,000	1.042
2014	13,190	13,846,250	1.087
2015	13,631	14,074,250	1.149
2016	13,988	13,332,300	1.235
2017	15,364	14,057,100	1.351
2018	15,949	13,586,400	1.515
2019	16,270	12,601,600	1.754
2020	16,468	10,118,900	2.128

	(4) = (2)(3)	(5) = $1.03^{(2020-RY)}$	(6) = (4)(5)/(1)	(7) = 1,357.30/(5)	(8) = (1)(7) Projected Ultimate based on Expected Method
R <sub>Y</sub>	Projected Ultimate Claims from Development Method	Trend	Trended Pure Premium	Detrended Pure Prem	
2013	13,518,908.00	1.230	1,319.25	1,104	13,908,760
2014	15,050,873.75	1.194	1,362.51	1,137	14,993,275
2015	16,171,313.25	1.159	1,375.32	1,171	15,959,403
2016	16,465,390.50	1.126	1,324.85	1,206	16,868,706
2017	18,991,142.10	1.093	1,350.70	1,242	19,083,924
2018	20,583,396.00	1.061	1,369.17	1,279	20,404,881
2019	22,103,206.40	1.030	1,399.28	1,318	21,440,029
2020	21,533,019.20	1.000	1,307.57	1,357	22,351,975
Total	144,417,249.20				145,010,953
Average Trended PP excluding 2020:					
		All Years	1,357.30		
		All Years excluding high/low	1,356.51		
		Latest 5	1,350.31		
		Selected	1,357.30		

- (g) Calculate ultimate claims using the Bornhuetter Ferguson method, where the a priori expected claims are the estimated ultimate claims from the expected method in part (f).

	(9) = $1 - 1/(3)$ Expected % Unreported	(10) = (8)(9) Expected Unreported Claims	(11) = (2) + (10) Ultimate Claims
Report Year			
2013	4.0%	560,622	13,534,622
2014	8.0%	1,200,014	15,046,264
2015	13.0%	2,069,583	16,143,833
2016	19.0%	3,209,835	16,542,135
2017	26.0%	4,958,147	19,015,247
2018	34.0%	6,936,313	20,522,713
2019	43.0%	9,216,523	21,818,123
2020	53.0%	11,848,227	21,967,127
Total		39,999,264	144,590,064

- (h) Evaluate the reasonableness of the inputs for the Bornhuetter Ferguson method in part (g).

**Commentary on Question:**

*The key point is to test and conclude on the reasonability of the input assumptions. Under this particular scenario (comparing Expected Method and Bornhuetter Ferguson Method), the difference in method ultimate claims will be the same as the difference in actual versus expected. However, the percentage difference by year must still relate to expected claims, not ultimate claims, otherwise the variability in immature years will be minimized.*

	(12) = (8) – (10)	(13) = (2) – (12)	(14) = (13)/(12)
Report Year	Expected Reported Claims	Difference Actual vs. Expected	Percentage Difference
2013	13,348,138	–374,138	–2.8%
2014	13,793,261	52,989	0.4%
2015	13,889,820	184,430	1.3%
2016	13,658,871	–326,571	–2.4%
2017	14,125,776	–68,676	–0.5%
2018	13,468,568	117,832	0.9%
2019	12,223,506	378,094	3.1%
2020	10,503,748	–384,848	–3.7%
Total	105,011,689	–420,889	–0.4%

The difference is reasonable in total. The largest difference is in the most recent two years, which is expected based on maturity.

## GIRR Fall 2021 Question 6 (LOs 1d, 1f, 3g, and 3j)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (1f) Demonstrate the importance of understanding key terminology and interrelationships.
- (3g) Estimate ultimate values using the methods cited in (3e).
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 16 and 22.

### Commentary on Question:

*This question tests the candidate's understanding of the evaluation and selection of estimated IBNR under various circumstances.*

### Solution:

- (a) Describe what an *actuarial central estimate* represents according to U.S. ASOPs.

An actuarial central estimate represents an expected value over the range of reasonably possible outcomes.

- (b) Assess the validity of the following statement:

“Credibility is not utilized in projecting unpaid claims for reserving.”

Invalid; credibility is often reflected implicitly when projecting ultimate claims.

- (c) Calculate the indicated IBNR as of December 31, 2020 for each of the frequency-severity method projections above.

Accident Year	(1) Reported Claims	(2) Development Based	(3) <u>Indicated IBNR</u> Claim Closure
2015	5,051,008	2,154	2,479
2016	5,453,150	55,306	53,536
2017	5,764,966	136,626	102,293
2018	5,967,139	275,802	337,862
2019	6,294,143	531,932	761,852
2020	5,980,004	1,173,792	1,398,061

Notes: (1) = (Earned Premium)(Reported Claim Ratio Triangle Latest Diagonal)

e.g., 2017:  $5,764,966 = 8,669,122 \times 66.5\%$

(2) = Ultimate Claims – (1)

e.g., 2017:  $136,626 = 5,901,592 - 5,764,966$

$$(3) = \text{Ultimate Claims} - (1)$$

e.g., 2017:  $102,293 = 5,867,259 - 5,764,966$

- (d) Critique the appropriateness of each method as a potential IBNR selection for accident year 2018.
- (i) Paid development method
  - (ii) Reported development method
  - (iii) Paid Bornhuetter Ferguson method
  - (iv) Reported Bornhuetter Ferguson method
- (i) Paid development is not appropriate because it is under-responsive to large claim.
- (ii) Reported development is not appropriate because it is over-responsive to large claim.
- (iii) Paid Bornhuetter Ferguson is not appropriate because it is under-responsive to large claim.
- (iv) Reported Bornhuetter Ferguson is an appropriate method because it is not distorted by large claim and also recognizes relative immaturity of a liability coverage.

## GIRR Fall 2021 Question 11 (LOs 3e, 3g)

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 15.

### Commentary on Question:

*This question tests the candidate's understanding of the development-based frequency-severity method for estimating ultimate claims.*

### Solution:

- (a) Calculate the ultimate claims for accident year 2020 using the development-based frequency-severity method. Justify any selections.

	(1)	(2)	(3)	(4)	(5)	(6) = (1)(3)(5)	(7) = (2)(4)
Accident Year (AY)	Frequency	Severity	Frequency Trend @0.5%	Severity Trend @4.7%	Change from Court Ruling	Trended Frequency	Trended Severity
2014	0.0424	28,830	1.0304	1.3173	1.06	0.0463	37,977
2015	0.0427	30,014	1.0253	1.2582	1.06	0.0464	37,762
2016	0.0429	31,554	1.0202	1.2017	1.06	0.0464	37,917
2017	0.0431	32,987	1.0151	1.1477	1.06	0.0464	37,860
2018	0.0436	34,257	1.0100	1.0962	1.06	0.0467	37,553
2019	0.0435	36,098	1.0050	1.0470	1.06	0.0463	37,795
2020	0.0452	37,317	1.0000	1.0000	1.00	0.0457	37,317
All years average						0.0463	37,740
Average excluding 2020						0.0464	37,811

e.g., (1) for 2020:  $0.452 = 431 / 9,542$   
 (2) for 2020:  $37,317 = 16,270,027 / 431$

Selected frequency = 0.0463 (both averages account for the court ruling change, so either is reasonable)

Selected severity = 37,740 (no outliers and no significant trend, therefore all years average is reasonable)



Ultimate claims =  $0.0463 \times 9,542 \times 37,740 = 16,680,290$ .

- (b) Calculate the percentage growth in accident year 2020 IBNR in changing from the development method to the development-based frequency-severity method.

IBNR from development method =  $16,270,027 - 5,778,161 = 10,491,866$

IBNR from F-S method =  $16,680,290 - 5,778,161 = 10,902,129$

Percent growth =  $10,902,129 / 10,491,866 - 1 = 3.91\%\%$

- (c) Explain why the accident year 2020 IBNR calculated using the development-based frequency-severity method is likely to be more appropriate than the IBNR calculated using the development method.

Any two of the following are acceptable:

- If we are confident in the expected increase in claim frequency for AY 2020 then the F-S method is more likely to be appropriate.
- Development method does not adjust for AY 2020 expected increase in claim frequency.
- This is seen by the fact the F-S method ultimate claims are 2.5% higher than the development method ultimate claims, and the F-S method IBNR is 3.91% higher than the development method IBNR.

**GIRR Fall 2021 Question 12 (LOs 3f, 3h, 3i)****Learning Outcomes:**

- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3h) Explain the effect of changing conditions on the projection methods cited in (3e).
- (3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 21 and 22.

**Commentary on Question:**

*This question tests the evaluation of the reasonableness of the various methods of projecting ultimate claims under specific circumstances as well as under changing conditions.*

**Solution:**

- (a) Describe two diagnostics that can be used to confirm the reasonableness of projected ultimate claims.

Any two of the following are acceptable:

- Claim ratio: ultimate claims divided by earned premiums
- Severity: ultimate claims divided by ultimate counts
- Pure Premium: ultimate claims divided by earned exposures
- IBNR: ultimate claims divided by earned exposures
- Total unpaid claims: ultimate claims less paid claims
- Average IBNR: IBNR divided by IBNR counts
- Average unpaid claims: total unpaid claims divide by the sum of open and IBNR counts

- (b) Explain what effect the tort reform is likely to have on reported claim development factors if the data is organized as follows:
  - (i) On an accident year basis.
  - (ii) On a report year basis.
- (i) Development factors in latest two calendar years (i.e., diagonals) will decrease.
- (ii) Development factors in latest two report years (i.e., rows) will decrease.
- (c) Recommend a preferred approach to estimating ultimate claims for each scenario in part (b). Justify your recommendation.

**Commentary on Question:**

*Other possible approaches are possible.*

- (i) Use a Berquist-Sherman adjustment to adjust historical triangle data to be consistent with the average severity in latest two calendar years.
  - (ii) Use a frequency-severity method and adjust the severity in the latest two report years to reflect the cap.
- (d) Assess the appropriateness of each selection (i) to (iv).
- (i) AY2013, Bornhuetter Ferguson method using reported claim ratio data: The Bornhuetter Ferguson method is most appropriate for immature years. Also, reported data may be distorted by the change in case adequacy. Conclusion: Not appropriate.
  - (ii) AY2016, Cape Cod method using paid claim data: The Cape Cod method is more appropriate for immature years, however, paid cumulative development factors still show some unpaid, so Cape Cod method is reasonable. Also, the paid data is not distorted by the change in case adequacy. Conclusion: Appropriate.
  - (iii) AY2019, Development method using paid claim data: Paid data is good to use, but the 2019 cumulative development factor is too highly leveraged to be reliable. Conclusion: Not appropriate.
  - (iv) AY2020, Expected method using reported claim ratio data: This method uses a priori data, which is not distorted by change in case adequacy. This method is good for immature accident years. Conclusion: Appropriate.

## GIRR Fall 2021 Question 16 (LOs 2a, 3c, 3d)

### Learning Outcomes:

- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 11 and 14.

### Commentary on Question:

*This question tests the candidate's understanding of creating a development triangle from detailed claims transaction data, identifying potential issues with data triangles, and diagnostic tests that can be used on data triangles.*

### Solution:

- (a) Define “maturity age” in the context of a claim development triangle.

The maturity age refers to the time interval from the beginning of the experience period to the valuation date of the claims.

- (b) Construct a development triangle of cumulative reported claims, by accident year, with maturity ages 6, 12, 18, 24, 30 and 36 months.

Accident Year (AY)	<u>Incremental Paid Claims at Maturity Age (in Months)</u>					
	6	12	18	24	30	36
2018	50	100	250	0	55	75
2019	265	0	30	185		
2020	0	275				

e.g., AY2019 at 6 months:  $265 = 190 + 75$

AY	<u>Cumulative Paid Claims at Maturity Age (in Months)</u>					
	6	12	18	24	30	36
2018	50	150	400	400	455	530
2019	265	265	295	480		
2020	0	275				

e.g., AY2019 at 18 months:  $295 = 265 + 0 + 30$

AY	<u>Case Estimates at Maturity Age (in Months)</u>					
	6	12	18	24	30	36
2018	150	200	75	390	410	350
2019	35	260	225	0		
2020	550	65				

e.g., AY2019 at 12 months:  $260 = 35 + 225 + 0$

AY	<u>Reported Claims at Maturity Age (in Months)</u>					
	6	12	18	24	30	36
2018	200	350	475	790	865	880
2019	300	525	520	480		
2020	550	340				

Reported claims = Cumulative paid claims + Case estimates

e.g., AY2019 at 12 months:  $525 = 265 + 260$

- (c) Select which line of business was the likely source for each of the following claims, providing a justification for each selection:
- (i) Claim 2 is likely Automobile physical damage as it has a short reporting delay and was settled within 6 months of claim occurrence.
  - (ii) Claim 3 is likely Medical malpractice claim as it has a long reporting delay and has not closed within 36 months of its occurrence.
  - (iii) Claim 7 is likely Workers' compensation claim as it was reopened after its initial settlement.
- (d) Identify two anomalies relating to this triangle.
- Any two of the following are acceptable:
- Reported pure premiums decreased in AYs 2015-2016, then increased again in AYs 2017 and subsequent accident years.
  - Reported pure premium for AY2014 increased significantly at 72 months, then decreased again at 84 months.
  - Reported pure premium development is increasing over time (i.e., development factors increase down each column).
- (e) Describe a business, operational, or environmental change that could cause each of the anomalies identified in part (d).

**Commentary on Question:**

*Only one change is needed for each anomaly identified in part (d).*

- Reported pure premiums decreased in AYs 2015-2016, then increased again in AYs 2017 and subsequent accident years:
  - Changes in policy terms (e.g., limits, deductibles) could cause PP to change over time.
  - Changes in the type of insureds (exposures) could cause PP to change over time.
- Reported pure premium for AY2014 increased significantly at 72 months, then decreased again at 84 months:
  - The reporting of a large claim (or case estimate) which then decreased/normalized could cause an increase, then decrease in reported pure premiums.
  - The reporting of a large claim, which was subsequently covered by reinsurance (or subrogation) could cause an increase, then decrease in reported pure premiums.
- Reported pure premium development is increasing over time (i.e., development factors increase down each column).
  - Change in policy terms (e.g., limits, deductibles) could cause development to change over time.
  - Change in the type of insureds (exposures) could cause development to change over time.
  - Change in case reserve adequacy (or claim settlement patterns) could cause development to change over time.

## GIRR Fall 2021 Question 18 (LOs 1d, 3f, 3g, 4a, and 4b)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).
- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 15, and 23.

### Commentary on Question:

*This question tests the candidate's understanding of selecting development factors and estimating a tail factor using Boor's algebraic method. It also tests the calculation of unpaid ULAE using the classical paid-to-paid method, as well as an understanding of the Kittel refinement to the classical paid-to-paid method and the Mango and Allen smoothing adjustment.*

### Solution:

- (a) Select age-to-age development factors for all periods excluding the tail factor. Justify your selections.

Adjusted Age-to-Age Development Factors Excluding the Large Claim							
Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96
2013	3.191	1.675	1.352	1.197	1.122	1.091	1.063
2014	3.058	1.673	1.305	1.201	1.141	1.094	
2015	2.846	1.691	1.334	1.218	1.131		
2016	2.858	1.700	1.321	1.198			
2017	2.727	1.726	1.332				
2018	2.732	1.729					
2019	2.716						
All Years Avg.	2.876	1.699	1.329	1.204	1.131	1.092	1.063
Avg. excl. high&low	2.844	1.698	1.329				
Volume Wtd. Avg.	2.861	1.699	1.329	1.204	1.131	1.092	
5 Year Avg.	2.776	1.704					
3 Year Avg.	2.725	1.718	1.329	1.206			
Selected	2.725	1.718	1.329	1.206	1.131	1.093	1.063

Justification for selection: Selected 3 years average to recognize trend down the columns.

Notes: Adjusted factors for large claim:

$$AY2017, 24-36 = 1.726 = (1,082 - 150)/540$$

$$AY2017, 36-48 = 1.332 = (1,391 - 150)/(1,082 - 150)$$

$$\text{Volume Wtd. Avg., 24-36: } 1.699 = (866 + 875 + 876 + 923 + 1,082 + 968 - 150)/(517 + 523 + 518 + 543 + 540 + 560)$$

$$\text{Volume Wtd. Avg., 36-48: } 1.329 = (1,171 + 1,142 + 1,169 + 1,219 + 1,391 - 150)/(866 + 875 + 876 + 923 + 1,082 - 150)$$

- (b) Derive a paid tail factor using Boor's algebraic method.

		(1)	(2)	(3) = (1)(2)	(4)	(5) = (4)/(3)
				Estimated Claims		
Accident Year	Actual Paid	Paid Development Factors		96	Ultimate Claims from Reported Development Method	Implied Tail Factor
		72-84	84-96			
2013	1,824			1,824	1,975	1.083
2014	1,712		1.063	1,820	1,974	1.085
2015	1,610	1.093	1.063	1,870	2,032	1.087
Selected:						1.085

- (c) Calculate ultimate claims using the paid development method and the tail factor of 1.072.

		(1)	(2)	(3)	(4) = (2)(3)
				Age-to-Ultimate Development Factors	Ultimate Claims
Accident Year	Actual Paid	Paid Claims Excluding Large Claim			
2013	1,824	1,824		1.072	1,955
2014	1,712	1,712		1.140	1,951
2015	1,610	1,610		1.245	2,004
2016	1,460	1,460		1.408	2,056
2017	1,391	1,241		1.698	2,257
2018	968	968		2.257	2,184
2019	573	573		3.877	2,222
2020	224	224		10.566	2,367
Total	9,762	9,612			16,997

e.g.,  $1,241 = 1,391 - 150$   
 $1.698 = 1.206 \times 1.131 \times 1.093 \times 1.063 \times 1.072$



- (d) Calculate the unpaid ULAE as of December 31, 2020 using the classical paid-to-paid method and a multiplier of 50%.

$$\text{Case outstanding} = 14,660 - 9,762 = 4,898$$

$$\text{IBNR} = 17,065 - 14,660 = 2,405$$

$$\text{Unpaid ULAE} = 0.08 \times 2,405 + 0.8 \times 0.5 \times 4,898 = 388.$$

- (e) Describe the Kittel refinement to the classical paid-to-paid method and the weakness it is designed to address.

Kittel method derives ULAE ratio by comparing paid ULAE to average of paid and reported claims (rather than paid to paid ratio used in Classical method).

Kittel's change addresses some of the distortion that can arise with increasing (changing) exposures because reported claims react quicker to exposure changes.

- (f) Describe the Mango and Allen smoothing adjustment.

The Mango and Allen Smoothing Adjustment uses expected claim in place of actual claims.

## GIRR Spring 2022 Question 2 (LOs 1d, 2a, 3c, 3d)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11 and 14.

### Commentary on Question:

*This question tests the candidate's understanding of creating a development triangle from detailed claims transaction data, and diagnostic tests that can be used on data triangles.*

### Solution:

- (a) Update both development triangles shown above to include the claim transactions not captured due to the system error.

Accident Year	Incremental Reported Claims - Missing (000)			
	12	24	36	48
2018		15		20
2019	75		-10	
2020		65		
2021				

Accident Year	Cumulative Reported Claims - Missing (000)			
	12	24	36	48
2018	0	15	15	35
2019	75	75	65	
2020	0	65		
2021	0			

Accident Year	Reported Claims (000)			
	12	24	36	48
2018	1,196	1,540	1,653	1,758
2019	1,344	1,682	1,973	
2020	1,294	1,772		
2021	1,451			

e.g.,  $1,344 = 1,269 + 75$

Accident Year	Incremental Reported Counts - Missing			
	12	24	36	48
2018		1		1
2019	1			
2020				
2021				

Accident Year	Cumulative Reported Counts - Missing			
	12	24	36	48
2018	0	1	1	2
2019	1	1	1	
2020	0	0		
2021	0			

Accident Year	Reported Counts			
	12	24	36	48
2018	230	251	261	267
2019	236	256	266	
2020	231	251		
2021	234			

- (b) Determine calendar year 2021 reported claims.

$$\begin{aligned} \text{Calendar year 2021 reported claims (000)} \\ = (1,451 + 1,772 + 1,973 + 1,758) - (1,294 + 1,682 + 1,653) = 2,325 \end{aligned}$$

- (c) Determine case reserves as of December 31, 2021, for accident year 2021 only.

$$\text{Accident Year 2021 case reserves (000)} = 1,451 - 800 = 651$$

- (d) Describe the investigative tests you would recommend using for the following independent situations:

- (i) The claim department implemented a new definition of claims to distinguish between reported incidents that are valid claims and incidents not covered under the insurance policy.
- (ii) The claim department implemented a new initiative to increase their use of partial settlements.
- (iii) Ratios of closed no pay counts to closed counts
- (iv) Any of the following is acceptable:
  - Ratios of paid claims to reported claims
  - Average paid claims (paid claims divided by closed counts)
  - Average paid claims on closed with payment counts (paid claims divided by counts closed with payment)
- (e) Provide two examples of company operational changes that could cause an increase in average reported claims without affecting reported counts.

**Commentary on Question:**

*Other answers are possible.*

Any two of the following are acceptable:

- Case reserve strengthening
- Increase in policy limits
- Expanded coverage
- Increase in defense costs, e.g., increased use of outside counsel

### GIRR Spring 2022 Question 8 (LOs 3c, 3d, 3e, 3g)

#### Learning Outcomes:

- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 14 and 20.

#### Commentary on Question:

*This question tests the candidate's understanding of Berquist-Sherman adjustments when there has been a change in case estimate adequacy and a change in claim settlement patterns.*

#### Solution:

- (a) Verify that the case estimates have increased for this line of business using one diagnostic test.

Change in average case is preferred as the ratios of paid to reported claims could be either due to a change in average case or a change in claim settlement patterns.

Accident Year	Average Case Estimates					
	12	24	36	48	60	72
2016	15,948	18,451	23,047	26,126	27,936	32,733
2017	16,881	19,537	24,087	27,664	32,429	
2018	17,816	20,541	25,486	32,125		
2019	18,881	21,761	29,339			
2020	19,690	25,185				
2021	22,360					

e.g., 2018 at 12 months:  $17,816 = (38,734,090 - 10,407,100) / (3,391 - 1,801)$

Accident Year	Change in Average Case Estimates				
	12	24	36	48	60
2016-2017	5.9%	5.9%	4.5%	5.9%	16.1%
2017-2018	5.5%	5.1%	5.8%	16.1%	
2018-2019	6.0%	5.9%	15.1%		
2019-2020	4.3%	15.7%			
2020-2021	13.6%				

e.g., 2018-2019 at 12 months:  $6.0\% = 18,881 / 17,816 - 1$

There is a significant increase along the most recent diagonal which is evidence of an increase in case estimates.

- (b) Describe a different diagnostic test from the test performed in part (a) that may indicate that case estimates have increased for this line of business.

The ratios of paid claims to reported claims could also indicate a possible change in case estimates.

- Calculate the triangle of paid claims to reported claims.
- In a stable environment, the values in each column should be consistent.
- A decrease in the ratios along the most recent diagonal could suggest a possible change in case estimates, however, a change in claim settlement pattern could also affect these ratios.

- (c) Evaluate the disposal rates for this line of business to confirm that the rate of claims settlement has increased.

Disposal rates = ratios of closed counts to ultimate counts.

Accident Year	Disposal ratios					
	12	24	36	48	60	72
2016	0.534	0.669	0.771	0.782	0.809	0.950
2017	0.504	0.658	0.731	0.783	0.915	
2018	0.488	0.657	0.758	0.896		
2019	0.480	0.696	0.898			
2020	0.486	0.772				
2021	0.533					

The increase in the latest diagonal is evidence of the increase in claim settlement.

- (d) Recommend disposal rates for each maturity age. Justify your recommendation.

Recommended disposal ratios: use the latest diagonal because that's where the rates increased.

12	24	36	48	60	72
0.533	0.772	0.898	0.896	0.915	0.950

- (e) Calculate the adjusted case estimate triangle for this line of business, adjusting for changes in both case estimates and settlement rates. Justify any selections you make.

Adjusted Average Case = last diagonal from part (a), trended to each AY at 5%:

Accident Year	Adjusted Average Case Estimates					
	12	24	36	48	60	72
2016	17,520	20,719	25,344	29,138	30,884	32,733
2017	18,396	21,755	26,611	30,595	32,429	
2018	19,316	22,843	27,942	32,125		
2019	20,282	23,985	29,339			
2020	21,296	25,185				
2021	22,360					

e.g.,  $21,296 = 22,360 / 1.05$

Adjusted Closed Counts:

- Latest diagonal from closed counts triangle
- Other values = selected disposal ratio  $\times$  ultimate counts

Accident Year	Adjusted Closed Counts					
	12	24	36	48	60	72
2016	1,990	2,885	3,353	3,346	3,418	3,548
2017	1,988	2,882	3,350	3,343	3,414	
2018	1,967	2,851	3,314	3,307		
2019	1,975	2,863	3,328			
2020	1,975	2,863				
2021	1,968					

e.g., 2018 at 12 months:  $1,967 = 0.533 \times 3,691$

Accident Year	Adjusted Open Counts = Reported Counts – Adjusted Closed Counts					
	12	24	36	48	60	72
2016	1,282	663	193	387	308	187
2017	1,287	631	258	350	308	
2018	1,424	619	296	364		
2019	1,296	554	248			
2020	1,369	614				
2021	1,322					

Adjusted Case Estimates = Adjusted Average Case Estimates × Adjusted Open Counts

Accident Year	Adjusted Case Estimates					
	12	24	36	48	60	72
2016	22,453,936	13,744,749	4,887,882	11,264,233	9,522,868	6,121,130
2017	23,671,034	13,737,756	6,877,722	10,713,283	9,987,990	
2018	27,506,953	14,148,080	8,280,886	11,693,510		
2019	26,275,582	13,287,871	7,275,990			
2020	29,143,944	15,463,340				
2021	29,560,500					



**GIRR Spring 2022 Question 15 (LOs 3e, 3g)****Learning Outcomes:**

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 18 and 19.

**Commentary on Question:**

*This question tests the candidate's understanding of the Cape Cod method for estimating ultimate claims.*

**Solution:**

- (a) Describe one situation in which the Cape Cod method might be preferred over the Bornhuetter Ferguson method.

Any one of the following is acceptable:

- When the actuary wants to derive an expected value based on historical data (or an objective approach, or a specified formula) rather than an independent a priori estimate (or professional judgement).
- When the actuary wants to assume that the cost per exposure unit is constant for all years in the experience period.

- (b) Describe one situation in which the Generalized Cape Cod method might be preferred over the Cape Cod method.

Any one of the following is acceptable:

- When the actuary wants to use a distinct expected claim ratio for each year in the experience period rather than a constant claim ratio for all years.
- When the actuary does not want to assume that the cost per exposure unit is constant for all years in the experience period.

(c) Calculate the adjusted expected claim ratio.

	(1)	(2)	(3) = (2) <sub>2021</sub> /(2) <sub>AY</sub>	(4) = (1)(3)	(5)
Accident Year (AY)	Earned Premiums (000)	Average Rate Level	Premium On-Level Factor	On-Level Earned Premium (000)	Cumulative Development Factors
2013	29,614	1.0000	0.9849	29,167	1.011
2014	27,371	1.0000	0.9849	26,958	1.028
2015	27,077	0.9900	0.9948	26,938	1.049
2016	28,792	0.9800	1.0050	28,936	1.090
2017	30,307	0.9800	1.0050	30,459	1.159
2018	29,053	0.9800	1.0050	29,198	1.305
2019	26,785	0.9800	1.0050	26,919	1.709
2020	25,618	0.9800	1.0050	25,746	2.399
2021	27,616	0.9849	1.0000	27,616	3.999
Total	252,233			251,936	

Notes: Column (2) average rate levels:

AY2015:  $0.99 = 0.5 \times 1.0 + 0.5 \times 0.98$

AY2021:  $0.9849 = 0.98 \times (7/8) + 0.98 \times 1.04 \times (1/8)$

	(6) = 1 / (5)	(7) = (4)(6)	(8)	(9) = 1.02 <sup>(2021-AY)</sup>	(10) = (8)(9)
AY	Expected % Developed	Used-Up On- Level Earned Premiums (000)	Reported Claims as of Dec. 31, 2021 (000)	Claims Trend Factor	Adjusted Claims at Dec. 31, 2021 (000)
2013	98.9%	28,849	15,795	1.172	18,506
2014	97.3%	26,223	14,119	1.149	16,218
2015	95.3%	25,679	17,998	1.126	20,269
2016	91.7%	26,547	17,630	1.104	19,465
2017	86.3%	26,280	16,178	1.082	17,512
2018	76.6%	22,374	15,699	1.061	16,660
2019	58.5%	15,751	11,231	1.040	11,685
2020	41.7%	10,732	7,963	1.020	8,122
2021	25.0%	6,906	4,910	1.000	4,910
		189,342	121,523		133,347

Note: AY2019 Reported Claims (column (8)) excludes the 3,000,000 unusual claim that is not expected again ( $11,231 = 14,231 - 3,000$ ).

Adjusted Expected Claim Ratio:  $133,347 / 189,342 = 70.4\%$

- (d) Calculate projected ultimate claims for all accident years.

	(11) = $70.4\% \times (4) / (9)$	(12) = $1 - (6)$	(13) = (11)(12) Expected Unreported	(14) = Reported Claims + (13) Projected Ultimate Claims
AY	Expected Claims (000)	Expected % Unreported	(000)	(000)
2013	17,532	1.1%	191	15,986
2014	16,528	2.7%	450	14,569
2015	16,846	4.7%	787	18,785
2016	18,457	8.3%	1,524	19,154
2017	19,817	13.7%	2,719	18,897
2018	19,377	23.4%	4,529	20,228
2019	18,222	41.5%	7,560	21,791
2020	17,777	58.3%	10,367	18,330
2021	19,449	75.0%	14,585	19,495
	164,005		42,711	167,234

- (e) Calculate expected claims for accident year 2021 using the Generalized Cape Cod approach and a decay factor of 80%.

	(7) Used-Up On- Level Earned Premiums (000)	(10) Adjusted Claims at Dec 31, 2021 (000)	(15) = (10) / (7) Claim Ratios	(16) = $0.8^{(2021-AY)}$ Decay Factors
AY				
2013	28,849	18,506	64.1%	16.8%
2014	26,223	16,218	61.8%	21.0%
2015	25,679	20,269	78.9%	26.2%
2016	26,547	19,465	73.3%	32.8%
2017	26,280	17,512	66.6%	41.0%
2018	22,374	16,660	74.5%	51.2%
2019	15,751	11,685	74.2%	64.0%
2020	10,732	8,122	75.7%	80.0%
2021	6,906	4,910	71.1%	100.0%
	189,342	133,347		

Expected claim ratio for AY2021:

$$\text{sumproduct}[(7), (15), (16)] / \text{sumproduct}[(7), (16)] = 71.8\%$$

$$\text{Expected claims for AY2021} = 71.8\% \times 27,616 \times 0.75 + 4,910 = 19,771$$

**GIRR Spring 2022 Question 18 (LOs 3h, 3i, 3j)****Learning Outcomes:**

- (3h) Explain the effect of changing conditions on the projection methods cited in (3e).
- (3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 21 and 22.

**Commentary on Question:**

*This question tests the candidate's understanding of the effect that changing conditions have on the estimates of ultimate claims.*

**Solution:**

- (a) Describe a data substitution that you would make in your analysis to mitigate the problem for each of the following independent scenarios.
  - (i) There is a change in policy limits between successive policy years.
  - (ii) Exposure growth during the past two years has caused a distortion in recent development factors due to significant shifts in the average accident date within each accident year.
  - (iii) A tort reform change two years ago reduced the expected severity of many newly reported claims.
  - (iv) There has been a change in the definition of claim count you typically use for diagnostics.
- (i) Substitute policy year data for accident year data.
- (ii) Substitute accident quarter data for accident year data.
- (iii) Substitute report year data for accident year data.
- (iv) Substitute earned exposures in place of claim counts.
- (b) Describe the effect you expect this shift to have on an accident year claim triangle using reported claims.

Since liability claims have a longer reporting tail than property claims, I expect to see an increase in development at later evaluations in the triangle.

(c) Describe an approach to estimating ultimate claims for this business.

Use a frequency-severity method and explicitly address the changing liability severity.

## GIRR Fall 2022 Question 6 (LOs 3g, 3j, 6b, 6c, 6d)

### Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).
- (6b) Identify the different types of rate regulatory approaches for general insurance.
- (6c) Describe the purpose of base rates and rating factors and explain how they are used to determine an insured's premium.
- (6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 17, 18, 19, 21, 23, and 27.

### Commentary on Question:

*This question tests the candidate's understanding of estimating ultimate claims using the frequency severity method, the expected method and the Bornhuetter Ferguson method. This question also tests the candidate's ability to estimate reported claims with an adjustment for case outstanding strengthening.*

### Solution:

- (a) Calculate ultimate claims using the development-based frequency-severity method.

	(1)	(2)	(3) = (2)/(1)	(4)	(5) = (3)(4)	(6)
Accident Year	Earned Exposures	Ultimate Counts	Reported Frequency	Frequency Trend @ 1%	Trended Frequency	Calculated Ultimate Counts
2015	11,090	1,230	0.11091	1.06152	0.11773	1,234
2016	11,250	1,270	0.11289	1.05101	0.11865	1,264
2017	11,460	1,305	0.11387	1.04060	0.11850	1,300
2018	11,770	1,349	0.11461	1.03030	0.11809	1,349
2019	12,070	1,381	0.11442	1.02010	0.11672	1,397
2020	12,360	1,447	0.11707	1.01000	0.11824	1,445
2021	12,480	1,480	0.11859	1.00000	0.11859	1,474
Average (all years)					0.11807	

Selected frequency: 0.11807

Rationale: no outliers and no significant trend, so simple average is reasonable.

(6) =  $0.11807 \times (1)/(4)$

	(7)	(8)	(9) = (7)(8)	(10)	(11) = (6)(10)
Accident Year	Ultimate Severity	Severity Trend @ 6.5%	Trended Severity	Calculated Ultimate Severity	Projected Ultimate Claims
2015	4,349	1.45914	6,345.81	4,502	5,552,843
2016	4,666	1.37009	6,392.82	4,794	6,059,090
2017	5,002	1.28647	6,434.90	5,106	6,639,119
2018	5,358	1.20795	6,472.19	5,438	7,334,547
2019	5,881	1.13423	6,670.38	5,791	8,090,495
2020	6,314	1.06500	6,724.41	6,167	8,911,632
2021	6,540	1.00000	6,540.00	6,568	9,678,863
Average (all years)			6,511.50		52,266,590
Average (latest 5 years)			6,568.38		

Selected severity: 6,568.38

Rationale: there has been an increase in the more recent years, so use average of latest 5 years.

$$(10) = 6,568.38 / (8)$$

- (b) Construct the reported claims triangle adjusted for the change in case adequacy.

Accident Year	Adjusted Average Case Estimates						
	12	24	36	48	60	72	84
2015	3,019.21	4,711.60	6,331.08	7,611.32	8,629.94	9,217.78	9,584.81
2016	3,215.46	5,017.85	6,742.60	8,106.05	9,190.89	9,816.94	
2017	3,424.46	5,344.01	7,180.87	8,632.95	9,788.30		
2018	3,647.05	5,691.37	7,647.62	9,194.09			
2019	3,884.11	6,061.31	8,144.72				
2020	4,136.58	6,455.30					
2021	4,405.45						

e.g., AY2021 at 12 months:  $4,405.45 = (3,175,077 - 1,082,487) / (875 - 400)$   
 AY2019 at 24 months:  $6,061.31 = 6,455.30 / 1.065$

Accident Year	Adjusted Reported Claims						
	12	24	36	48	60	72	84
2015	1,930,388	2,761,294	3,589,678	4,284,121	4,884,010	5,284,288	5,274,875
2016	2,073,457	3,013,099	3,948,018	4,735,629	5,294,541	5,763,708	
2017	2,251,286	3,199,812	4,277,015	5,120,705	5,759,272		
2018	2,489,201	3,627,479	4,653,380	5,558,325			
2019	2,692,962	3,900,733	5,107,412				
2020	2,908,798	4,364,690					
2021	3,175,077						

e.g., AY2019 at 24 months:  $3,900,733 = 6,061.31 \times (975 - 618) + 1,736,844$

- (c) Recommend the revised annual claim severity trend. Justify your recommendation.

**Commentary on Question:**

*Other selections are acceptable as long as the justification matches the data.*

Accident Year	Ultimate Reported Severities	Year-to- Year Change
2015	4,316.59	
2016	4,561.67	5.68%
2017	4,813.61	5.52%
2018	5,066.25	5.25%
2019	5,441.62	7.41%
2020	5,802.31	6.63%
2021	5,990.39	3.24%

Average all years: 5.62%

Average excluding high & low: 5.77%

Average excluding last year: 6.10%

Recommended: 5.77%

Justification: select average excluding high & low to eliminate the variability.

- (d) Explain why you might expect the answer to part (c) to be lower than the original annual severity trend of 6.5%.

Due to the increase in the average case in the most recent diagonal, this will tend to overstate the annual severity trend. By adjusting the historical case estimates for the change, this will increase those values, which will tend to decrease the indicated annual reported severity trend.



- (e) Calculate ultimate claims using the ultimate counts provided and ultimate reported severities adjusted for the change in case adequacy.

Accident Year	Ultimate Counts	Ultimate Reported Severities	Ultimate Claims
2015	1,230	4,316.59	5,309,406
2016	1,270	4,561.67	5,793,321
2017	1,305	4,813.61	6,281,761
2018	1,349	5,066.25	6,834,371
2019	1,381	5,441.62	7,514,877
2020	1,447	5,802.31	8,395,943
2021	1,480	5,990.39	8,865,777

e.g., AY2019:  $7,514,877 = 1,381 \times 5,441.62$

- (f) Calculate expected claims for all accident years using the expected method and your recommended annual claim severity trend from part (c). Justify any selections.

Annual claim trend =  $(1 + 0.01)(1 + 0.0577) - 1 = 6.83\%$

Accident Year	Claim Trend @6.83%	Trended Pure Premiums Based on Reported	Expected Claims
2015	1.48623	711.54	5,311,155
2016	1.39125	716.44	5,755,608
2017	1.30234	713.87	6,263,319
2018	1.21911	707.89	6,871,912
2019	1.14120	710.52	7,528,174
2020	1.06827	725.66	8,235,350
2021	1.00000	710.40	8,882,995
Average all years (excl. 2021)		714.32	48,848,513
Average (excluding 2020)		711.78	
Selected 2021 level pure premium		711.78	

Justification: 2020 appears to be an outlier, so use average of all years excluding 2020.

e.g., AY2019:

$$710.52 = 7,514,877 \times 1.14120 / 12,070$$

$$7,528,174 = 711.78 \times 12,070 / 1.14120$$

- (g) Calculate ultimate claims for all accident years using the Bornhuetter Ferguson method.

Accident Year	(12) Reported Claims	(13): part (e) Ultimate Claims	(14) = (13)/(12) Age-to-Ult Factor	(15): part (f) Expected Claims	(16) = (12) + (15)[1 – 1/(14)] BF Estimate Ultimate Claims
2015	5,274,875	5,309,406	1.00655	5,311,155	5,309,417
2016	5,763,708	5,793,321	1.00514	5,755,608	5,793,128
2017	5,759,272	6,281,761	1.09072	6,263,319	6,280,227
2018	5,558,325	6,834,371	1.22957	6,871,912	6,841,381
2019	5,107,412	7,514,877	1.47137	7,528,174	7,519,137
2020	4,364,690	8,395,943	1.92361	8,235,350	8,318,835
2021	3,175,077	8,865,777	2.79230	8,882,995	8,876,829
					48,938,953

- (h) Recommend the selected ultimate claims for accident year 2021 for this line of business. Justify your recommendation.

Recommend using average of part (e), part (f) and part (g) estimates = 8,875,200

Justification:

- Development method (9,678,673) and part (a) estimate (9,678,863) do not adjust for the change in case outstanding, so both are inappropriate.
- Parts (e), (f) and (g) estimates all adjust for the change in case outstanding so are all reasonable methods.
- Recommend the average of all 3 since they are all close in value.

**GIRR Fall 2022 Question 7 (LOs 1j, 3c, and 3d)****Learning Outcomes:**

- (1j) Describe qualitative information required for actuarial work.
- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 5 and 14.

**Commentary on Question:**

*This question tests investigative analysis of various development triangles.*

**Solution:**

- (a) Describe two operational changes that could have caused this decrease.
  - A change in systems or process for reporting counts could cause a decrease in frequency.
  - A change in the definition of claim counts could cause a decrease in frequency.
- (b) Describe one external environmental change that could have caused this decrease.

Any one of the following is acceptable:

  - Legislative change implementing tort reform which reduces claims filed.
  - Court interpretation clarifying (confirming) a coverage exclusion.
- (c) Identify a change in pattern in this triangle.

There is a significant decrease along the latest diagonal.
- (d) Describe two possible operational changes that could have caused the pattern change identified in part (b).

**Commentary on Question:**

*This part of the question incorrectly referenced part (b) instead of part (c). The following solution relates to reference to the pattern change identified in part (c). Candidates who answered based on part (b) were graded on that basis.*

- This could be the result of a decrease from slowing down of the payment of claims (claim settlement).
  - Alternatively, it could be a result of increasing from a significant change in case estimates.
- (e) Describe an additional test to further investigate the change in pattern identified in part (b).

**Commentary on Question:**

*This part of the question incorrectly referenced part (b) instead of part (c). The following solution relates to reference to the pattern identified in part (c). Candidates who answered based on part (b) were graded on that basis.*

*Candidates can choose to either refer to the change in claim settlement or the change in case adequacy.*

Change in claim settlement could be confirmed by evaluating the ratios of closed to reported counts to see if a similar pattern is evident (i.e., significant decrease along the latest diagonal).

Change in case adequacy could be confirmed by evaluating average case estimates to see if there is a significant increase along the most recent diagonal.

### **GIRR Fall 2022 Question 10 (LOs 3j)**

#### **Learning Outcomes:**

(3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

#### **Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 22.

#### **Commentary on Question:**

*This question tests the candidate's understanding of the evaluation and selection of estimated ultimate claims under various circumstances.*

#### **Solution:**

- (a) Describe two weaknesses in selecting each of the following methods to estimate ultimate claims for these accident years.
- (i) Development Method using reported data.
  - (ii) Generalized Cape Cod Method using reported data.
    - (i)
      - For a long-tailed line, applying large development factors to immature years can create volatile (highly leveraged) estimates.
      - Development factors will be distorted by the change in case reserve adequacy.
    - (ii)
      - Responsiveness to claim deterioration is reduced by using expected claims. (i.e., the Generalized Cape Cod (GCC) method is not responsive enough)
      - The GCC method still relies on development factors for the expected claim ratio (ECR), but development factors are distorted by change in case adequacy.
      - The calculation of the ECR in the GCC method gives more weight to years with more exposure, and more weight to years with maturity. This means more weight will be given to older accident years, which may not reflect the recent claim deterioration.
- (b) Evaluate the appropriateness of selecting the Expected Method using reported pure premium data to estimate ultimate claims for the two most recent accident years.

Any two of the following statements is acceptable:

- The expected method is good for immature years.
- Pure premium trend (claim deterioration) can be evaluated and explicitly considered.
- The latest year of data can be ignored in selecting expected claims, so case reserve adequacy does not cause a distortion.

### GIRR Fall 2022 Question 13 (LOs 3h, 3i)

#### Learning Outcomes:

- (3h) Explain the effect of changing conditions on the projection methods cited in (3e).
- (3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 21.

#### Commentary on Question:

*This question tests the candidate's understanding of the effect that changing conditions have on the estimates of ultimate claims.*

#### Solution:

- (a) Critique each of the two methods used for the analysis. Your critique should indicate any potential bias in the methods.
  - The loss development factors under both methods include both development and currency movement.
  - This makes the factors less reliable, and the currency exchange rate does not follow a pattern.
  - Because currency B has been losing value since CY 2019, the development factors for the last 3 calendar years will be understated, and so will the ultimate estimates.
  - The expected claim ratio for the BF method may not be accurate as the premiums, paid claims and case estimates are converted at different times (and rates).
- (b) Propose an alternative approach or method for analyzing this data that should produce more accurate results. Justify your proposal.
  - Claims should be analyzed separately by currency without conversion.
  - In order to deal with the low volume for claims in currency B, one could credibility weight the currency B development factors with currency A development factors.
  - For the BF method, can use 65% for currency A and 60% for currency B.
  - For financial reporting, the claim liabilities would be the claim liabilities for currency A plus the exchange rate at the financial reporting date times the claim liabilities for currency B.
- (c) Describe how your responses to parts (a) and (b) would be affected if this were a short-tail line rather than a liability line.
  - Short tail versus long tail does not change the intermingling of development with currency exchange.

- However, short tail lines have smaller development factors and development factors that reach 1 sooner, therefore the bias should have a smaller effect (i.e., smaller bias on total claim liabilities).

## GIRR Fall 2022 Question 15 (LOs 3d, 3e, 3g)

### Learning Outcomes:

- (3d) Analyze development triangles for investigative testing.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 15 and 16.

### Commentary on Question:

*This question tests the candidate's understanding of the development method of estimating ultimate claims as well as understanding how a change in the rate of claim settlement can affect development patterns.*

### Solution:

- (a) Estimate ultimate claims using paid claims and your colleague's selected age-to-age factors.

Accident Year	Age-to- Age Factors	Age-to-Ult Factors	Paid Claims	Ultimate Claims
2014	1.0000	1.0000	3,150,859	3,150,859
2015	1.0420	1.0420	3,334,361	3,474,404
2016	1.0730	1.1181	3,340,680	3,735,101
2017	1.1350	1.2690	3,211,463	4,075,362
2018	1.2430	1.5774	3,005,560	4,740,890
2019	1.3530	2.1342	2,385,228	5,090,520
2020	1.8270	3.8992	1,491,676	5,816,280
2021	2.6810	10.4536	766,038	8,007,886
Total			20,685,865	38,091,301

- (b) State two concerns with your colleague's selected age-to-age factors.

### Commentary on Question:

*Other answers are possible. Concerns need to be specific to the colleague's selected factors and not about factors in general. For example, high leverage is influenced by the line of business and not by the colleague's selections.*

- There is concern with using straight average.



- There was no tail factor selected.
- (c) Explain your rationale for each of the concerns identified in part (b).
- Concern with using straight average: This is clearly a growing line of business, so more weight should be given to more recent years.
  - No tail factor: There should be a tail factor as there is still development up to 96 months.
- (d) Recommend alternative selected age-to-age factors for the following. Justify your recommendations.
- (i) 12-24
- (ii) 36-48

**Commentary on Question:**

*Other answers are possible.*

- (i) 12-24: Latest 3 years will give more weight to the decreasing trend
- (ii) 36-48: Remove AY 2016 as it appears to be an anomaly
- (e) Estimate ultimate claims using reported claims and your colleague's selected age-to-age factors.

AY	Age-to-Age Factors	Age-to-Ult Factors	Reported Claims	Ultimate Claims
2014	1.01000	1.01000	3,161,268	3,192,881
2015	1.01000	1.02010	3,454,115	3,523,543
2016	1.03100	1.05172	3,684,648	3,875,229
2017	1.06700	1.12219	3,787,476	4,250,262
2018	1.09600	1.22992	3,878,344	4,770,048
2019	1.13100	1.39104	3,997,935	5,561,279
2020	1.35700	1.88764	3,596,409	6,788,720
2021	1.63900	3.09384	3,028,985	9,371,194
Total			28,589,180	41,333,156

- (f) Provide two reasons why the ultimate claims from part (e) are higher than the ultimate claims from part (a).
- There is no tail factor for paid claims and there is a tail factor for reported.
  - The latest diagonal of age-to-age factors for reported claims is much higher and this will tend to make the reported claims higher.

(g) Evaluate your colleague's conclusion.

- An increase in claim settlement could possibly increase the latest diagonal of the reported triangle.
- This increase would also show up in the paid age-to-age triangle.
- Since the pattern is not in the paid triangle, it is therefore likely that the cause of the increase in the reported triangle was not due to an increase in claim settlement pattern.

**GIRR Fall 2022 Question 17 (LOs 3e, 3f, 3g, 3j)****Learning Outcomes:**

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 15, 19, and 22.

**Commentary on Question:**

*This question tests the candidate's understanding of Cape Cod method for estimating ultimate ALAE.*

**Solution:**

- (a) Provide two reasons an actuary may want to estimate ultimate ALAE separate from ultimate indemnity.

Any two of the following are acceptable:

- The relationship between ALAE and claims is changing over time
- A change in ALAE handling (e.g., change in legal billing)
- A change in payment/reporting pattern for indemnity
- Trends for indemnity and ALAE are different
- ALAE is material and credible
- The company wants to understand ALAE cost drivers separate from indemnity

- (b) Calculate the adjusted expected pure premium for ALAE (i.e., ALAE cost per exposure) by accident year and in total using the Cape Cod method.

$$\text{Annual pure premium trend} = (1 - 0.015)(1 + 0.040) - 1 = 2.44\%$$

	(1)	(2)	(3)	(4) = 1 / (3)
Accident Year	Earned Exposures	Reported ALAE as of Dec. 31, 2021	Reported ALAE Cumulative Development Factors	Expected % Reported
2014	24,282	3,617	1.000	100.0%
2015	25,414	4,159	1.011	98.9%
2016	26,264	2,256	1.053	95.0%
2017	26,950	2,410	1.114	89.8%
2018	28,044	2,051	1.234	81.0%
2019	29,110	2,672	1.411	70.9%
2020	29,880	4,900	1.922	52.0%
2021	30,606	2,699	3.574	28.0%
Total	220,550	24,764		

	(5) = (1)(4)	(6)	(7)	(8) = (2)(6)(7)	(9) = (8)/(5)
Accident Year	Used-Up Earned Exposures	Pure Premium Trend	Tort Reform	Adjusted Reported ALAE as of 12/31/21	Adjusted Expected Pure Premium
2014	24,282	1.1838	0.90	3,854	0.1587
2015	25,137	1.1556	0.90	4,326	0.1721
2016	24,942	1.1281	0.90	2,290	0.0918
2017	24,192	1.1012	0.90	2,389	0.0987
2018	22,726	1.0750	0.90	1,984	0.0873
2019	20,631	1.0494	0.95	2,664	0.1291
2020	15,546	1.0244	1.00	5,020	0.3229
2021	8,564	1.0000	1.00	2,699	0.3152
Total	166,020			25,225	0.1519

e.g., AY 2019 Pure Premium Trend factor:  $1.0494 = 1.0244^2$   
 AY 2019 Tort reform factor:  $0.95 = 0.5 \times 0.90 + 0.5 \times 1.00$

- (c) Comment on whether or not the results from part (b) are consistent with the key assumption of the Cape Cod method.

The adjusted expected pure premium shows significant variation by accident year. This is not consistent with the underlying assumption of the Cape Cod method, which assumes relatively constant pure premium for all years in the experience period.

- (d) Calculate the projected ultimate ALAE by accident year using the Cape Cod method.

$$(10) = [0.1519 \times (1)] / [(6)(7)]$$

$$(11) = 1 - (4)$$

$$(12) = (10)(11)$$

$$(13) = (2) + (12)$$

Accident Year	Expected ALAE	Expected Unreported %	Expected Unreported	Projected Ultimate ALAE
2014	3,463	0.0%	0	3,617
2015	3,713	1.1%	40	4,199
2016	3,930	5.0%	198	2,454
2017	4,132	10.2%	423	2,833
2018	4,404	19.0%	835	2,886
2019	4,437	29.1%	1,292	3,964
2020	4,432	48.0%	2,126	7,026
2021	4,650	72.0%	3,349	6,048
Total	33,160		8,264	33,028

- (e) Compare actual ALAE as of December 31, 2021 to expected ALAE from the Cape Cod method.

$$(14) = (2) - (12)$$

$$(15) = (2) - (14)$$

$$(16) = (15) / (2)$$

Accident Year	Reported ALAE as of Dec. 31, 2021	Expected Reported	Test Actual vs. Expected	Actual vs. Expected as a % of Actual
2014	3,617	3,463	154	4%
2015	4,159	3,672	487	12%
2016	2,256	3,733	(1,477)	-65%
2017	2,410	3,709	(1,299)	-54%
2018	2,051	3,569	(1,518)	-74%
2019	2,672	3,144	(472)	-18%
2020	4,900	2,306	2,594	53%
2021	2,699	1,301	1,398	52%
Total	24,764	24,897	(133)	-1%

- (f) Assess the actual versus expected results from part (e).
- The actual vs. expected appears reasonable overall, however, variation by accident year is significant.
  - Need to investigate/research/analyze further (or, need to perform additional diagnostics).
- (g) Describe a scenario where an actuary would likely choose to apply the Generalized Cape Cod method over the Cape Cod method.

Any one of the following is acceptable:

- Want to vary the expected claims (or pure premium) by year. The Cape Cod method assumes constant expected claims (or pure premium) for all years in experience period.
- Want to blend development method (experience-based) and Cape Cod method into one method. A decay of 1.0 is the Cape Cod method. A decay of 0 is the development method.

### **GIRR Spring 2023 Question 3 (LOs 3e, 3g)**

#### **Learning Outcomes:**

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

#### **Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 15.

#### **Commentary on Question:**

*This question tests the candidate's understanding of development method for estimating ultimate claims.*

#### **Solution:**

- (a) State the two key assumptions of the development method.
- Historical experience is predictive of future experience.
  - Activity observed to date is relevant for projecting future activity.
- (b) Describe an advantage of using paid claims instead of reported claims when applying the development method.
- Paid development patterns are not influenced by changes in philosophy or processes regarding case estimates.
- (c) Describe an advantage of using reported claims instead of paid claims when applying the development method.
- Reported claims are often used instead of paid claims as there tends to be less volatility and more credibility associated with the selection of development factors for reported claims.
- (d) Describe one way you might account for the presence of large claims in the data when applying the development method.
- Remove the large claims from the data triangle so that the development pattern is not distorted by the presence of large claims.
- (e) Describe two ways you might account for limited credibility of the data when applying the development method.

- When the credibility of the data is more limited, use longer-term averages, which frequently demonstrate greater stability than shorter-term averages.
- Could look to industry data for development patterns.

(f) Calculate projected ultimate claims for all accident years using the paid development method.

AY	Paid Claims Age-to-age factors						
	12-24	24-36	36-48	48-60	60-72	72-84	84-ult
2016	2.339	1.481	1.306	1.218	1.133	1.047	
2017	1.931	1.613	1.332	1.186	1.131		
2018	2.767	1.510	1.317	1.213			
2019	2.774	1.517	1.353				
2020	2.269	1.598					
2021	2.210						
2022							
Simple 3	2.418	1.542	1.334	1.206	1.132	1.047	
Simple All	2.382	1.544	1.327	1.206	1.132	1.047	
Vol Wtd 3	2.385	1.542	1.334	1.205	1.132	1.047	
Vol Wtd 5	2.352	1.544	1.328	1.205	1.132	1.047	
Vol Wtd All	2.350	1.544	1.328	1.205	1.132	1.047	
Selected:	2.350	1.544	1.328	1.205	1.132	1.047	1.015
Age-to-Ult.	6.983	2.972	1.925	1.450	1.203	1.062	1.015

Recommend volume-weighted average of all years to address the variability.

#### *Algebraic Method for Paid Claims*

*Tail Factor:*

AY	Ultimate Reported Claims	Paid Claims Developed to 84 months	Implied Tail Factor
2016	2,513,084	2,487,315	1.010
2017	2,665,698	2,625,300	1.015
2018	2,809,772	2,760,204	1.018
Average			1.015



Paid Claims	CDF	Ultimate Paid Claims
2,487,315	1.015	2,523,552
2,507,208	1.062	2,663,547
2,328,436	1.203	2,800,417
2,091,115	1.450	3,031,578
1,650,625	1.925	3,177,499
1,140,537	2.972	3,389,654
408,139	6.983	2,850,185
12,613,375		20,436,433

## GIRR Spring 2023 Question 6 (LOs 3e, 3g)

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 20.

### Commentary on Question:

*This question tests the candidate's understanding of the Cape Cod method for estimating ultimate claims.*

### Solution:

- (a) Describe why the Cape Cod method may not be appropriate for coverages such as property or automobile collision.

The development factor may be less than 1, which will result in used-up exposures that are greater than the original exposures.

- (b) Calculate projected ultimate claims using the Cape Cod method applied to paid claims.

	(1)	(2)	(3) = 1/(2)	(4) = (1)(3)	(5)
Accident Year	On-Level Earned Premiums	Paid Cumulative Development Factors	Expected % Paid	Used-Up On- Level Earned Premiums	Actual Paid Claims
2017	14,304,922	1.048	95.42%	13,649,735	8,573,426
2018	14,662,414	1.097	91.16%	13,365,920	8,699,818
2019	14,826,526	1.326	75.41%	11,181,392	7,732,920
2020	15,064,165	1.847	54.14%	8,156,018	5,857,706
2021	15,448,284	3.146	31.79%	4,910,453	3,561,183
2022	15,630,481	9.473	10.56%	1,650,003	1,395,852
Total				52,913,520	35,820,905

Accident Year	(6) Claim Trend Factors	(7) Tort Reform Factors	(8) = (5)(6)(7) Adjusted Claims	(9) = (A)(1)/[(6)(7)] Expected Claims	(10) = (5) + (9)[1 – (3)] Ultimate Claims
2017	1.2763	0.800	8,753,684	9,438,074	9,005,704
2018	1.2155	0.800	8,459,747	10,157,636	9,597,986
2019	1.1576	0.800	7,161,457	10,784,894	10,384,410
2020	1.1025	0.950	6,135,215	9,688,961	10,300,884
2021	1.0500	1.000	3,739,242	9,911,179	10,321,955
2022	1.0000	1.000	1,395,852	10,529,475	10,813,802
Total			35,645,197	60,510,219	60,424,742

Adjusted Expected Claim Ratio =  $35,645,197 / 52,913,520 = 67.37\%$  (A)

e.g., 2020 tort reform factor:  $25\%(0.8) + 75\%(1.0) = 0.95$

- (c) Describe two situations that could result in such a difference in Cape Cod projections.

**Commentary on Question:**

*Other situations are possible.*

Any two of the following are acceptable:

- Decrease in the adequacy of case reserves in the latest diagonal
- Change in the settlement rates resulting in higher paid claims than in past
- Unusual payment of large claims where the case is low
- Change in environment (internal or external) that is reflected in case estimates but not yet seen in paid claims that lag the reporting of claims

## GIRR Spring 2023 Question 7 (LOs 3i, 3j)

### Learning Outcomes:

- (3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 21.

### Commentary on Question:

*This question tests the candidate's understanding of the appropriateness of various methods of estimating ultimate claims under changing conditions.*

### Solution:

Recommend a different estimation method to use with each of the following four independent books of business. Justify your recommendations.

### Commentary on Question:

*The Berquist-Sherman adjustments are adjustments to data and not a method. Candidates needed to also recommend the estimation method if they recommended Berquist-Sherman adjustments.*

- (i) A long-tailed book where the case estimates were strengthened in 2018.

The paid development is responsive to the case change, but it will be too leveraged for 2022 so avoid this method. Recommend adjusting for the case change using Berquist-Sherman approach and then use the reported development method to estimate ultimate claims.

- (ii) A book that has unstable development patterns and experience that has been improving.

Recommend using the frequency-severity method to separately analyze claim counts and average severity. This might give better insights as to what patterns are changing and where the deterioration is coming from (i.e., frequency or severity or both).

- (iii) A quickly growing book of business that has only been writing business for three years.

### Commentary on Question:

*The frequency-severity method is also acceptable, but not if already selected in part (ii), as the question asks for a different method for each part.*

Any of expected, Bornhuetter Ferguson or Cape Cod methods for a new line of business and also the significant growth.

- (iv) A medium-tailed book of business where the policy limit was increased from 2 million to 3 million, effective January 1, 2019.

The expected method (if the expected method was not chosen in part (iii)) because both the pattern and experience will change at mature (in the tail) periods which will take many years to figure out.

## GIRR Spring 2023 Question 11 (LOs 3e, 3g)

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 20.

### Commentary on Question:

*This question tests the candidate's understanding of a Berquist-Sherman adjustment for a change in claims settlement.*

### Solution:

- (a) Identify two possible reasons for a delay in claims processing.

Any two of the following are acceptable:

- a recent change in the claims processing system
- an increase in volume that creates a backlog of processing claims
- a change in claims personnel

- (b) Calculate the disposal ratio triangle for this line of business.

Accident Year	Disposal Ratios (closed counts / ultimate counts)					
	12	24	36	48	60	72
2017	0.291	0.532	0.735	0.886	0.991	0.996
2018	0.314	0.574	0.799	0.961	0.991	
2019	0.341	0.617	0.824	0.922		
2020	0.334	0.609	0.784			
2021	0.350	0.581				
2022	0.325					

e.g., for AY2020 at 12 months development:  $0.334 = 459 / 1,373$

- (c) Interpret the results from part (b).

Accident Year	Change in Disposal Ratios				
	12	24	36	48	60
2017-2018	7.7%	7.7%	8.8%	8.4%	0.0%
2018-2019	8.9%	7.5%	3.1%	-4.0%	
2019-2020	-2.1%	-1.3%	-4.8%		
2020-2021	4.8%	-4.5%			
2021-2022	-7.3%				

The ratios down the column should show noticeable decrease if there is a slowing in settlement patterns. There appears to be a noticeable decrease in the most recent diagonal.

(d) Calculate the adjusted paid claims triangle.

Selected disposal ratios (most recent diagonal):

12	24	36	48	60	72
0.325	0.581	0.784	0.922	0.991	0.996

Accident Year	Adjusted Closed Counts						Ultimate Counts
	12	24	36	48	60	72	
2017	445	797	1,075	1,265	1,359	1,365	1,371
2018	432	773	1,043	1,227	1,318		1,330
2019	427	764	1,032	1,213			1,315
2020	446	798	1,077				1,373
2021	462	826					1,421
2022	459						1,413

e.g., for AY2020 at 12 months:  $446 = 0.325 / 1,373$

Accident Year	Average Claim Cost					
	12	24	36	48	60	72
2017	3,448	4,443	4,997	5,426	5,714	6,125
2018	3,620	4,665	5,247	5,697	6,000	
2019	3,801	4,898	5,510	5,982		
2020	3,991	5,143	5,785			
2021	4,190	5,400				
2022	4,400					

e.g., AY2020 at 12 months:  $3,991 = 4,400 \times 1.05^{-2}$

Accident Year	Adjusted Paid Claims = (Adjusted Closed Counts)(Average Claim Cost)					
	12	24	36	48	60	72
2017	1,535,374	3,540,462	5,374,253	6,861,836	7,763,600	8,360,625
2018	1,563,931	3,606,313	5,474,211	6,989,463	7,908,000	
2019	1,623,608	3,743,923	5,683,096	7,256,166		
2020	1,779,980	4,104,507	6,230,445			
2021	1,934,318	4,460,400				
2022	2,019,600					
e.g., AY2020 at 12 months: $1,779,980 = 446 \times 3,991$						



### GIRR Spring 2023 Question 13 (LOs 3c, 3d)

#### Learning Outcomes:

- (3c) Identify the types of development triangles that can be used for investigative testing.  
 (3d) Analyze development triangles for investigative testing.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 14.

#### Commentary on Question:

*This question tests the candidate's understanding of the types of development triangles that can be used for investigative testing and analyzing development triangles for investigative testing.*

#### Solution:

- (a) Analyze this data for evidence of a change in case reserve adequacy, using two different investigative tests. Justify your conclusion.

Change in average case estimates:

Accident Year	Average Case Estimates = Case Estimates / Open Counts					
	12	24	36	48	60	72
2017	4,401	5,011	5,618	6,147	9,947	0
2018	4,771	5,421	5,923	6,477	0	
2019	5,041	5,844	6,452	0		
2020	5,345	6,083	7,575			
2021	5,636	7,466				
2022	6,801					

Accident Year	Change in average case estimates:				
	12	24	36	48	60
2017-2018	8.4%	8.2%	5.4%	5.4%	n/a
2018-2019	5.7%	7.8%	8.9%	n/a	
2019-2020	6.0%	4.1%	17.4%		
2020-2021	5.4%	22.7%			
2021-2022	20.7%				

Change in average reported claims:

Accident Year	Average Reported Claims					
	12	24	36	48	60	72
2017	3,764	4,448	4,801	5,094	5,204	5,231
2018	4,064	4,585	5,074	5,426	5,498	
2019	4,163	4,878	5,333	5,642		
2020	4,601	5,152	5,836			
2021	4,912	5,748				
2022	5,436					

Accident Year	Change in average reported:				
	12	24	36	48	60
2017-2018	8.0%	3.1%	5.7%	6.5%	5.7%
2018-2019	2.4%	6.4%	5.1%	4.0%	
2019-2020	10.5%	5.6%	9.4%		
2020-2021	6.7%	11.6%			
2021-2022	10.7%				

Based on the significant increase in the most recent diagonal of both triangles, there is an indication of a strengthening of case estimates.

- (b) Critique your colleague's conclusion.

The case strengthening could cause the most recent diagonal to increase, but a deterioration in claims experience could also cause the increase.

- (c) Describe why an increase in the most recent diagonal of the ratios of paid to reported claims triangle may not give a clear indication of this change.

A decrease in the overall adequacy of case estimates, which would decrease the reported claims, could also be driving the increase in the ratios of paid to reported claims.

- (d) Analyze this data for evidence of a change in claim settlement patterns, using an investigative test other than the test described in part (c). Justify your conclusion.

Change in ratios of closed to reported counts:

Accident	Closed to Reported Counts					
Year	12	24	36	48	60	72
2017	0.516	0.719	0.839	0.927	0.994	1.000
2018	0.510	0.720	0.839	0.928	1.000	
2019	0.507	0.717	0.842	1.000		
2020	0.507	0.719	0.893			
2021	0.507	0.768				
2022	0.547					

Accident	Change in ratios of closed to reported counts:				
Year	12	24	36	48	60
2017-2018	-1.2%	0.1%	0.0%	0.1%	n/a
2018-2019	-0.6%	-0.4%	0.3%	n/a	
2019-2020	0.1%	0.3%	6.1%		
2020-2021	0.0%	6.9%			
2021-2022	7.8%				

Based on the significant increase in the most recent diagonal, there is an indication of an increase in claim settlement patterns.

### GIRR Spring 2023 Question 14 (LOs 3g, 5c, 5d)

#### Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 26.

#### Commentary on Question:

*This question tests the development-based frequency-severity method for estimating ultimate claims.*

#### Solution:

- (a) Recommend an annual claim frequency trend to use for the development-based frequency-severity method. Justify your recommendation.

	(1)	(2)	(3) = (2) / (1)	(4) = (3) <sub>i</sub> / (3) <sub>i-1</sub> - 1
	Ultimate Counts Based on			
Accident Year	Earned Exposures	Development Method	Indicated Frequency	Year-to-Year Change
2017	11,434	1,235	10.80%	
2018	11,635	1,247	10.72%	-0.773%
2019	11,681	1,249	10.69%	-0.234%
2020	11,821	1,260	10.66%	-0.314%
2021	12,044	1,256	10.43%	-2.163%
2022	12,240	1,301	10.63%	1.924%
Average:				-0.312%
Selected frequency trend:				-0.312%

Justification: The year-to-year changes are quite erratic, with an overall decrease over the period. The average of all years provides a reasonable measure of the overall trend.

- (b) Estimate ultimate claims for all accident years using the development-based frequency-severity method.

Accident Year	(5) Frequency Trend @ -0.312%	(6) = (3)(5) Trended Frequency	(7) = 10.59%×(1)/(5) Calculated Ultimate Counts
2017	0.98450	10.63%	1,230
2018	0.98758	10.58%	1,248
2019	0.99067	10.59%	1,249
2020	0.99377	10.59%	1,260
2021	0.99688	10.40%	1,279
2022	1.00000	10.63%	1,296

Average excluding 2022

- all years	10.56%
- latest 3 years	10.53%
- excl. hi-lo	10.59%

Selected freq. at 2022 cost level 10.59%

Justification: Excluding high and low values excludes the outlier value in 2021.

Accident Year	(8) Ultimate Severity	(9) Severity Trend @ 7.5%	(10) = (8)(9) Trended Severity	(11) = 5,900.79/(9) Calculated Ultimate Severity	(12) = (7)(11) Projected Ultimate Claims
2017	4,104	1.43563	5,891.82	4,110.25	5,055,292
2018	4,384	1.33547	5,854.70	4,418.52	5,512,721
2019	4,751	1.24230	5,902.15	4,749.90	5,931,044
2020	5,066	1.15563	5,854.40	5,106.15	6,432,161
2021	5,531	1.07500	5,945.83	5,489.11	7,023,037
2022	5,897	1.00000	5,897.00	5,900.79	7,648,692

Average excluding 2022

- all years	5,889.78
- latest 3 years	5,900.79
- excl. hi-lo	5,882.89

Selected severity at 2022 cost level 5,900.79

Justification: Latest 3 years gives more consideration to the increasing more recent experience.

- (c) Describe two scenarios when projections from the frequency-severity method are preferred.

Any two of the following are acceptable:

- For immature periods (i.e., most recent accident years)
- Following the introduction of new GI products when limited or no historical experience is available
- Following entry into a new geographical area for which limited or no historical data exists
- If there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future

## GIRR Fall 2023 Question 1 (LOs 1d, 2a, 3d)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3d) Analyze development triangles for investigative testing.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11, and 14.

### Commentary on Question:

*This question tests the candidate's understanding of claim triangles and identifying anomalies in the data.*

### Solution:

- (a) Update both triangles to include the missing transactions.

Claim ID 100 – Changes to accident year (AY) 2019 row of each triangle:

Paid Claims				
AY	12	24	36	48
2019		0	6	6

Case Estimates				
AY	12	24	36	48
2019		5	0	0

Reported Claims				
AY	12	24	36	48
2019	0	5	6	6

Claim ID 200 – Changes to accident year (AY) 2020 row of each triangle:

Paid Claims			
AY	12	24	36
2020	0	6	6

Case Estimates			
AY	12	24	36
2020	17	4	4

Reported Claims			
AY	12	24	36
2020	17	10	10

Claim ID 300 – Changes to accident year (AY) 2021 row of each triangle:

Paid Claims		
AY	12	24
2021	0	11

Case Estimates		
AY	12	24
2021	29	29

Reported Claims		
AY	12	24
2021	29	40

Revised triangles:

Accident	Reported Claims (000)			
Year	12	24	36	48
2019	1,148	1,788	2,532	3,416
2020	3,444	4,903	6,857	
2021	5,739	12,210		
2022	8,035			

Accident	Paid Claims (000)			
Year	12	24	36	48
2019	138	466	888	1,431
2020	413	1,275	3,154	
2021	689	4,151		
2022	1,286			



- (b) Identify an anomaly in the triangle of ratios of paid claims to reported claims based on the corrected triangles from part (a).

Accident Year	Ratios of Paid Claims to Reported Claims			
	12	24	36	48
2019	0.12	0.26	0.35	0.42
2020	0.12	0.26	0.46	
2021	0.12	0.34		
2022	0.16			

For calendar year 2022 (i.e., the latest diagonal), the ratios have increased significantly.

- (c) Describe two operational changes that could have caused the anomaly you identified in part (b).

**Commentary on Question:**

*Only operational changes were given credit. Noting a decrease in the adequacy of case estimates is not sufficient without the explanation of what operational change could lead to a decrease in the adequacy of case estimates.*

- The insurer implemented new processes to speed-up the settlement of claims.
- A change to the approval process that decreased case estimates.

- (d) Calculate incurred claims for calendar year 2021.

Reported claims as of Dec. 31, 2021:  $5,739 + 4,903 + 2,532 = 13,174$

(i.e., the 2021 calendar year diagonal in the revised reported claims triangle)

Reported claims as of Dec. 31, 2020:  $3,444 + 1,788 = 5,232$

Ultimate claims as of Dec. 31, 2021:  $13,174 + 38,476 = 51,650$

Ultimate claims as of Dec. 31, 2020:  $5,232 + 17,722 = 22,954$

CY2021 incurred claims:

= Ultimate claims as of Dec. 31, 2021 – Ultimate claims as of Dec. 31, 2020

=  $51,650 - 22,954 = 28,696$

## GIRR Fall 2023 Question 2 (LOs 3j)

### Learning Outcomes:

(3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 22.

### Commentary on Question:

*This question tests the candidate's understanding of evaluating the selecting ultimate claims based on various methods.*

### Solution:

- (a) Evaluate the reasonableness of each of the following methods and datasets for estimating ABC Insurance's ultimate claims:
- (i) Expected method based on paid claims for AY 2017
  - (ii) Bornhuetter Ferguson method based on paid claims for AY 2020
  - (iii) Reported development method for AY 2022
- (i) - The expected method is more often used for immature periods, so possibly not appropriate for AY2017  
- Paid methods are affected by change in settlement, so not recommended  
- No change in case adequacy so paid methods are reasonable  
- Ultimate claims estimate is less than paid claims, so not recommended  
Conclusion: this value is not appropriate because ultimate claims are less than paid claims
- (ii) - BF method is reasonable to use for AY2020  
- Paid methods are affected by the change in settlement, but change occurred in most recent CY so may not affect AY2020 as much for BF  
- No change in case adequacy so paid methods are reasonable  
Conclusion: this method is likely reasonable.
- (iii) - Reported method is not affected by change in settlement  
- No change in case adequacy so reported methods are reasonable  
- Leveraged effect for AY2022:  $6,654,576 / 944,060 = 7.05$ , which is very high and therefore too much uncertainty  
Therefore, this method is likely not appropriate due to the high leverage.

- (b) Recommend ultimate claims from a method and dataset for AY 2021. Justify your recommendation.

Recommendation: Any reported method, or combination of any reported methods, is acceptable.

Justification:

- (i) Reported methods is not affected by change in settlement
  - (ii) Paid methods are affected by change in settlement, so are not recommended
  - (iii) Leveraged effect for AY2021:  $6,159,764 / 1,772,745 = 3.47$ . This is likely a reasonable amount of leverage.
- (c) Evaluate the reasonableness of the AY 2021 ultimate claims estimate using the paid development method after adjustment.
- Paid methods are reasonable with the adjustment for the change in settlement
  - No change in case adequacy so paid methods are reasonable
  - Leveraged effect for AY2021 without adjustment:  $4,747,208 / 841,930 = 5.64$ .
    - Ultimate claims with the adjustment are likely higher, so the leveraged effect would be even higher, therefore not recommended.
  - Conclusion: this value is likely not appropriate because of the leveraged effect.

### GIRR Fall 2023 Question 5 (LOs 3g)

#### Learning Outcomes:

(3g) Estimate ultimate values using the methods cited in (3e).

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 19.

#### Commentary on Question:

*This question tests the candidate's understanding of estimating IBNR using the Cape Cod and Generalized Cape Cod methods.*

#### Solution:

(a) Calculate the IBNR for all accident years using the Cape Cod method.

Accident Year	On-Level Earned Premiums	Reported CDFs	Expected % Reported	Used-Up On-Level Earned Premiums	Reported Claims
2019	15,700	1.100	90.91%	14,273	8,200
2020	15,200	1.500	66.67%	10,133	6,200
2021	15,800	2.200	45.45%	7,182	3,500
2022	16,300	4.000	25.00%	4,075	1,500
Total				35,663	19,400

Accident Year	Claim Trend Factors	Tort Reform Factors	Adjusted Claims
2019	1.0927	1.100	9,856
2020	1.0609	1.100	7,235
2021	1.0300	1.000	3,605
2022	1.0000	1.000	1,500
Total			22,197

Adjusted Expected Claim Ratio =  $22,197 / 35,663 = 62.24\%$

Accident Year	Expected Claims	Expected % Unreported	Expected Unreported Claims	Ultimate Claims	IBNR
2019	8,130	9.09%	739	8,939	739
2020	8,107	33.33%	2,702	8,902	2,702
2021	9,548	54.55%	5,208	8,708	5,208
2022	10,145	75.00%	7,609	9,109	7,609
Total	35,929			35,658	16,258

- (b) Calculate the accident year 2021 IBNR using the Generalized Cape Cod method and a decay factor of 0%.

Generalized Cape Cod with 0 decay factor = Development Method

2021 IBNR:	3,500	Reported claims
	2.20	CDF
	<u>7,700</u>	Ultimate claims
	4,200	IBNR (Ultimate – Reported)

### GIRR Fall 2023 Question 7 (LOs 3h, 3i)

#### Learning Outcomes:

- (3h) Explain the effect of changing conditions on the projection methods cited in (3e).
- (3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 21.

#### Commentary on Question:

*This question tests the candidate's understanding of the appropriateness of various methods of estimating ultimate claims under changing conditions.*

#### Solution:

- (a) Explain whether the Bornhuetter Ferguson method or Cape Cod method is more responsive to a deterioration in claims experience.

In the Bornhuetter Ferguson method, the expected claims are based on an a priori estimate and do not change unless the actuary deliberately makes a change. In the Cape Cod method, the expected claims are a function of the reported claims to date and will reflect the deterioration somewhat. Thus, the Cape Cod method is more responsive to a change in claims experience.

- (b) Describe how this change affects the reported claims development triangle evaluated as of December 31, 2022, assuming the following:
  - (i) The court decision affects only new claims.
  - (ii) The court decision affects new and open claims.
  - (i) The change affecting all new claims would occur on a row (accident year) basis and would be immediate with the effective date as claim adjusters estimate new claims that occurred after the effective date.
  - (ii) The change affecting all open claims would occur on a diagonal (or calendar year) basis and would have more of a phased-in effect as all claim estimates get re-evaluated by the claim department over time.
- (c) Describe why the Cape Cod method could be appropriate when estimating claims under scenario (b)(i) above.

The Cape Cod method allows for tort reform adjustments, so the benefit level change can be treated as tort reform. This would adjust prior accident years to the current benefit level.

- (d) Describe why a Berquist-Sherman data adjustment could be appropriate when estimating claims under scenario (b)(ii) above.

The benefit change on a diagonal is similar to the effect of a case adequacy change. The Berquist-Sherman adjustment uses the latest diagonal to restate prior calendar year data (diagonals) consistent with current benefit level.

**GIRR Fall 2023 Question 8 (LOs 3g, 5c, 5d, 5e)****Learning Outcomes:**

- (3g) Estimate ultimate values using the methods cited in (3e).
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 26.

**Commentary on Question:**

*This question tests the development-based frequency-severity method for estimating ultimate claims.*

**Solution:**

- (a) Explain why a linear trend model may not be appropriate when trend is decreasing.

If the trend is decreasing, as frequency trends often are, then eventually the application of a linear trend will result in a negative value, which cannot occur for GI frequency, severity, or pure premium.

- (b) Recommend an annual claim frequency trend to use for this line of business. Justify your recommendation.

Recommended trend:  $-1.11\%$

Justification: the increase for 2022 might be an anomaly, so exclude that year from the average.

- (c) Calculate projected ultimate claims using the development-based frequency-severity method and your recommended annual claim frequency trend.



Accident Year	Indicated Frequency	Frequency Trend	Trended Frequency	Calculated Ultimate Counts
2016	9.170%	0.935221	8.576%	1,472.57
2017	9.000%	0.945718	8.511%	1,482.03
2018	8.960%	0.956334	8.569%	1,476.93
2019	8.900%	0.967068	8.607%	1,465.38
2020	8.720%	0.977923	8.527%	1,468.39
2021	8.650%	0.988900	8.554%	1,486.51
2022	8.760%	1.000000	8.760%	1,462.54

Average, excluding 2022

All years 8.557%

Latest 3 years 8.563%

Selected frequency at 2022 cost level 8.684%

Accident Year	Ultimate Severity	Severity Trend	Trended Ultimate Severity	Calculated Ultimate Severity	Ultimate Claims
2016	3,750.00	1.418519	5,319.45	3,764.58	5,543,602
2017	3,993.00	1.338226	5,343.53	3,990.45	5,913,955
2018	4,230.00	1.262477	5,340.28	4,229.88	6,247,220
2019	4,489.00	1.191016	5,346.47	4,483.67	6,570,290
2020	4,679.00	1.123600	5,257.32	4,752.69	6,978,783
2021	5,048.00	1.060000	5,350.88	5,037.85	7,488,816
2022	5,409.00	1.000000	5,409.00	5,340.12	7,810,150

Average, excluding 2022

All years 5,326.32

Latest 3 years 5,318.23

Selected frequency at 2022 cost level 5,340.12

46,552,817

**GIRR Fall 2023 Question 10 (LOs 3e, 3g)****Learning Outcomes:**

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 17 and 18.

**Commentary on Question:**

*This question tests the estimation of ultimate claims using the expected method and the Bornhuetter Ferguson method.*

**Solution:**

- (a) Describe two situations where the expected method is most often used when estimating ultimate claims.

Any two of the following four situations:

- For immature experience periods, particularly in the case of long-tail lines of business
- Following the introduction of new GI products when limited or no historical experience is available
- Following entry into a new geographical area for which limited or no historical data exists
- If there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future

- (b) Describe the primary assumption of the expected method.

The primary assumption of the expected method is that actuaries can better project ultimate values based on an a priori estimate than from the experience observed to date.

- (c) Calculate the expected claim ratios for each year at the 2022 cost level using reported claims.

Accident Year	Earned Premium	Ultimate Claims Based on Reported	Claim Trend Factors at 3%	Premium On Level Factors	On-Level Earned Premium	Trended Ultimate Claims Based on Reported	Trended On-Level Claim Ratio
2018	14,750	11,753	1.1255	1.103	16,269	13,228	81.31%
2019	15,895	13,006	1.0927	1.098	17,453	14,212	81.43%
2020	17,400	14,507	1.0609	1.060	18,444	15,390	83.44%
2021	18,705	15,836	1.0300	1.034	19,341	16,311	84.33%
2022	20,010	16,544	1.0000	1.000	20,010	16,544	82.68%

- (d) Calculate the pure premiums for each year at the 2022 cost level using reported claims.

Accident Year	Trended Pure Premium
2018	67.84
2019	69.33
2020	68.40
2021	69.41
2022	70.10

- (e) Calculate the accident year 2021 ultimate claims using the Bornhuetter Ferguson method and:

(i) A selected expected claim ratio of 82% at the 2022 cost level

(ii) A selected pure premium of 69 at the 2022 cost level

Implicit development factor (Ultimate/Reported)	1.412
Expected % undeveloped $(1 - 1/1.412) =$	29.2%

(i) Using expected claim ratio

2021 Earned Premium	18,705
Claim ratio at 2021 cost level $= 82\% \times 1/1.03 =$	0.823
2021 Expected Claims $= 0.823 \times 18,705 =$	15,398
BF estimate of ultimate claims $= 11,213 + 15,398 \times 0.292 =$	15,708

(ii) Using expected pure premium

$$2021 \text{ Expected Claims} = 69 \times 235 / 1.03 = 15,743$$

$$\text{BF estimate of ultimate claims} = 11,213 + 15,743 \times 0.292 = 15,809$$

**GIRR Fall 2023 Question 13 (LOs 3e, 3f, 3g)****Learning Outcomes:**

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 15.

**Commentary on Question:**

*This question tests the candidate's understanding of estimating ultimate claims using the development method.*

**Solution:**

- (a) Identify a potential problem with your colleague's recommendation.

Due to the large claims in 2019 and 2021, the 12-24 and 24-36 age-to-age factors are too high, therefore ultimate claims would be overstated.

- (b) Describe an alternative approach to your colleague's recommendation.

Recommend adjusting for the large claims (i.e., removing them from the development factor analysis).

*{alternatively, could use average of the factors that exclude the AY2021 12-24 and AY2019 24-36 factors.}*

- (c) Estimate total ultimate claims based on the development method and your alternative from part (b).

Construction of right triangle that excludes large claims:

Accident Year	12	24	Reported Claims (000)					
			36	48	60	72	84	
2016	1,826	2,742	2,948	3,174	3,239	3,248	3,248	
2017	2,296	3,656	3,928	4,230	4,458	4,506		
2018	3,064	4,932	5,465	6,104	6,373			
2019	2,327	3,675	<b>4,022</b>	<b>4,624</b>				
2020	2,691	4,495	4,924					
2021	2,497	<b>4,025</b>						
2022	3,740							

e.g., AY2021 @ 24 months: 4,025 = 5,025 – 1,000

Development factors:

	12-24	24-36	36-48	48-60	60-72	72-84
	1.5016	1.0751	1.0767	1.0205	1.0028	1.0000
	1.5923	1.0744	1.0769	1.0539	1.0108	
	1.6097	1.1081	1.1169	1.0441		
	1.5793	1.0944	1.1497			
	1.6704	1.0954				
	1.6119					
Simple average	1.5942	1.0895	1.1050	1.0395	1.0068	1.0000
Volume-weighted average	1.6002	1.0916	1.1081	1.0416	1.0074	1.0000
Selected:						
- Age-to-age:	1.5942	1.0895	1.1050	1.0395	1.0068	1.0000
- Age-to-ultimate	2.0086	1.2599	1.1564	1.0465	1.0068	1.0000

AY	Reported Claims	Age-to-Ultimate Factors	Ultimate Claims
2016	3,248	1.0000	3,248
2017	4,506	1.0000	4,506
2018	6,373	1.0068	6,416
2019	4,624	1.0465	4,839
2020	4,924	1.1564	5,694
2021	4,025	1.2599	5,071
2022	3,740	2.0086	7,512
Ultimate claims, excluding large claims			37,287
Ultimate claims, including large claims:			38,787

- (d) Describe how you would adjust for the large claims when estimating ultimate claims based on the paid development method for this line of business.

The ultimate values would need to include the case estimates for large claims.

### GIRR Fall 2023 Question 14 (LOs 3c, 3d)

#### Learning Outcomes:

- (3c) Identify the types of development triangles that can be used for investigative testing.
- (3d) Analyze development triangles for investigative testing.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 14 and 20.

#### Commentary on Question:

*This question tests investigative analysis of various development triangles.*

#### Solution:

- (a) Identify two other examples of actions that could result in shifts in a reported claim pattern.

Any two of the following are acceptable:

- new procedures for the payment of claims such as direct deposit to a claimant's bank account instead of issuance of checks
- changes in the distribution of policy limits purchased by insureds; (or offered by the company)
- changes in the distribution of deductibles purchased by insureds; (or offered by the company)
- changes in the use of partial settlements or ex gratia payments
- shifts in the attitude toward defense of questionable claim files
- change in the definition of reported claims

- (b) Verify your colleague's assumption.

First, need to determine if the case adequacy was strengthened in calendar year (CY) 2021: Analyze change in average case estimates.

Accident Year	Average Case Estimates					
	12	24	36	48	60	72
2017	5,141	6,014	6,700	7,120	8,155	28,343
2018	5,670	6,456	6,931	8,510	8,670	
2019	5,821	6,742	8,372	9,033		
2020	6,158	7,923	8,828			
2021	7,588	8,303				
2022	8,159					



Accident Year	Change in Average Case Estimates					
	12	24	36	48	60	72
2017-2018	10.3%	7.3%	3.4%	<b>19.5%</b>	6.3%	
2018-2019	2.7%	4.4%	<b>20.8%</b>	6.1%		
2019-2020	5.8%	<b>17.5%</b>	5.4%			
2020-2021	<b>23.2%</b>	4.8%				
2021-2022	7.5%					

Analysis: There appears to have been strengthening in CY 2021 to support colleague's recommendation.

(c) Critique your colleague's recommendation.

Any one of the following is acceptable:

- even though the change was in CY 2021, could still use the most recent diagonal to reflect the most recent data point
- common practice to use most recent diagonal, so could use that
- even though it is more common to use most recent diagonal, using CY 2021 still acceptable

(d) Construct a reported claims triangle adjusted for the change in case adequacy, basing the adjustments on the calendar year 2022 diagonal.

Accident Year	Adjusted Average Case Estimates					
	12	24	36	48	60	72
2017	6,011	6,503	7,349	7,994	8,157	28,343
2018	6,390	6,912	7,812	8,497	8,670	
2019	6,792	7,348	8,305	9,033		
2020	7,220	7,811	8,828			
2021	7,675	8,303				
2022	8,159					

Accident Year	Adjusted Reported Claims					
	12	24	36	48	60	72
2017	3,602,331	4,775,496	5,805,686	6,784,638	7,372,495	7,702,277
2018	3,848,406	5,063,164	6,447,061	7,430,019	8,060,259	
2019	4,198,283	5,567,827	6,905,880	8,051,684		
2020	4,622,567	5,976,718	7,664,425			
2021	4,890,709	6,611,842				
2022	5,320,155					

## GIRR Spring 2024 Question 2 (LOs 3e, 3g)

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 15.

### Commentary on Question:

*This question tests the development method for estimating ultimate claims where there is seasonality. In addition, it tests the candidate's understanding of expected paid and reported claims for an interim period between actuarial analyses as well as tail factors.*

### Solution:

- (a) Calculate the ultimate claims for accident year 2023 using the development method. Justify your selections.

Accident Half-Year	Age-to-Age Factors							
	6-12	12-18	18-24	24-30	30-36	36-42	42-48	48-Ult
2020-1	1.022	1.049	1.007	1.010	1.005	1.001	1.000	
2020-2	1.053	1.024	1.014	1.011	1.006	1.001		
2021-1	1.027	1.043	1.008	1.012	1.006			
2021-2	1.046	1.028	1.016	1.010				
2022-1	1.025	1.037	1.007					
2022-2	1.055	1.009						
2023-1	1.018							
AHY-1 Avg	1.023	1.043	1.007	1.011				
AHY-2 Avg	1.051	1.020	1.015	1.010				
All years Avg	1.035	1.032	1.010	1.011	1.006	1.001	1.000	
AHY-1 Selected Factors:								
Age-to-age	1.023	1.043	1.007	1.011	1.006	1.001	1.000	1.000
Age-Ult	1.093	1.069	1.025	1.017	1.007	1.001	1.000	1.000
AHY-2 Selected Factors:								
Age-to-age	1.051	1.020	1.015	1.011	1.006	1.001	1.000	1.000
Age-Ult	1.108	1.054	1.033	1.017	1.007	1.001	1.000	1.000

Accident Year 2023:

Accident Half-Year	Reported Claims	Age-Ultimate Factor	Ultimate Claims
2023-1	2,283,355	1.069	2,439,958
2023-2	2,451,221	1.108	2,715,998
Total			5,155,957

- (b) Calculate the accident year 2023 expected reported claims from December 31, 2023 to June 30, 2024.

Accident Half-Year	Reported Claims	Incremental Dev. Factor	Expected Claims
2023-1	2,283,355	0.043	97,784
2023-2	2,451,221	0.051	125,554
Total			223,337

- (c) Describe one disadvantage of the Bondy method.

Its primary disadvantage is the potential to greatly underestimate the remaining development for long-tail lines.

- (d) State one advantage and one disadvantage of Boor's algebraic method.

Advantage: It is based entirely on data in triangles, so no need for additional data.

Disadvantage: Need reliable estimates of ultimate claims for most mature periods, and that is not always available.

### GIRR Spring 2024 Question 4 (LOs 3e, 3f)

#### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 17.

#### Commentary on Question:

*This question tests the candidate's understanding of the expected method of estimating ultimate claims.*

#### Solution:

- (a) Provide one reason why the expected method is preferred over the development method when estimating ultimate claims for a new line of business.

The expected method is preferred when there is limited or no historical experience available.

- (b) Explain why a pure premium approach is preferred over an expected claim ratio approach when developing expected claims for self-insurers.

A self-insurer does not typically have earned premiums in the same way that an insurer does.

- (c) Provide two reasons why the trended on-level claim ratio for accident year 2023 might be excluded when selecting the 2023 cost level expected claim ratio.

Any two of the following are acceptable:

- By definition, a priori is “presupposed by experience” and “formed or conceived beforehand” and therefore would exclude 2023
- Accident year 2023 might provide significantly different results than the rest of the experience period
- When the cumulative development factors are highly leveraged for the latest years’ experience

- (d) Explain the steps you would follow to apply the expected method to estimate ultimate salvage received for a collision line of business.
- 1 Create a triangle of ratios of salvage received to paid claims
  - 2 Develop the ratios to ultimate values using a development method approach
  - 3 Selected an ultimate salvage ratio
  - 4 Multiply the ultimate salvage ratio to the ultimate claims to estimate ultimate salvage

## GIRR Spring 2024 Question 7 (LOs 1d, 3e, 3f, 3g)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 15, 17, 18.

### Commentary on Question:

*This question tests the candidate's understanding of the development method, the Bornhuetter Ferguson method, and the Benktander method of estimating IBNR.*

### Solution:

- (a) Calculate the IBNR for each AY as of December 31, 2023 using:
  - (i) the Development method,
  - (ii) the Bornhuetter Ferguson method, and
  - (iii) two iterations of the Benktander method.
- (i)

AY	Reported Claims	CDF	Development Method Ultimate Claims	Development Method IBNR
2021	5,613,235	1.2556	7,047,851	1,434,616
2022	4,682,692	1.5958	7,472,822	2,790,130
2023	3,554,432	2.3060	8,196,475	4,642,043

(ii)

AY	Historical Earned Premiums	Claim Trend Factor @6.1%	Premium On-Level Factor	Claim Ratio at Each AY Cost Level	Expected Claims Based on Claim Ratio
2021	10,119,409	1.1257	1.034	69.81%	7,064,127
2022	10,552,425	1.0610	1.020	73.06%	7,709,934
2023	10,850,455	1.0000	1.000	76.00%	8,246,346

AY	Ultimate Claims BF Method	BF Method IBNR
2021	7,051,164	1,437,929
2022	7,561,352	2,878,660
2023	8,224,719	4,670,287

(iii)

AY	BK Method (Ultimate Claims)		BK Method (IBNR)	
	Iteration 1	Iteration 2	Iteration 1	Iteration 2
2021	7,048,525	7,047,988	1,435,290	1,434,753
2022	7,505,877	7,485,164	2,823,185	2,802,472
2023	8,212,471	8,205,534	4,658,039	4,651,102

- (b) Explain if this business is performing better or worse than expected for AY 2023 using the methods above.

2023 claim ratio for each method:

Development method 75.5%

BF method 75.8%

BK 2<sup>nd</sup> iteration 75.6%

Expected claim ratio: 76.0%

Since all claim ratios are lower than the expected claim ratio, all are performing better than expected.

- (c) Identify one other weakness of the Benktander method.

There is not a clear sense as to the improvement in the estimation of ultimate claims from additional iterations.

## GIRR Spring 2024 Question 9 (LOs 3e, 3j)

### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 19 and 22.

### Commentary on Question:

*This question tests the candidate's understanding of the Cape Cod method for estimating ultimate claims.*

### Solution:

- (a) Describe two differences between the Cape Cod method and the Generalized Cape Cod method for estimating ultimate claims.

Any two of the following are acceptable:

- The Generalized Cape Cod (GCC) method uses a judgmentally selected decay factor to assign different weights to each year in the experience period.
- In the Cape Cod method, expected claims for each year in the experience period are derived from the same expected claim ratio. In the GCC method, a distinct expected claim ratio is obtained for each year in the experience period.
- The GCC method takes into account the relationship between the variance and trending, which if not considered could cause excessive weight to be given to years that are out of date.

- (b) Describe two major differences between the Bornhuetter Ferguson and Cape Cod methods.

Any two of the following are acceptable:

- The difference between the two methods is in the determination of the expected value input.
- The derivation of the expected value for the Cape Cod method is prescribed by the method itself and is not an independent a priori estimate as in the Bornhuetter Ferguson method.
- Whereas the expected value used with the Bornhuetter Ferguson method can incorporate significant professional judgment, the expected value used in the Cape Cod method is determined by a formula; professional judgment does not typically play a role.



- (c) Describe two advantages that blended methods provide when evaluating and selecting estimates of ultimate claims.

Any two of the following are acceptable:

- The Bornhuetter Ferguson and Cape Cod methods are easy to apply and relatively easy to explain to non-actuarial users.
- Blending expected claims with actual claims is intuitively appealing; as a year matures, more weight will be given to actual claims instead of expected claims
- Because future claim emergence is tied to exposures instead of historical claim experience, external information can be readily incorporated into the analysis. For example, rate level changes and trend can be used in blended methods. Even changes in the distribution of business, such as shifts in exposures by class, territory, or limit, could be factored into the analysis.

## GIRR Spring 2024 Question 10 (LOs 3d, 3e, 3g)

### Learning Outcomes:

- (3d) Analyze development triangles for investigative testing.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 14 and 20.

### Commentary on Question:

*This question tests the candidate's understanding of Berquist-Sherman adjustments when there has been a change in claim settlement patterns.*

### Solution:

- (a) Perform two diagnostic tests to confirm that there was a change in claim settlement patterns in 2023.

Ratio of Paid Claims to Reported Claims:

AY	12	24	36	48	60	72
2018	0.579	0.756	0.866	0.939	0.996	<b>1.000</b>
2019	0.601	0.766	0.864	0.938	<b>0.999</b>	
2020	0.590	0.764	0.863	<b>0.977</b>		
2021	0.591	0.758	<b>0.942</b>			
2022	0.569	<b>0.829</b>				
2023	<b>0.628</b>					

- if there has been a speed up in claim settlement in 2023, expect the latest diagonal to show noticeable increase in the ratios
- there is evidence of a speed up in this case

Ratio of Closed Counts to Reported Counts:

AY	12	24	36	48	60	72
2018	0.648	0.757	0.824	0.869	0.905	<b>1.000</b>
2019	0.655	0.760	0.822	0.869	<b>0.981</b>	
2020	0.651	0.764	0.822	<b>0.949</b>		
2021	0.652	0.758	<b>0.923</b>			
2022	0.646	<b>0.834</b>				
2023	<b>0.696</b>					

- if there has been a speed up in claim settlement in 2023, expect the latest diagonal to show noticeable increase in the ratios
- there is evidence of a speed up in this case

- (b) Perform one diagnostic test to determine whether there was a change in case adequacy in 2023.

Change in average case:

Accident Year	Average Case					
	12	24	36	48	60	72
2018	3,442	3,435	2,880	1,880	185	-
2019	3,652	3,553	2,998	1,966	194	
2020	3,861	3,756	3,231	2,075		
2021	4,062	4,005	3,378			
2022	4,340	4,205				
2023	4,508					

Accident Year	Change in Average Case					
	12	24	36	48	60	72
2018-2019	6.1%	3.4%	4.1%	4.6%	5.2%	
2019-2020	5.7%	5.7%	7.8%	5.5%		
2020-2021	5.2%	6.6%	4.6%			
2021-2022	6.9%	5.0%				
2022-2023	3.9%					
Average:	5.5%	5.2%	5.5%	5.1%	5.2%	

- evidence of change in case adequacy would show up as a change in one of the diagonals significantly different than 5%
- there is no evidence of a significant change in case adequacy in this situation

- (c) Calculate the adjusted paid claims triangle.

**Commentary on Question:**

*Solution needs to use ultimate counts from reported only because reported counts are not affected by the settlement change but closed counts are.*

**Ratio of Closed Counts to Ultimate Counts:**

AY	12	24	36	48	60	72	Ultimate Counts (from reported)
2018	0.387	0.580	0.721	0.815	0.888	0.991	1,485
2019	0.395	0.578	0.702	0.810	0.962		1,492
2020	0.388	0.575	0.711	0.888			1,499
2021	0.395	0.564	0.798				1,503
2022	0.398	0.630					1,474
2023	0.420						1,491
Selected	0.420	0.630	0.798	0.888	0.962	0.991	

**Adjusted Closed Counts:**

AY	12	24	36	48	60	72	Ultimate Counts (from reported)
2018	623	935	1,185	1,319	1,429	1,471	1,485
2019	626	939	1,190	1,325	1,436		1,492
2020	629	944	1,196	1,331			1,499
2021	631	946	1,199				1,503
2022	619	928					1,474
2023	626						1,491

**Adjusted Paid Claims:**

AY	12	24	36	48	60	72
2018	2,743,316	4,113,672	5,212,419	5,801,704	6,288,756	6,472,400
2019	2,756,247	4,133,063	5,236,989	5,829,052	6,318,400	
2020	2,769,179	4,152,454	5,261,560	5,856,400		
2021	2,776,568	4,163,534	5,275,600			
2022	2,722,995	4,083,200				
2023	2,754,400					

- (d) Describe an alternative approach that could be used for determining ratios of paid claims to cumulative closed counts.

Instead of a fixed ratio that does not vary by accident year and development period, determine a mathematical curve to approximate the relationship between cumulative closed counts and cumulative paid claims.

- (e) Describe a possible problem with the alternative approach identified in part (d).

In some situations, a mathematical relationship may not even exist.

- (f) Critique your colleague's recommendation.

This line of business did not have a change in case adequacy, so an adjustment for that is not needed. However, adjusting for both a change in case adequacy and a change in claim settlement should not significantly affect the results, as adjusting for the change in case adequacy should have little, if any, effect.

**GIRR Spring 2024 Question 11 (LOs 3h, 3i)****Learning Outcomes:**

- (3h) Explain the effect of changing conditions on the projection methods cited in (e)
- (3i) Assess the appropriateness of the projection methods cited in (e) in varying circumstances

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 21.

**Commentary on Question:**

*This question tests the candidate's understanding of ultimate claims when conditions are changing.*

**Solution:**

- (a) Describe how this reform would affect the reported claims development triangle evaluated as of December 31, 2023, assuming the following:
  - (i) The reform affected only new claims.
  - (ii) The reform affected new and open claims.
  - (i) The change affecting all new claims would occur on a row (accident year) basis and would be immediate with the effective date as claim adjusters estimate new claims that occurred after the effective date.
  - (ii) The change affecting all open claims would occur on a diagonal (or calendar year) basis and would have more of a phased-in effect as all claim estimates get re-evaluated by the claim department over time.

- (b) Describe why the expected method could be well-suited to estimate claims under scenario (a)(i) above.

The expected method allows for tort reform adjustments, so would adjust prior accident years to the current benefit level.

- (c) Describe why a Berquist-Sherman data adjustment could be well-suited to estimate claims under scenario (a)(ii) above.

The benefit change on a diagonal is similar to the effect of a case adequacy change. The Berquist-Sherman adjustment uses the latest diagonal to restate prior calendar year data (diagonals) consistent with current benefit level.

- (d) Describe whether this reform would affect indemnity, ALAE, ULAE, or some combination.

Likely effect is change in indemnity and no change to ALAE and ULAE.

- (e) Describe whether this reform would affect paid data, reported data, or both paid and reported data.

Change in claims affects both paid and reported data.

## GIRR Spring 2024 Question 12 (LOs 3g, 5c, 5d)

### Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 26.

### Commentary on Question:

*This question tests the development-based frequency-severity method for estimating ultimate claims.*

### Solution:

- (a) Describe two options to consider when experience is not fully credible for trending.

Any two of the following are acceptable:

- Rely on industry data for a similar line of business in a similar jurisdiction.
- Combine the insurer's experience in specific states or provinces with the experience of a larger region.
- Combine the insurer's experience with that of other insurers in a group under common ownership.

- (b) Recommend the annual claim frequency trend to use for this line of business. Justify your recommendation.

Accident Year	Earned Exposures	Ultimate Counts	Indicated Frequency	Annual Change in Frequency
2018	16,451	1,485	9.027%	
2019	16,557	1,492	9.011%	-0.172%
2020	16,815	1,499	8.915%	-1.072%
2021	16,915	1,503	8.886%	-0.326%
2022	17,147	1,474	8.596%	-3.256%
2023	17,461	1,491	8.539%	-0.666%
Average:				-1.098%
Exponential fitted:				-1.200%
Selected:				-1.200%

Justification: use all years due to erratic changes.



- (c) Calculate the ultimate counts using the development-based frequency-severity method with your selected frequency trend from part (b). Justify any selections.

Accident Year	Freq trend @- 1.2%	Trended Frequency	F-S Ultimate Counts
2018	0.941431	8.498%	1,498
2019	0.952864	8.587%	1,489
2020	0.964436	8.598%	1,494
2021	0.976148	8.674%	1,485
2022	0.988002	8.493%	1,487
2023	1.000000	8.539%	1,496

Average trended frequency at 2023 cost level excluding 2023

all years 8.570%

excluding hi-lo 8.561%

Selected frequency @ 2023 level: 8.570%

Justification for selected frequency: No significant trend; no significant outliers

- (d) State one other influence that the trend rate should also recognize.

Social influences, (i.e., the impact on insurance costs of societal changes such as changes in claim consciousness, court practices, and legal precedents, as well as in other noneconomic factors).

- (e) Calculate the ultimate claims using the development-based frequency-severity method. Justify any selections.

Accident Year	Severity Trend @5.0%	Trended Reported Severity	F-S Ultimate Severity	F-S Ultimate Claims
2018	1.276282	6,022.77	4,966.93	7,438,122
2019	1.215506	6,493.23	5,215.28	7,766,041
2020	1.157625	6,503.54	5,476.05	8,182,046
2021	1.102500	6,457.34	5,749.85	8,538,549
2022	1.050000	6,219.15	6,037.34	8,979,399
2023	1.000000	6,168.00	6,339.21	9,485,828

Average trended severity at 2023 cost level excluding 2023

all years 6,339.21

excluding hi-lo 6,389.91

Selected severity @ 2023 level: 6,339.21

Justification for selected severity: No significant trend; no significant outliers.

**GIRR Fall 2024 Question 2 (LOs 3e, 3f, 3g)****Learning Outcomes:**

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 15 and 17.

**Commentary on Question:**

*This question tests the estimation of ultimate claims using the expected method.*

**Solution:**

- (a) Describe one advantage of using the pure premium approach rather than the claim ratio approach when using the expected method.

Any one of the following is acceptable:

- No adjustment is required for premium rate changes.
- It may be possible to select a pure premium exposure base that is a leading indicator of claims experience.
- It may be possible to choose an exposure base that requires no adjustments.

- (b) Provide one reason why the expected method might be preferred over the development method in this scenario for analyzing accident year 2023 claims.

The age-to-ultimate factor at 12 months is very large (or highly leveraged) implying a very immature accident year. The expected method is better for immature years.

- (c) Calculate the expected claims for accident year 2023 using the expected method with the following approaches:
  - (i) Claim ratio
  - (ii) Pure premium

Accident Year	Project Ultimate Claims from Paid Development Method	Claim Ratio Trend Factor	Trended On- Level Claim Ratios	Trended Pure Premiums
2017	39,794,820	1.194	85.39%	601.90
2018	38,874,654	1.159	81.68%	575.94
2019	40,100,870	1.126	82.42%	580.87
2020	42,901,092	1.093	88.20%	621.93
2021	42,491,743	1.061	80.60%	579.88
2022	43,661,907	1.030	77.78%	588.86
2023	46,092,453	1.000		
Average:			82.68%	591.56

	(i)	(ii)
Expected claims for accident year 2023:	46,988,824	44,408,679

- (d) Estimate accident year 2023 claims expected to be paid between December 31, 2023 and December 31, 2024 using your results from part (c)(ii).

% Paid @ 12 months:	15.7%
% Paid @ 24 months:	26.1%
% Paid between 12 to 24 months:	10.4%
AY 2023 expected paid between 12 to 24 months:	4,619,299

**GIRR Fall 2024 Question 7 (LOs 3h, 3i, 3j)****Learning Outcomes:**

- (3h) Explain the effect of changing conditions on the projection methods cited in (3e).
- (3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 21 and 22.

**Commentary on Question:**

*This question tests the candidate's understanding of changing conditions on different projection methods.*

**Solution:**

- (a) Provide two reasons why actuaries use multiple methods to estimate ultimate claims.

Any two of the following are acceptable:

- where required by actuarial standards
- each method has different underlying assumptions, none of which are usually perfectly true
- to allow the results of different methods to be compared
- to better reflect the complexities of the business being modelled
- to identify sensitivity to the underlying assumptions

- (b) Provide two areas in which an actuary can exercise professional judgement in estimating ultimate claims, other than the selection of methods.

Any two of the following are acceptable:

- reviewing reasonableness of results
- choosing assumptions (e.g., trend)
- assessing reasonableness of information provided
- evaluating estimates from different methods
- final selection of estimates
- determining relevance of information
- what relevant information is sufficient
- whether to supplement available information
- whether to assess the needs of users
- the level and detail of documentation

- (c) Explain how effective each of the following projection methods will be in responding to the recent changes at XYZ:

(i) Paid development method

change in claim ratio	responsive
volume change	responsive
change in reporting pattern	will distort factors, so not responsive
overall	should be mostly responsive

(ii) Expected method

change in claim ratio	not responsive
volume change	responsive
change in reporting pattern	not responsive, but results will be fine if expected ultimate claim ratio is adjusted.
overall	likely not responsive

(iii) Reported Bornhuetter Ferguson method

change in claim ratio	not responsive
volume change	responsive
change in reporting pattern	not responsive
overall	likely not responsive

**GIRR Fall 2024 Question 10 (LOs 2a, 3e, 3g)****Learning Outcomes:**

- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 16 and 22.

**Commentary on Question:**

*This question tests the candidate's understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies. The candidate also needs to understand earned premiums adjusted to current rate level.*

**Solution:**

- (a) Provide an example of another line of business that often has a long lag between the occurrence date and the report date.

Any of the following are acceptable:

- Errors & Omissions
- Medical malpractice
- Any type of bodily injury liability only coverage

- (b) Provide an example of a line of business where claim files are commonly reopened.

Any of the following are acceptable:

- Workers compensation
- Any type of bodily injury liability only coverage

- (c) Construct a revised cumulative paid claims triangle adjusted for the legislative change.

**Commentary on Question:**

*It is necessary to start with incremental paid claims, as the reform affects claims paid after a certain date and not the cumulative of all claims paid to that date.*

Accident Year	Incremental Paid Claims					
	12	24	36	48	60	72
2018	1,518,006	1,766,528	1,553,804	1,308,213	798,483	204,638
2019	1,582,770	1,969,314	1,523,378	1,064,621	903,118	
2020	1,573,601	2,034,384	1,315,593	1,284,989		
2021	1,608,502	1,795,820	1,492,737			
2022	1,448,977	1,890,519				
2023	1,791,306					

Accident Year	Adjustment Factors for Tort Reform					
	12	24	36	48	60	72
2018	0.80	0.80	0.80	0.90	1.00	1.00
2019	0.80	0.80	0.90	1.00	1.00	
2020	0.80	0.90	1.00	1.00		
2021	0.90	1.00	1.00			
2022	1.00	1.00				
2023	1.00					

Accident Year	Adjusted Incremental Paid Claims = Incremental Paid Claims × Adjustment Factors					
	12	24	36	48	60	72
2018	1,214,405	1,413,222	1,243,043	1,177,392	798,483	204,638
2019	1,266,216	1,575,451	1,371,040	1,064,621	903,118	
2020	1,258,881	1,830,946	1,315,593	1,284,989		
2021	1,447,652	1,795,820	1,492,737			
2022	1,448,977	1,890,519				
2023	1,791,306					

Accident Year	Adjusted Cumulative Paid Claims					
	12	24	36	48	60	72
2018	1,214,405	2,627,627	3,870,670	5,048,062	5,846,545	6,051,183
2019	1,266,216	2,841,667	4,212,707	5,277,328	6,180,446	
2020	1,258,881	3,089,826	4,405,419	5,690,408		
2021	1,447,652	3,243,472	4,736,209			
2022	1,448,977	3,339,496				
2023	1,791,306					

- (d) Verify the projected ultimate claims for accident years 2024 and 2025.

Accident Year	Projected Frequency	Projected Counts	Projected Severity	Projected Ultimate Claims
2024	10.57%	1,120.23	6,342.50	7,105,054
2025	10.54%	1,128.46	6,818.19	7,694,043

e.g.,  $10.57\% = 10.6\% \times (1 - 0.3\%)$   
 $1,120.23 = 10.57\% \times 10,600$   
 $6,342.50 = 5,900 \times (1 + 7.5\%)$   
 $7,105,054 = 1,120.23 \times 6,342.50$

- (e) Calculate the claims expected to be paid in calendar years 2024 and 2025, using the results from part (c).

**Commentary on Question:**

*Age-to-ultimate factors are calculated by dividing the given ultimate claims by cumulative paid claims to date (i.e., the latest diagonal).*

Accident Year	Cumulative Paid Claims						Projected Ultimate Claims
	12	24	36	48	60	72	
2018	1,518,006	3,284,534	4,838,338	6,146,551	6,945,034	7,149,672	7,149,672
2019	1,582,770	3,552,084	5,075,462	6,140,083	7,043,201	7,289,724	7,289,724
2020	1,573,601	3,607,985	4,923,578	6,208,567	7,231,724	7,484,846	7,484,846
2021	1,608,502	3,404,322	4,897,059	6,280,054	7,314,992	7,571,028	7,571,028
2022	1,448,977	3,339,496	4,873,746	6,250,157	7,280,168	7,534,985	7,534,985
2023	1,791,306	4,087,339	5,965,167	7,649,810	8,910,480	9,222,361	9,222,361
2024	1,380,051	3,148,951	4,595,660	5,893,535	6,864,776	7,105,054	7,105,054
2025	1,494,453	3,409,990	4,976,627	6,382,093	7,433,847	7,694,043	7,694,043

	12-24	24-36	36-48	48-60	60-72	72-84
Age-to-age:	2.282	1.459	1.282	1.165	1.035	1.000
Age-to-ult:	5.148	2.256	1.546	1.206	1.035	1.000

e.g.,  $5.148 = 9,222,361 / 1,791,306$   
 $2.282 = 5.148 / 2.256$



Accident Year	Incremental Paid Claims					
	12	24	36	48	60	72
2018	1,518,006	1,766,528	1,553,804	1,308,213	798,483	204,638
2019	1,582,770	1,969,314	1,523,378	1,064,621	903,118	246,523
2020	1,573,601	2,034,384	1,315,593	1,284,989	1,023,157	253,122
2021	1,608,502	1,795,820	1,492,737	1,382,995	1,034,938	256,036
2022	1,448,977	1,890,519	1,534,250	1,376,411	1,030,011	254,817
2023	1,791,306	2,296,033	1,877,828	1,684,643	1,260,671	311,881
2024	1,380,051	1,768,900	1,446,709	1,297,876	971,241	240,278
2025	1,494,453	1,915,537	1,566,637	1,405,466	1,051,754	260,196

CY2024 paid claims: 7,863,009

CY2025 paid claims: 7,805,652

## GIRR Fall 2024 Question 12 (LOs 3f, 3g, 3j)

### Learning Outcomes:

- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 15, 18, 19, and 22.

Actuarial Standards of Practice, Actuarial Standards Board of the American Academy of Actuaries, No. 25, Credibility Procedures, 2013.

### Commentary on Question:

*This question tests the candidate's understanding of estimating ultimate claims using the development method, the Bornhuetter Ferguson method, and the Cape Cod method, including adjustments because of a large claim.*

### Solution:

- (a) Calculate projected ultimate claims for all accident years using the development method.

Accident Year (AY)	Reported Claims, Adjusted for Large Claim				
	12	24	36	48	60
2019	540,061	575,731	648,087	683,622	702,734
2020	554,275	591,019	665,056	701,405	
2021	567,907	<b>606,134</b>	<b>681,837</b>		
2022	581,936	621,002			
2023	596,836				

AY	Reported Claims Age-to-Age factors				
	12-24	24-36	36-48	48-60	60-ultimate
2019	1.066	1.126	1.055	1.028	
2020	1.066	1.125	1.055		
2021	1.067	1.125			
2022	1.067				
Simple All Years Avg.	1.067	1.125	1.055	1.028	
Vol. Wtd. Avg.	1.067	1.125	1.055	1.028	
Selected	1.067	1.125	1.055	1.028	1.028
Age-to-ult. (CDF)	1.338	1.254	1.115	1.057	1.028

AY	Reported Claims Excluding Large Claim	Age-to-Ultimate Development Factors	Ultimate Claims Excluding Large Claim	Ultimate Claims Including Large Claim
2019	702,734	1.028	722,380	722,380
2020	701,405	1.057	741,172	741,172
2021	681,837	1.115	759,935	<b>809,935</b>
2022	621,002	1.254	778,838	778,838
2023	596,836	1.338	798,464	798,464
Total	3,303,814		3,800,789	3,850,789

- (b) Critique the appropriateness of selecting the development method for this line of business.

The method seems appropriate after adjusting for large loss because development factors are relatively stable.

- (c) Calculate projected ultimate claims for all accident years using the Bornhuetter Ferguson method.

**Commentary on Question:**

*The claim ratio to use is the given expected claim ratio based on industry data of 65% for all accident years.*

AY	% Claims Unreported	Reported Claims	Earned Premiums	Claim Ratio	BF Method Ultimate Claims
2019	0.027	702,734	1,000,000	65%	720,412
2020	0.054	701,405	1,040,000	65%	737,675
2021	0.103	731,837	1,082,000	65%	804,115
2022	0.203	621,002	1,125,000	65%	769,194
2023	0.253	596,836	1,170,000	65%	788,877
Total		3,353,814	5,417,000		3,820,273

e.g.,  $804,115 = 731,837 + 0.103 \times 1,082,000$

- (d) Critique the appropriateness of selecting the Bornhuetter Ferguson method for this line of business.

The BF method may not be appropriate as management is uncertain that expected claim ratio from industry data is representative of this book of business. Alternatively, the BF method correctly reflects large claim.

- (e) Calculate projected ultimate claims for all accident years using the Cape Cod method.

	Rate Level	Percent Premium Earned in Each CY at Rate Level				
	Index	2019	2020	2021	2022	2023
	1.00000	100.00%	100.00%	100.00%	50.00%	-
	0.95000	-	-	-	50.00%	100.00%
	Total	100.00%	100.00%	100.00%	100.00%	100.00%
Average rate level in each CY:		1.0000	1.0000	1.0000	0.9750	0.9500
On-level factors:		0.9500	0.9500	0.9500	0.9744	1.0000

	(1)	(2)	(3) = (1)(2)	(4)	(5) = 1/(4)	(6) = (3)(5)
	Earned	On-level	On-Level	Reported	Expected	Used Up On-
AY	Premiums	Factors	Earned	CDF	% Claims	Level Earned
			Premiums		Reported	Premiums
2019	1,000,000	0.9500	950,000	1.028	97.3%	924,163
2020	1,040,000	0.9500	988,000	1.057	94.6%	934,990
2021	1,082,000	0.9500	1,027,900	1.115	89.7%	922,263
2022	1,125,000	0.9744	1,096,154	1.254	79.7%	874,012
2023	1,170,000	1.0000	1,170,000	1.338	74.7%	874,552
Total	5,417,000		5,232,054			4,529,980

	(7)	(8)	(9)=(7)(8)	(10) = 0.6995×(3)/(8)
AY	Reported Claims Excluding Large Claim	Claim Trend Factors	Adjusted Claims	Expected Claims
2019	702,734	0.9224	648,179	720,426
2020	701,405	0.9412	660,157	734,258
2021	681,837	0.9604	654,836	748,632
2022	621,002	0.9800	608,582	782,376
2023	596,836	1.0000	596,836	818,381
Total	3,303,814		3,168,590	3,804,073

Adjusted expected claim ratio =  $3,168,590 / 4,529,980 = 0.6995$

AY	(11) = 1 – (5) Expected % Unreported	(12) = (10)(11) Expected Unreported Claims	(13) = (7) + (12) Ultimate Claims Including Large Claim
2019	2.7%	19,593	722,327
2020	5.4%	39,396	740,801
2021	10.3%	76,937	<b>808,774</b>
2022	20.3%	158,553	779,555
2023	25.3%	206,657	803,493
Total		501,136	3,854,950

Note: for AY2021 = 681,837 + 76,937 + 50,000 = 808,774

- (f) Critique the appropriateness of selecting the Cape Cod method for this line of business.

Any of the following is acceptable:

- Method is appropriate for newer lines of business (immature experience periods)
- Method is better than BF because expected claim ratio is experience-based
- Method is appropriate because it explicitly adjusts for trend
- Method is appropriate but usually used for longer-tailed lines

### GIRR Fall 2024 Question 13 (LOs 3e, 3g)

#### Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 20.

#### Commentary on Question:

*This question tests the candidate's understanding of frequency-severity method for estimating ultimate claims.*

#### Solution:

- (a) Recommend an annual claim frequency trend.

Accident Year (AY)	Frequency	Change from Court Ruling	Court Ruling Adjusted Frequency	Annual Trend
2017	0.04263	1.05	0.0448	
2018	0.04284	1.05	0.0450	0.51%
2019	0.04305	1.05	0.0452	0.48%
2020	0.04327	1.05	0.0454	0.51%
2021	0.04349	1.05	0.0457	0.50%
2022	0.04370	1.05	0.0459	0.49%
2023	0.04611	1.00	0.0461	0.49%
All years average				0.50%
Average excluding 2023				0.50%
Recommended Trend				0.50%

- (b) Calculate the projected ultimate claims for all accident years using the development-based frequency-severity method.

AY	Frequency	Severity	Frequency Trend @0.5%	Severity Trend @4.0%	Change from Court Ruling	Trended Frequency	Trended Severity	Ultimate Claims
2017	0.04263	28,747	1.0304	1.2653	1.05	0.0461	36,374.31	5,011,553
2018	0.04284	29,953	1.0253	1.2167	1.05	0.0461	36,442.61	5,451,088
2019	0.04305	31,137	1.0202	1.1699	1.05	0.0461	36,425.31	5,732,349
2020	0.04327	32,388	1.0151	1.1249	1.05	0.0461	36,432.17	6,219,950
2021	0.04349	33,732	1.0100	1.0816	1.05	0.0461	36,484.55	6,502,557
2022	0.04370	35,093	1.0050	1.0400	1.05	0.0461	36,496.90	7,148,701
2023	0.04611	36,175	1.0000	1.0000	1.00	0.0461	36,175.18	7,909,279
All years average:						0.0461	36,404.44	
Average excluding 2023:						0.0461	36,442.64	
Selected:						0.0461	36,442.64	

e.g., Ultimate claims for AY2022:

$$7,148,701 = (0.0461)(4,668)(36,442.64)/(1.0050 \times 1.0400 \times 1.05)$$

- (c) Describe how to calculate the *proportion of closed counts* triangle when using the frequency-severity closure method.

Instead of using development triangles with cumulative data, the closure method relies on triangles of incremental counts and claims. The proportion is the percent of counts closed of the remaining open counts.

- (d) Describe how to calculate the triangle of *disposal ratios* when using the Berquist-Sherman adjustment for changing settlement rates.

The proportion closed counts is equal to the ratio of the counts closed at each maturity age to the counts remaining as of the prior maturity age. The disposal ratio is the percent closed of ultimate counts.

## GI 101 – LEARNING OBJECTIVE 4

### 4. Topic: Financial Reporting

The candidate will understand financial reporting of claim liabilities with respect to unpaid unallocated loss adjustment expenses.



## GIRR Fall 2020 Question 15 (LOs 1d, 1i, 4b, and 4c)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (1i) Describe how and why data are segregated and aggregate.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 4, and 23.

### Commentary on Question:

*This question tests the candidate's understanding of unpaid ALAE and unpaid ULAE.*

### Solution:

- (a) Describe one way a reinsurer might assess the reasonableness of an estimate of unpaid ULAE.

Either one of the following is acceptable:

- Consider the reinsurer from a run-off perspective.
- Estimate the number of years to run-off the claim liabilities and the estimated cost per year.

- (b) Recommend one of the two approaches from the table above to use in estimating unpaid ULAE. Justify your recommendation.

Either one of the following is acceptable:

- Kittel refinement because it incorporates reported claims which reduces distortion from exposure growth.
- Kittel refinement because the classical paid-to-paid overstates the ULAE ratio (numerator) when exposure is growing.

- (c) Estimate unpaid ULAE as of December 31, 2019 using the approach you selected in part (b).

Ratio of ULAE to claims (Kittel refinement): average of 2018 and 2019 = 7.20%

For the ULAE ratio selection, use the average of the most recent 2 years to reflect the growing exposure base.

$$\begin{aligned}
 \text{Unpaid ULAE} &= (\text{ULAE ratio} \times \text{pure IBNR}) + [\text{ULAE ratio} \times \text{multiplier} \times (\text{case estimates} + \\
 &\quad \text{development on case estimates})] \\
 &= (0.072 \times 1,600,000 \times 0.2) + 0.072 \times 0.75 \times (3,510,000 + 0.8 \times 1,600,000) \\
 &= 281,700.
 \end{aligned}$$

- (d) Determine calendar year 2019 incurred ULAE.

$$\begin{aligned}\text{CY 2019 incurred ULAE} &= \text{2019 paid ULAE} + \text{Change in outstanding in 2019} \\ &= 880,000 + 281,700 - 270,000 = 891,700.\end{aligned}$$

- (e) Critique your colleague's recommendation.

Any two of the following are acceptable:

- ALAE shouldn't be evaluated on a calendar year basis because ALAE reflect development over time.
- ALAE is more directly related to the size of a claim and should be evaluated like claim experience.
- ALAE are directly attributable to claims and should be analyzed similar to claims while ULAE are general and not assigned to claims.
- Accident year detail is recorded for ALAE which allows a deeper analysis.
- ALAE reporting requires accident year detail.

**GIRR Spring 2021 Question 3 (LOs 3g, 4a, 4b, 4c, 5b, 5c, 5d, 5e)****Learning Outcomes:**

- (3g) Estimate ultimate values using the methods cited in (3e).
- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.
- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16, 23, and 26.

**Commentary on Question:**

*This question tests the candidate's understanding of claims trend analysis and selection as well as estimating ultimate claims using the development-based frequency-severity method. This question also tests the candidate's understanding of estimating unpaid ULAE using the classical paid-to-paid method with the Mango-Allen smoothing adjustment.*

**Solution:**

- (a) Explain why this may happen when using the development-based frequency-severity method.  
  
For the development-based frequency-severity method, the severity would be developed to an ultimate value separately, which might not equal the developed ultimate claims divided by the developed ultimate counts.
- (b) Recommend a claim frequency at the accident year 2020 cost level. Justify your recommendation.

Accident Year	(1) Earned Exposures	(2) Projected Ultimate Counts	(3) Based on Development Method Claims	(4) Severity
2015	25,200	2,088	9,028,629	4,324
2016	26,700	2,194	9,779,132	4,458
2017	25,300	2,063	9,477,056	4,594
2018	24,500	1,983	9,375,491	4,733
2019	23,900	1,933	8,987,726	4,724
2020	24,200	1,709	7,810,473	4,749
Total	149,800	11,970	54,458,507	

Accident Year	(5) = (2)/(1) Indicated Frequency	(6) <sub>t</sub> = (5) <sub>t</sub> / (5) <sub>t-1</sub> Year-to-year Change	(7) Frequency Trend @ -0.78%	(8) = (5)(7) Trended Frequency
2015	0.082857		0.961604	0.079676
2016	0.082172	-0.008266	0.969163	0.079638
2017	0.081542	-0.007676	0.976782	0.079648
2018	0.080939	-0.007392	0.984461	0.079681
2019	0.080879	-0.000743	0.992200	0.080248
2020	0.070620	-0.126842	1.000000	0.070620

Frequency trend selection: (column 6): Average of 2016-2018 = -0.78%  
(2019 & 2020 are outliers)

Recommended 2020 cost level frequency (column 8): average excluding 2020  
= 0.0798  
(all other years are stable and 2020 is an outlier)

- (c) Calculate ultimate claims using the development-based frequency-severity method and the recommended claim frequency from part (b).

Accident Year	(4) Indicated Severity	(9) <sub>t</sub> = (4) <sub>t</sub> / (4) <sub>t-1</sub> Year-to-year Change	(10) Severity Trend @ 3.06%	(11) = (4)(10) Trended Severity
2015	4,324		1.162655	5,027.32
2016	4,458	0.030990	1.128134	5,029.22
2017	4,594	0.030507	1.094638	5,028.77
2018	4,733	0.030257	1.062136	5,027.09
2019	4,724	-0.001902	1.030600	4,868.55
2020	4,749	0.005292	1.000000	4,749.00

Severity trend selection: (column 9): Average of 2016-2018 = 3.06%  
(2019 & 2020 are outliers)

Recommended 2020 cost level frequency (column 11): Average of 2016-2018  
= 5,028.10  
(2019 & 2020 are outliers)

	(12) = (1)×0.0798/(7)	(13) = 5.028.10/(10)	(14) = (12)(13)
Accident Year	Ultimate Counts	Ultimate Severity	Ultimate Claims
2015	2,090.83	4,324.67	9,042,137
2016	2,198.00	4,457.01	9,796,505
2017	2,066.50	4,593.39	9,492,263
2018	1,985.55	4,733.95	9,399,499
2019	1,921.82	4,878.81	9,376,179
2020	1,930.76	5,028.10	9,708,066
Total			56,814,649

- (d) Calculate the expected claims paid for calendar years 2017 through 2020.

	12	24	36	48	60	72
Cumulative paid claims development factors by maturity age (CDF)	11.245	2.017	1.228	1.063	1.010	1.000
% Cumulative Paid (1/CDF)	8.9%	49.6%	81.4%	94.1%	99.0%	100.0%
% Incremental Paid	8.9%	40.7%	31.9%	12.6%	4.9%	1.0%

e.g., % incremental paid at 24 months = 40.7% = 49.6% – 8.9%

Accident Year	Ultimate Claims from Part (c)	Projected in Calendar Year			
		2017	2018	2019	2020
2015	9,042,137	2,880,340	1,142,940	446,367	89,526
2016	9,796,505	3,985,781	3,120,642	1,238,293	483,607
2017	9,492,263	844,132	<b>3,861,998</b>	3,023,727	1,199,837
2018	9,399,499		835,883	3,824,256	2,994,177
2019	9,376,179			833,809	3,814,768
2020	9,708,066				863,323
Total		7,710,253	8,961,462	9,366,451	9,445,237

e.g., Accident year 2017 expected paid claims in calendar year 2018

$$= 0.407 \times 9,492,263 = \mathbf{3,861,998}$$

- (e) Recommend a ULAE ratio using the classical paid-to-paid method with the Mango-Allen smoothing adjustment. Justify your recommendation.

Calendar Year	Paid ULAE	Expected Claims from Part (d)	Ratio ULAE to Claims
2017	738,905	7,710,253	9.58%
2018	851,350	8,961,462	9.50%
2019	883,245	9,366,451	9.43%
2020	879,224	9,445,237	9.31%
Total	3,352,724	35,483,403	9.45%

Recommended ULAE ratio = total of all years = 9.45%, as there are no significant outliers.

**Commentary on Question:**

*Candidates could also recommend a ULAE ratio that considered the downward trend.*

- (f) Calculate the unpaid ULAE.

$$\begin{aligned} \text{Calculated unpaid ULAE} &= 9.45\% \times 4,351,459 \times (1 - 0.25) + 9.45\% \times 11,117,813 \\ &= 1,358,858. \end{aligned}$$

### GIRR Spring 2021 Question 4 (LOs 3i, 4a)

#### Learning Outcomes:

- (3i) Assess the appropriateness of the projection methods cited in (e) in varying circumstances.
- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 22 and 23.

#### Commentary on Question:

*This question tests the candidate's ability to evaluate and justify selections of ultimate values based on various methods. In addition, this question tests the candidate's understanding of estimating unpaid unallocated loss adjustment expenses.*

#### Solution:

- (a) Explain why the development method may not be appropriate for estimating unpaid claims for this coverage.

#### Commentary on Question:

*Any two of the following are acceptable.*

- The development method is not appropriate for immature experience periods (i.e., the data is less than five years).
  - The development method is not appropriate when limited or no historical experience is available.
  - The development method is not appropriate when conditions are changing (i.e., tort reform will distort development).
- (b) Recommend an appropriate method for estimating unpaid claims for this coverage. Justify your recommendation.

#### Commentary on Question:

*Although the Cape Cod method is the most appropriate recommendation, other methods are acceptable if the justification is appropriate for the circumstances. Justification should include at least three explanations.*

The Cape Cod method is recommended. Justification:

- Good for immature experience periods
- Good when limited or no historical experience is available
- Good for long-tailed coverages
- Allows for explicit trend adjustment

- Allows for explicit tort reform adjustment
  - Industry development (experience) can be used to supplement company development (which is limited to five years)
  - Cape Cod method uses actual experience
  - Cape Cod method adds stability
  - Can be applied to paid and/or reported data
- (c) Explain why the classical paid-to-paid method may not be appropriate for estimating unpaid ULAE for this coverage.

**Commentary on Question:**

*Any two of the following are acceptable.*

- Tort reform may change the relationship between payments for ULAE and payments for claims.
  - Experience period has not reached a steady-state (only five years but coverage is long-tailed).
  - Classical paid-to-paid method is not appropriate if significant changes in exposure are occurring (growth in this case).
- (d) Recommend an appropriate method for estimating unpaid ULAE for this coverage. Justify your recommendation.

**Commentary on Question:**

*Although the Mango & Allen smoothing adjustment is the most appropriate recommendation, other methods are acceptable if the justification is appropriate for the circumstances. Justification should include at least two explanations.*

The Mango & Allen smoothing adjustment is recommended. Justification:

- Appropriate for long-tail coverages
- Appropriate for changing exposure volume
- Appropriate for relatively new insurer/coverage



## GIRR Fall 2021 Question 18 (LOs 1d, 3f, 3g, 4a, and 4b)

### Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).
- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 15, and 23.

### Commentary on Question:

*This question tests the candidate's understanding of selecting development factors and estimating a tail factor using Boor's algebraic method. It also tests the calculation of unpaid ULAE using the classical paid-to-paid method, as well as an understanding of the Kittel refinement to the classical paid-to-paid method and the Mango and Allen smoothing adjustment.*

### Solution:

- (a) Select age-to-age development factors for all periods excluding the tail factor. Justify your selections.

Adjusted Age-to-Age Development Factors Excluding the Large Claim							
Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96
2013	3.191	1.675	1.352	1.197	1.122	1.091	1.063
2014	3.058	1.673	1.305	1.201	1.141	1.094	
2015	2.846	1.691	1.334	1.218	1.131		
2016	2.858	1.700	1.321	1.198			
2017	2.727	1.726	1.332				
2018	2.732	1.729					
2019	2.716						
All Years Avg.	2.876	1.699	1.329	1.204	1.131	1.092	1.063
Avg. excl. high&low	2.844	1.698	1.329				
Volume Wtd. Avg.	2.861	1.699	1.329	1.204	1.131	1.092	
5 Year Avg.	2.776	1.704					
3 Year Avg.	2.725	1.718	1.329	1.206			
Selected	2.725	1.718	1.329	1.206	1.131	1.093	1.063

Justification for selection: Selected 3 years average to recognize trend down the columns.

Notes: Adjusted factors for large claim:

$$AY_{2017}, 24-36 = 1.726 = (1,082 - 150)/540$$

$$AY2017, 36-48 = 1.332 = (1,391 - 150)/(1,082 - 150)$$

$$Volume\ Wtd.\ Avg.,\ 24-36: 1.699 = (866 + 875 + 876 + 923 + 1,082 + 968 - 150)/(517 + 523 + 518 + 543 + 540 + 560)$$

$$Volume\ Wtd.\ Avg.,\ 36-48: 1.329 = (1,171 + 1,142 + 1,169 + 1,219 + 1,391 - 150)/(866 + 875 + 876 + 923 + 1,082 - 150)$$

- (b) Derive a paid tail factor using Boor's algebraic method.

		(1)	(2)	(3) = (1)(2)	(4)	(5) = (4)/(3)
				Estimated Claims		
Accident Year	Actual Paid	Paid Development Factors		96	Ultimate Claims from Reported Development Method	Implied Tail Factor
		72-84	84-96			
2013	1,824			1,824	1,975	1.083
2014	1,712		1.063	1,820	1,974	1.085
2015	1,610	1.093	1.063	1,870	2,032	1.087
Selected:						1.085

- (c) Calculate ultimate claims using the paid development method and the tail factor of 1.072.

		(1)	(2)	(3)	(4) = (2)(3)
				Age-to-Ultimate Development Factors	Ultimate Claims
Accident Year	Actual Paid	Paid Claims Excluding Large Claim			
2013	1,824	1,824		1.072	1,955
2014	1,712	1,712		1.140	1,951
2015	1,610	1,610		1.245	2,004
2016	1,460	1,460		1.408	2,056
2017	1,391	1,241		1.698	2,257
2018	968	968		2.257	2,184
2019	573	573		3.877	2,222
2020	224	224		10.566	2,367
Total	9,762	9,612			16,997

e.g.,  $1,241 = 1,391 - 150$   
 $1.698 = 1.206 \times 1.131 \times 1.093 \times 1.063 \times 1.072$

- (d) Calculate the unpaid ULAE as of December 31, 2020 using the classical paid-to-paid method and a multiplier of 50%.

$$\text{Case outstanding} = 14,660 - 9,762 = 4,898$$

$$\text{IBNR} = 17,065 - 14,660 = 2,405$$

$$\text{Unpaid ULAE} = 0.08 \times 2,405 + 0.8 \times 0.5 \times 4,898 = 388.$$

- (e) Describe the Kittel refinement to the classical paid-to-paid method and the weakness it is designed to address.

Kittel method derives ULAE ratio by comparing paid ULAE to average of paid and reported claims (rather than paid to paid ratio used in Classical method).

Kittel's change addresses some of the distortion that can arise with increasing (changing) exposures because reported claims react quicker to exposure changes.

- (f) Describe the Mango and Allen smoothing adjustment.

The Mango and Allen Smoothing Adjustment uses expected claim in place of actual claims.

## GIRR Spring 2022 Question 12 (LOs 4b, 4c)

### Learning Outcomes:

- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 23.

### Commentary on Question:

*This question tests the candidate's understanding of estimating unpaid ULAE using the classical paid-to-paid method, as well as the Kittel refinement with the Mango and Allen smoothing adjustment.*

### Solution:

- (a) Estimate unpaid ULAE as of December 31, 2021, using the classical paid-to-paid method with a simple four-year average of historical experience, and a pure IBNR refinement.

Calendar Year	Paid ULAE	Actual Paid Claims	Ratio of Paid ULAE to Paid Claims
2018	16,172,450	176,261,530	9.18%
2019	16,807,540	184,338,130 <sup>(1)</sup>	9.12%
2020	17,831,120	187,853,340	9.49%
2021	19,284,360	197,358,720	9.77%
Total	70,095,470	745,811,720	9.39%

Note: (1):  $184,338,130 = 195,338,130 - 11,000,000$  (adjustment for the large closed claim)

$$\text{Unpaid ULAE} = 9.39\% \times 26,803,900 + 9.39\% \times (1 - 0.3) \times (95,171,300 + 43,591,100) = 11,636,593$$

- (b) Estimate unpaid ULAE as of December 31, 2021 using the Kittel refinement with the Mango and Allen smoothing adjustment, a simple four-year average of historical experience, and a pure IBNR refinement.

Calendar Year	(1) Paid ULAE	(2) Expected Paid Claims	(3) Expected Reported Claims	(4) = (1) / Avg[(2),(3)] ULAE Ratio
2018	16,172,450	181,712,920	179,693,890	8.95%
2019	16,807,540	188,100,130	190,637,250	8.88%
2020	17,831,120	195,680,570	206,174,180	8.87%
2021	19,284,360	205,582,000	222,977,380	9.00%
Total	70,095,470	771,075,620	799,482,700	8.92%

$$\text{Unpaid ULAE} = 8.92\% \times 26,803,900 + 8.92\% \times (1 - 0.3) \times (95,171,300 + 43,591,100) = 11,061,217$$

- (c) Critique the appropriateness of each result from (a) and (b).

Since exposures are growing, the paid-to-paid ratio in part (b) will overstate ULAE, because the paid ULAE in ratio numerator will react to exposure growth faster than paid claims in ratio denominator.

The Kittel adjustment helps adjust for exposure growth and the Mango and Allen smoothing adjustment is useful for exposure growth.

The Mango and Allen smoothing adjustment is good for volatile lines (or lines with large claims, or lines with low-frequency, high-severity).

**GIRR Fall 2022 Question 12 (LOs 4a, 4b, 4c)****Learning Outcomes:**

- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 23.

**Commentary on Question:**

*This question tests the candidate's understanding of estimating unpaid ULAE using the Wendy Johnson count-based method.*

**Solution:**

- (a) Describe one such special study.

The Wendy Johnson method relies on selected weights required for maintaining, opening and closing a claim. In practice the weights would typically come from special studies (e.g., workload studies, time studies) from an insurer's claims department.

- (b) Recommend an average ULAE per weighted count. Justify your recommendation.

**Commentary on Question:**

*Newly reported, open, and closed counts can be determined directly or by rearranging the triangles by year end instead of by development.*

**Directly determining calendar year (CY) counts (e.g., 2018):**

CY 2018 newly reported counts =  $1,122 + 32 + 26 = 1,180$

CY 2018 closed counts =  $694 + 263 + 87 = 1,044$

Cumulative reported counts to end of 2018 =  $1,033 + 1,081 + 1,122 + 28 + 32 + 26 = 3,322$

Cumulative closed counts to end of 2018 =  $636 + 650 + 694 + 210 + 263 + 87 = 2,540$

CY 2018 open counts =  $3,322 - 2,540 = 782$

***Rearranging data triangles:***

**Reported Counts by Year End**

CY	2016-12-31	2017-12-31	2018-12-31	2019-12-31	2020-12-31	2021-12-31
2016	1,033	28	26	1	0	0
2017		1,081	32	16	0	0
2018			1,122	59	8	0
2019				828	41	25
2020					799	34
2021						806
incremental	1,033	1,109	1,180	904	848	865
cumulative	1,033	2,142	3,322	4,226	5,074	5,939

**Closed Counts by Year End**

CY	2016-12-31	2017-12-31	2018-12-31	2019-12-31	2020-12-31	2021-12-31
2016	636	210	87	21	4	1
2017		650	263	64	10	0
2018			694	274	71	12
2019				521	222	69
2020					511	210
2021						530
incremental	636	860	1,044	880	818	822
cumulative	636	1,496	2,540	3,420	4,238	5,060

Open counts

	(1)	(2)	(3)	(4)	(5)
	Counts				
Calendar Year	Paid ULAE	Newly Reported	Open	Closed	Weighted Total
2018	718,960	1,180	782	1,044	933.90
2019	738,400	904	806	880	845.30
2020	746,800	848	836	818	835.40
2021	787,600	865	879	822	864.10

e.g., (5) for 2018:  $933.90 = 0.25 \times 1,180 + 0.55 \times 782 + 0.20 \times 1,044$

Calendar Year	(6) = (1) / (5) Avg ULAE Per Weighted Count	(7) Trending Period in Years	(8) = 1.02 <sup>(7)</sup> Trend to 2022 at 2.0%	(9) = (6)(8) Avg ULAE Trended to 2022
2018	769.85	4	1.0824	833.31
2019	873.54	3	1.0612	927.00
2020	893.94	2	1.0404	930.06
2021	911.47	1	1.0200	929.70

Recommended average ULAE per weighted count = 928.92

Rationale: 2018 appears to be an outlier, so use average of 2019 to 2021

- (c) Calculate estimated unpaid ULAE as of December 31, 2021.

Calendar Year	(10) Newly Reported	(11) Open	(12) Closed	(13) Weighted Total
	Counts			
2022	208	559	528	465.05
2023	69	278	350	240.15
2024	5	133	150	104.40
2025	0	25	108	35.35
2026	0	0	25	5.00

e.g., for 2018:

$$(11): 559 = 879 + 208 - 528$$

$$(13): 465.05 = 0.25 \times 208 + 0.55 \times 559 + 0.20 \times 528$$

Calendar Year	(14) Trending Period in Years	(15) = 1.02 <sup>(14)</sup> Trend from 2022 at 2.0%	(16) = 928.92 × (15) Trended Average ULAE	(17) = (13)(16) Estimated Unpaid ULAE
2022	0	1.0000	928.92	431,994
2023	1	1.0200	947.50	227,542
2024	2	1.0404	966.45	100,897
2025	3	1.0612	985.78	34,847
2026	4	1.0824	1,005.49	5,027
Total				800,308

e.g., for 2018: (16) = 928.92 × (15)



## GIRR Spring 2023 Question 9 (LOs 4a, 4b, 4c)

### Learning Outcomes:

- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 23.

### Commentary on Question:

*This question tests the candidate's understanding of estimating unpaid ULAE.*

### Solution:

- (a) Explain why the classical paid-to-paid method may not be appropriate for estimating unpaid ULAE in this case.

The significant inflation in this case will cause a relatively higher increase in the calendar paid ULAE than the calendar paid claims, since inflation can more quickly affect the underlying costs of ULAE, including the salaries of claims adjusters, rent, and utilities. This will overstate the paid ULAE to paid claims ratio, thus overestimating the unpaid ULAE.

- (b) Calculate the ULAE ratio for each year using the Mango and Allen smoothing adjustment based on paid and reported claims data.

### Commentary on Question:

*It is recommended to solve this part of the question by displaying the details of how the expected reported claims for CY 2021 & 2022 are determined to ensure that no report years are excluded.*

First need to calculate the CY2021 & CY2022 expected reported claims:

	Maturity Age in months				
	12	24	36	48	60
Reported CDF	2.306	1.479	1.137	1.023	1.000
% Cumulative Reported	43.4%	67.6%	88.0%	97.8%	100.0%
% Incremental Reported	43.4%	24.2%	20.3%	9.8%	2.2%

e.g.,  $43.4\% = 1/2.306$

Report Year	Selected Ultimate Claims	Expected Reported Claims in Calendar Years	
		2021	2022
2017	8,297,960	186,562	
2018	9,230,643	904,692	207,532
2019	10,390,684	2,113,205	1,018,387
2020	11,357,111	2,753,886	2,309,752
2021	12,811,927	5,555,909	3,106,651
2022	14,531,428		6,301,573
Total		11,514,254	12,943,895

e.g., for Report Year 2019:

$$2,133,205 = 10,390,684 \times 20.3\%$$

$$1,018,387 = 10,390,684 \times 9.8\%$$

Calendar Year	Paid ULAE	Expected Claims		Ratio of Paid ULAE to Average of Paid and Reported Claims
		Paid	Reported	
2019	725,000	8,950,624	9,323,021	7.93%
2020	825,176	9,921,833	10,304,355	8.16%
2021	935,423	11,058,159	11,514,254	8.29%
2022	1,062,610	12,393,344	12,943,895	8.39%
Total	3,548,209	42,323,960	44,085,525	8.21%

$$\text{e.g., } 8.29\% = 935,423 / ((11,058,159 + 11,514,254) / 2)$$

- (c) Recommend a ULAE ratio to use for this line of business. Justify your recommendation.

Recommended ratio: 8.28%

Justification: using the average of the latest 3 years to reflect the increasing trend and remove the 2019 low outlier value.

- (d) Calculate unpaid ULAE as of December 31, 2022 using the recommended ratio from part (c).

$$\begin{aligned} \text{Calculated unpaid ULAE} &= 8.28\% \times 4,965,557 \times (1 - 0.40) + 8.28\% \times 13,974,912 \\ &= 1,403,552 \end{aligned}$$

## GIRR Fall 2023 Question 4 (LOs 4b)

### Learning Outcomes:

- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 23.

### Commentary on Question:

*This question tests the Wendy Johnson count-based method to calculate unallocated loss adjustment expenses.*

### Solution:

- (a) Verify that the forecasted incremental reported count for AY 2021 at 36 months is 95.

### Commentary on Question:

*The correct formula for determining the AY 2021 cumulative counts to 36 months is  $(820 - 536)(0.770 - 0.654) / (1 - 0.654) + 536 = 631.2$ . Using the formula  $0.770 \times 820 = 631.4$  is not correct, even though the answer is close to 631.2.*

AY 2021 cumulative counts to 24 months: 536

AY 2021 cumulative counts to 36 months:

$$(820 - 536)(0.770 - 0.654) / (1 - 0.654) + 536 = 631$$

AY 2021 incremental reported counts at 36 months =  $631 - 536 = 95$

- (b) Estimate unpaid ULAE as of December 31, 2022 using a simple three-year average of historical experience.

Calendar Year	Weighted Counts	Avg ULAE Per Wtd. Count	Trended Period (Yrs.)	Trend to 2023 @2%	Avg ULAE Trended to 2023
2020	789.10	811.05	3	1.06121	860.69
2021	806.35	837.11	2	1.04040	870.92
2022	813.90	850.23	1	1.02000	867.23
Selected average ULAE per weighted count at 2023 level					866.28

Projection of Unpaid ULAE Using Count-Based ULAE Method:

Calendar Year	Counts				Trending Period in Years	Trend from 2023 at 3.0%	Trended Average ULAE	Estimated Unpaid ULAE
	Newly Reported	Open	Closed	Weighted Total				
2023	416	558	674	534.10	0	1.0000	866.28	462,682
2024	282	336	504	339.30	1	1.0300	892.27	302,748
2025	190	173	353	195.25	2	1.0609	919.04	179,443
2026	124	72	225	100.30	3	1.0927	946.61	94,945
2027	75	22	125	45.55	4	1.1255	975.01	44,412
2028	25	0	47	10.95	5	1.1593	1,004.26	10,997
Total								1,095,226

### GIRR Spring 2024 Question 8 (LOs 4a, 4b, 4c)

#### Learning Outcomes:

- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 23.

#### Commentary on Question:

*This question also tests the candidate's understanding of estimating unpaid ULAE using the classical paid-to-paid method with the Mango-Allen smoothing adjustment.*

#### Solution:

- (a) Provide another reason why the classical paid-to-paid method overstates unpaid ULAE, even in a steady state environment.

For most insurance portfolios, the average size of claims remaining open at the valuation date is greater than the average size of claims opened, and claims closed over the prior calendar year. This is the case even where there is no inflation and no growth in the exposure base.

- (b) Describe two situations where the Mango and Allen smoothing adjustment is particularly valuable in producing a more reasonable estimate of unpaid ULAE.

Any two of the following are acceptable:

- Long-tail lines of business
- Changing exposure volume
- When large claims result in significant distortions to the calendar year paid and reported claims from year to year
- Where there are few claims paid or reported per year with great variability in the average claim value (i.e., low frequency and highly variable severity)
- Relatively new insurer who does not have a significant volume of credible paid or reported claims
- Sparse or volatile data

- (c) Calculate the ULAE ratio for each year using the Mango and Allen smoothing adjustment based on paid and reported claim data.

		Maturity Age in Months				
		12	24	36	48	60
Reported CDF		3.505	2.020	1.765	1.420	1.165
% Cumulative Reported		28.5%	49.5%	56.7%	70.4%	85.8%
% Incremental Reported		28.5%	21.0%	7.2%	13.8%	15.4%

Accident Year	Selected Ultimate Claims	Projected in Calendar Year				
		2019	2020	2021	2022	2023
<i>Expected Reported Claims</i>						
2019	5,331,195	1,521,026	1,118,180	381,302	733,856	821,770
2020	4,622,596		1,318,858	969,556	330,621	636,315
2021	5,116,924			1,459,893	1,073,238	365,976
2022	5,524,846				1,576,276	1,158,797
2023	6,060,412					1,729,076
Total	26,655,973	1,521,026	2,437,037	2,810,751	3,713,990	4,711,934

*ULAE Ratio based on Mango and Allen Smoothing Adjustment:*

Calendar Year	Paid ULAE	Expected Claims		Ratio ULAE to Claims
		Paid	Reported	Average of Paid and Reported
2019	278,480	991,462	1,521,026	22.2%
2020	323,800	1,170,742	2,437,037	18.0%
2021	369,200	1,573,118	2,810,751	16.8%
2022	448,080	2,346,706	3,713,990	14.8%
2023	675,994	3,297,712	4,711,934	16.9%
Total	1,817,074	8,388,278	13,673,712	16.5%

- (d) Recommend a ULAE ratio to use for this line of business. Justify your recommendation.

Recommendation is to use the average of all years of 16.5%. The justification is to use the average as this is a new line of business and there probably isn't yet the stability in the numbers.

- (e) Calculate unpaid ULAE as of December 31, 2023 using the recommended ratio from part (d).

% of ULAE opening a claim file: 30%

$$\text{IBNR} = 5,750,000 - 3,250,000 = 2,500,000$$

$$\text{Unpaid ULAE} = 16.5\% \times 3,250,000 \times (1 - 0.30) + 16.5\% \times 2,500,000 = 786,559$$

**GIRR Fall 2024 Question 9 (LOs 4a, 4b)****Learning Outcomes:**

- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 23.

**Commentary on Question:**

*This question tests the candidate's understanding of estimating unpaid ULAE using the classical paid-to-paid method, as well as the Wendy Johnson count-based method.*

**Solution:**

- (a) Describe why unallocated loss adjustment expenses (ULAE) are usually analyzed on a calendar year basis.

ULAE is not assigned to specific claims, therefore, no accident year can be assigned.

- (b) Describe a weakness of the classical paid-to-paid method that the Kittel refinement is intended to address.

ULAE are not simply associated with the payment of claims, but other activity as well.

- (c) Describe these two major drawbacks.

Ratio-based methods do not recognize that the amount of ULAE does not depend solely on the magnitude of the total claims in the portfolio.

ULAE from ratio-based methods will fluctuate in response to changes in the estimates of unpaid claims.

- (d) Estimate unpaid ULAE as of December 31, 2023 using the classical paid-to-paid method.



Calendar Year	Paid Claims	Paid ULAE	Paid-to-Paid
			ULAE Ratio
2021	30,400,000	1,489,600	4.9%
2022	31,698,113	1,680,000	5.3%
2023	28,000,000	1,596,000	5.7%
	90,098,113	4,765,600	5.3%

Selected: 5.5%

	As of Dec. 31, 2023	Expense Multiplier	Unpaid ULAE
Case Reserves	19,507,585	75%	804,688
IBNER	7,861,668	75%	324,294
IBNYR	4,812,040	100%	264,662
	41,919,318		1,393,644

- (e) Demonstrate that the projected open counts for calendar years 2024, 2025, and 2026 are calculated correctly based on newly reported claims and closed claims.

Calendar Year	Projected Open Counts
2024	1,044
2025	323
2026	0

e.g.,  $1,044 = 1,402 + 1,067 - 1,425$

- (f) Estimate unpaid ULAE as of December 31, 2023 using the Wendy Johnson method.

Calendar Year	Paid ULAE	Counts			Weighted Total	Avg ULAE Per Weighted Count
		Newly Reported	Open	Closed		
Weights:		30%	50%	20%		
2021	1,489,600	2,325	1,336	2,370	1,840	810
2022	1,680,000	2,550	1,391	2,495	1,960	857
2023	1,596,000	2,528	1,402	2,517	1,963	813

Selected Average ULAE per Weighted Count: 827

e.g.,  $810 = 1,489,600 / 1,840$

Calendar Year	Counts				Trending Period in Years	Prospective Trend Factor	Trended Avg. ULAE	Estimated Unpaid ULAE
	Newly Reported	Open	Closed	Weighted Total				
2024	1,067	1,044	1,425	1,127	1	1.0200	843	950,475
2025	122	323	843	367	2	1.0404	860	315,420
2026	-	-	323	65	3	1.0612	877	56,678
Total								1,322,572

e.g.,  $843 = 827 \times 1.02$   
 $950,475 = 843 \times 1,127$

## GI 101 – LEARNING OBJECTIVE 5

### 5. Topic: Trending

The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

### GIRR Fall 2020 Question 16 (LOs 2d, 5b, 5e, 6g)

#### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 26, 27, and 32.

#### Commentary on Question:

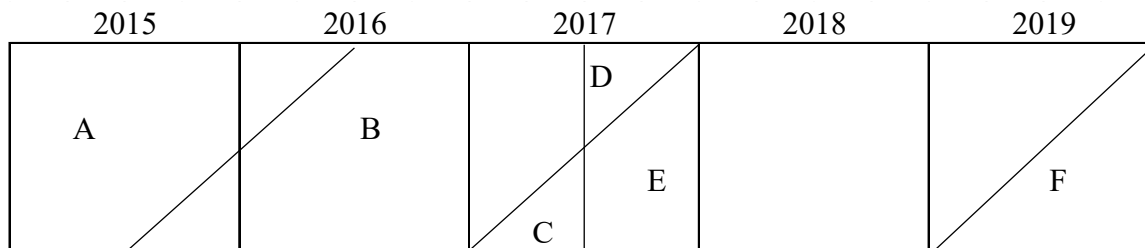
*This question tests basic ratemaking using a claim ratio approach. The candidate also needs to understand earned premiums adjusted to current rate level for ratemaking purposes.*

#### Solution:

- (a) Calculate premium on-level factors for accident years 2015-2019 to use for ratemaking purposes.

#### Commentary on Question:

*The diagram is helpful to solve the question but not required for credit.*



		<u>Percent Premium Earned in Each CY at Rate</u>				
	<u>Rate Level</u>	<u>Level</u>				
Level	Index	2015	2016	2017	2018	2019
A	1.00000	87.5%	12.5%			
B	1.08000	12.5%	87.5%	37.5%		
C	1.18800			12.5%		
D	0.86400			12.5%		
E	0.95040			37.5%	100.0%	50.0%
F	0.99792					50.0%
Total		100.0%	100.0%	100.0%	100.0%	100.0%

Average rate level in each CY:      1.01000   1.07000   1.01790   0.95040   0.97416

On-level factors for ratemaking:      0.9880   0.9326   0.9804   1.0500   1.0244

e.g.,     $0.97416 = 0.5 \times 0.95040 + 0.5 \times 0.99792$   
 $1.2044 = 0.99792 / 0.97416$

- (b) Calculate the trended on-level claim ratios for each accident year.

Trend from the average accident date in each AY (i.e., July 1) to the average accident date in future rating period.

Average accident date in future rating period: November 1, 2021

Accident Year	Trending Period in Years	Earned Premiums	<u>Premium Adj. Factors</u>		Trended Earned Prem. at Current Rate Level
			Trend at 1.00%	On-Level	
2015	6.333	11,755,570	1.0650	0.9880	12,370,486
2016	5.333	11,864,520	1.0545	0.9326	11,668,350
2017	4.333	12,406,530	1.0441	0.9804	12,698,923
2018	3.333	12,492,860	1.0337	1.0500	13,559,877
2019	2.333	12,394,530	1.0235	1.0244	12,995,072

e.g., for AY2019:     $1.0235 = 1.01^{2.333}$   
 $12,995,072 = 12,394,530 \times 1.0235 \times 1.0244$

Accident Year	Ultimate Claims	Pure Premium Trend Factor at 4.00%	Regulation Adjustment to Claims	Trended Claims	Trended Claim Ratio
2015	8,130,150	1.2820	0.80	8,338,086	67.40%
2016	7,970,110	1.2327	0.80	7,859,570	67.36%
2017	7,781,380	1.1853	0.90	8,300,615	65.36%
2018	8,001,680	1.1397	1.00	9,119,247	67.25%
2019	7,995,960	1.0958	1.00	8,762,239	67.43%

e.g., for AY 2019:  $1.0958 = 1.04^{2.333}$   
 $8,762,239 = 7,995,960 \times 1.0958 \times 1.00$   
 $67.43\% = 8,762,239 / 12,995,072$

- (c) Recommend a trended claim ratio to use for ratemaking. Justify your recommendation.

Accident Year	Trended Claim Ratio	Accident Year Weights
2015	67.40%	10%
2016	67.36%	15%
2017	65.36%	20%
2018	67.25%	25%
2019	67.43%	30%

Weighted average trended claim ratio = 66.96%

Justification: No significant outliers, so average of all years with more weight to more recent experience.

- (d) Calculate the indicated rate change.

Weighted average trended claim ratio	66.96%
Ratio of ULAE to claims	10.00%
Weighted average trended claim ratio including ULAE = $0.6696 \times (1 + 0.10) =$	73.65%
Fixed expenses as ratio to premiums at current rate level	6.00%
Variable expenses (ratio to premiums)	19.00%
Profit and contingencies ratio to premiums	5.00%
Permissible claim ratio = $(1 - 0.19 - 0.05) / (1 + 0.06/0.7365) =$	70.28%
Indicated rate change = $0.7365 / 0.7028 - 1 =$	4.81%

- (e) Explain why an indicated rate increase of 5% is not necessarily indicative of deteriorating experience.

We are told that rates were adequate at the time of the rate change. Therefore, if experience does not get better or worse after the change, then experience should change with expected net trend.

Net trend = (claim trend)/(premium trend) – 1 =  $(1 + 0.04) / (1 + 0.01) - 1 = 2.97\%$

Time from the change to the effective date of the new rates = 1.5 years

Therefore, experience should change with respect to net trend =  $(1 + 0.0297)^{1.5} - 1 = 4.5\%$

Since this is close to the rate change implemented at that time, this is as expected and does not indicate deteriorating experience.

## GIRR Fall 2020 Question 20 (LOs 5a, 5b, 5c, 5d, 5e)

### Learning Outcomes:

- (5a) Identify and describe the influences of portfolio changes on claim frequency and severity.
- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 27.

### Commentary on Question:

*This question tests the candidate's understanding of trend on premiums for ratemaking purposes.*

### Solution:

- (a) Calculate the annual premium trend due to the shift in policy limits for each year.

### Commentary on Question:

*The increased limits factors in effect starting on November 1, 2020 need to be used to calculate the weighted average ILFs, as they represent the current rating factors.*

Weighted Average ILF	Annual Trend Due to Shift in ILF
1.0238	
1.0281	0.42%
1.0372	0.89%
1.0465	0.90%
1.0532	0.64%
1.0599	0.64%

- (b) Recommend the annual premium trend due to the shift in policy limits to use for ratemaking. Justify your recommendation.

Average all years: 0.70%  
Average excluding high/low: 0.72%

Recommend annual trend: 0.72%  
Justification: exclude the high and low values because of the volatility.



- (c) Explain why the annual premium trend due to a shift in policy limits tends to be positive while the annual premium trend due to a shift in deductibles tends to be negative.

Over time, policy limits tend to shift to higher limits. The higher limits have higher factors, which results in more premium to the insurer, meaning positive trend.

Over time, deductibles tend to shift to higher deductibles. The higher deductibles have lower factors, which results in less premium to the insurer, meaning negative trend.

- (d) Calculate the calendar year 2017 on-level earned premium trended for ratemaking purposes.

Average earned premium date in future rating period: December 1, 2021

Total premium trend =  $(1 + 0.0072)(1 - 0.004) - 1 = 0.3176\%$

Experience period trend factor (2017 to 2019) =  $1.0599 / 1.0465 = 1.0128$

Forecast period from July 1, 2019 to Dec. 1, 2021: 29 months

CY2017 on-level EP trended to future rating period:

$$= 17,808,000 \times 1.0128 \times (1 + 0.003176)^{(29/12)} = 18,174,778.$$

### GIRR Spring 2021 Question 3 (LOs 3g, 4a, 4b, 4c, 5b, 5c, 5d, 5e)

#### Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.
- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16, 23, and 26.

#### Commentary on Question:

*This question tests the candidate's understanding of claims trend analysis and selection as well as estimating ultimate claims using the development-based frequency-severity method. This question also tests the candidate's understanding of estimating unpaid ULAE using the classical paid-to-paid method with the Mango-Allen smoothing adjustment.*

#### Solution:

- (a) Explain why this may happen when using the development-based frequency-severity method.  
  
For the development-based frequency-severity method, the severity would be developed to an ultimate value separately, which might not equal the developed ultimate claims divided by the developed ultimate counts.
- (b) Recommend a claim frequency at the accident year 2020 cost level. Justify your recommendation.

Accident Year	(1) Earned Exposures	(2) Projected Ultimate Counts	(3) Based on Development Method Claims	(4) Severity
2015	25,200	2,088	9,028,629	4,324
2016	26,700	2,194	9,779,132	4,458
2017	25,300	2,063	9,477,056	4,594
2018	24,500	1,983	9,375,491	4,733
2019	23,900	1,933	8,987,726	4,724
2020	24,200	1,709	7,810,473	4,749
Total	149,800	11,970	54,458,507	

Accident Year	(5) = (2)/(1) Indicated Frequency	(6) <sub>t</sub> = (5) <sub>t</sub> / (5) <sub>t-1</sub> Year-to-year Change	(7) Frequency Trend @ -0.78%	(8) = (5)(7) Trended Frequency
2015	0.082857		0.961604	0.079676
2016	0.082172	-0.008266	0.969163	0.079638
2017	0.081542	-0.007676	0.976782	0.079648
2018	0.080939	-0.007392	0.984461	0.079681
2019	0.080879	-0.000743	0.992200	0.080248
2020	0.070620	-0.126842	1.000000	0.070620

Frequency trend selection: (column 6): Average of 2016-2018 = -0.78%  
(2019 & 2020 are outliers)

Recommended 2020 cost level frequency (column 8): average excluding 2020  
= 0.0798  
(all other years are stable and 2020 is an outlier)

- (c) Calculate ultimate claims using the development-based frequency-severity method and the recommended claim frequency from part (b).

Accident Year	(4) Indicated Severity	(9) <sub>t</sub> = (4) <sub>t</sub> / (4) <sub>t-1</sub> Year-to-year Change	(10) Severity Trend @ 3.06%	(11) = (4)(10) Trended Severity
2015	4,324		1.162655	5,027.32
2016	4,458	0.030990	1.128134	5,029.22
2017	4,594	0.030507	1.094638	5,028.77
2018	4,733	0.030257	1.062136	5,027.09
2019	4,724	-0.001902	1.030600	4,868.55
2020	4,749	0.005292	1.000000	4,749.00

Severity trend selection: (column 9): Average of 2016-2018 = 3.06%  
(2019 & 2020 are outliers)

Recommended 2020 cost level frequency (column 11): Average of 2016-2018  
= 5,028.10  
(2019 & 2020 are outliers)

	(12) = (1)×0.0798/(7)	(13) = 5.028.10/(10)	(14) = (12)(13)
Accident Year	Ultimate Counts	Ultimate Severity	Ultimate Claims
2015	2,090.83	4,324.67	9,042,137
2016	2,198.00	4,457.01	9,796,505
2017	2,066.50	4,593.39	9,492,263
2018	1,985.55	4,733.95	9,399,499
2019	1,921.82	4,878.81	9,376,179
2020	1,930.76	5,028.10	9,708,066
Total			56,814,649

(d) Calculate the expected claims paid for calendar years 2017 through 2020.

	12	24	36	48	60	72
Cumulative paid claims development factors by maturity age (CDF)	11.245	2.017	1.228	1.063	1.010	1.000
% Cumulative Paid (1/CDF)	8.9%	49.6%	81.4%	94.1%	99.0%	100.0%
% Incremental Paid	8.9%	40.7%	31.9%	12.6%	4.9%	1.0%

e.g., % incremental paid at 24 months = 40.7% = 49.6% - 8.9%

Accident Year	Ultimate Claims from Part (c)	Projected in Calendar Year			
		2017	2018	2019	2020
2015	9,042,137	2,880,340	1,142,940	446,367	89,526
2016	9,796,505	3,985,781	3,120,642	1,238,293	483,607
2017	9,492,263	844,132	<b>3,861,998</b>	3,023,727	1,199,837
2018	9,399,499		835,883	3,824,256	2,994,177
2019	9,376,179			833,809	3,814,768
2020	9,708,066				863,323
Total		7,710,253	8,961,462	9,366,451	9,445,237

e.g., Accident year 2017 expected paid claims in calendar year 2018

$$= 0.407 \times 9,492,263 = \mathbf{3,861,998}$$

- (e) Recommend a ULAE ratio using the classical paid-to-paid method with the Mango-Allen smoothing adjustment. Justify your recommendation.

Calendar Year	Paid ULAE	Expected Claims from Part (d)	Ratio ULAE to Claims
2017	738,905	7,710,253	9.58%
2018	851,350	8,961,462	9.50%
2019	883,245	9,366,451	9.43%
2020	879,224	9,445,237	9.31%
Total	3,352,724	35,483,403	9.45%

Recommended ULAE ratio = total of all years = 9.45%, as there are no significant outliers.

**Commentary on Question:**

*Candidates could also recommend a ULAE ratio that considered the downward trend.*

- (f) Calculate the unpaid ULAE.

$$\begin{aligned} \text{Calculated unpaid ULAE} &= 9.45\% \times 4,351,459 \times (1 - 0.25) + 9.45\% \times 11,117,813 \\ &= 1,358,858. \end{aligned}$$

## GIRR Spring 2021 Question 12 (LOs 5b, 5c, 5d, 5e, 6g, 6h)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6h) Demonstrate the use of credibility in ratemaking.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27 and 32.

### Commentary on Question:

*This question tests the candidate's understanding of premium trend. This question also tests basic ratemaking using a claim ratio approach incorporating the complement of credibility.*

### Solution:

- (a) Recommend the annual premium trend to use for ratemaking. Justify your recommendation.

### Commentary on Question:

*Change in annual written premium is needed to analyze the trend.*

Calendar Year	Average On-Level Written Premium (OLWP)	Year-to-Year Change in Average OLWP
2011	540.00	
2012	546.48	1.20%
2013	552.71	1.14%
2014	560.01	1.32%
2015	572.21	2.18%
2016	579.54	1.28%
2017	587.30	1.34%
2018	593.65	1.08%
2019	601.07	1.25%
2020	608.52	1.24%
Average all years		1.34%
Average excluding 2015 outlier		1.23%

Recommended annual trend = 1.23%.

Justification: Annual trend is reasonably stable except for 2015, which appears to be an outlier.

- (b) Calculate the trended claim ratio for each accident year.

Average earned premium date in the future rating period = 9 months after August 1, 2021 = May 1, 2022

Accident Year	Average Earned Premium Date		Trending Period (months)	Trended On-Level Earned Premium @1.23%	Trended Claim Ratios
	Experience Period	Forecast Period			
2016	2016-07-01	2022-05-01	70	9,065,912.50	75.42%
2017	2017-07-01	2022-05-01	58	8,888,948.54	72.76%
2018	2018-07-01	2022-05-01	46	8,419,705.00	69.45%
2019	2019-07-01	2022-05-01	34	8,166,989.29	70.21%
2020	2020-07-01	2022-05-01	22	8,435,636.28	67.27%
All year average:					71.02%
Excluding high/low:					70.81%
Average (2018-2020):					68.98%

- (c) Recommend a trended claim ratio to use for ratemaking. Justify your recommendation.

**Commentary on Question:**

*Other recommendations acceptable as long as the justification matches the data.*

Selected weighted average trended experience claim ratio: 70.81%.

Justification: Exclude high and low years to smooth the erratic values. No clear trend.

- (d) Calculate the claim ratio to use for the complement of credibility.

Indicated rate change for policies effective January 1, 2021 through June 30, 2021	4%
Approved rate change for policies effective January 1, 2021 through June 30, 2021	2%
Permissible claim ratio for policies effective January 1, 2021 through June 30, 2021	55%
Pure premium trend	5.0%
Premium trend	1.23%
Average accident date of prior filing	01-Apr-21
Average accident date of forecast period	01-May-22
Trending period in months	13
Complement of credibility claim ratio = $1.04/1.02 \times 0.55 \times (1.05/1.0123)^{(13/12)}$	58.34%

(e) Calculate the indicated rate change.

Selected trended claim ratio	70.81%
Credibility assigned to the experience claim ratio	77.00%
Complement of credibility	58.34%
Credibility weighed claim ratio	67.94%
<hr/>	
Indicated rate change = $(67.94\% + 15\%)/(1 - 11\% - 4\%) - 1 =$	-2.42%



### GIRR Fall 2021 Question 4 (LOs 5b, 5e, 6d, 6e, 6g)

#### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6d) Calculate loadings for catastrophes and large claims.
- (6e) Apply loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27, 31, and 32.

#### Commentary on Question:

*This question tests the candidate's understanding of basic ratemaking, including the application of a loading for catastrophes in ratemaking.*

#### Solution:

- (a) Calculate the pure premium for the earthquake endorsement.

Midpoint of future rating period:	July 1, 2023
Exposure trend period (months): July 1, 2020 to October 1, 2020	3
Exposure trend = $(1.035^{(3/12)}) =$	1.00864
Severity trend period (months): October 1, 2020 to July 1, 2023	33
Severity trend = $(1.07^{(33/12)}) =$	1.20450
Trended modeled catastrophe claims = $225,000 \times 1.009 \times 1.204 =$	273,352.46
Trended exposures = $15,000 \times (1 + 0.035)^{((3 + 33)/12)} =$	16,630.77
Pure premium = $273,352.46 / 16,630.77 =$	16.44

- (b) Calculate the premium for the earthquake endorsement.

$$\text{Endorsement premium} = (16.44 + 5) / (1 - 0.1 - 0.25) = 30.77$$

- (c) Calculate the indicated rate for the basic homeowners coverage. Justify any selections.

	(1)	(2)	(3)	(4)	(5)	(6) = (5)/(4)
Accident Year	On Level Earned Premium (OLEP)	Ultimate Claims	Trend Period (years)	Trended OLEP	Trended Ultimate Claims	Claim Ratio
2018	15,500,000	9,000,000	5	17,113,252	12,622,966	0.7376
2019	16,250,000	8,000,000	4	17,589,523	10,486,368	0.5962
2020	17,000,000	8,200,000	3	18,040,536	10,045,353	0.5568

Notes: (3) For 2020: July 1, 2020 to July 1, 2023 = 3 years

(4) For 2020:  $18,040,536 = 17,000,000 \times 1.02^3$

(5) For 2020:  $10,045,353 = 8,200,000 \times 1.07^3$

Selected claim ratio = 0.5765 (average of 2019 and 2020 is used as 2018 is an outlier)

Indicated rate change =  $0.5765 / 0.57 - 1 = 0.0114$

Indicated rate =  $1,050 \times 1.0114 = 1,061.97$

- (d) State whether you agree with management's proposal. Justify your response.

**Commentary on Question:**

*Other justification responses are possible.*

Do not agree.

Justification: There is additional administrative cost related to this optional add-on, such as the mid-term addition or cancellation.

## GIRR Fall 2021 Question 5 (LOs 5b, 5c)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 27.

### Commentary on Question:

*This question tests the candidate's understanding of premium trend.*

### Solution:

- (a) Provide two circumstances in which exposure and premium trend adjustments need to be considered for a ratemaking analysis.
- When working with inflation-sensitive exposures.
  - Where a change (or shift) in the mix of exposures and rating characteristics results in a corresponding change in premiums over time.
- (b) Calculate and select the annual premium trend due to the change in discount level. Justify your selection.

Calendar Year	5% discount	10% discount	Average Discount	Annual Change
2016	5.2%	9.3%	98.81%	
2017	5.0%	10.0%	98.75%	−0.06%
2018	4.5%	11.0%	98.68%	−0.08%
2019	4.5%	12.0%	98.58%	−0.10%
2020	6.5%	25.0%	97.18%	−1.42%
Average excluding 2020:				−0.08%

e.g.,  $98.81\% = 1 - 0.05 \times 0.052 - 0.1 \times 0.093$   
 $-0.06\% = 98.75\% / 98.81\% - 1$

Select −0.08% as the annual premium trend. Justification is that 2020 should be excluded as this is assumed to be a one-time change and the annual change should therefore return to historical levels after 2020.

- (c) Calculate the premium trend factor to be used for 2018 using earned premium for the trending analysis and incorporating the annual trend selected in part (b).

2018 average earned date	July 1, 2018
2020 average earned date	July 1, 2020
Trending Period 1	24 months
Effective date of new rates	February 1, 2022
Average earned date of forecast period	February 1, 2023
Trending Period 2 (July 1, 2020 to February 1, 2023)	31 months

Trending factor period 1:  $((1 - 0.0008)(1 + 0.0075))^{(24/12)} = 1.01345$

Trending factor period 2:  $((1 - 0.002)(1 + 0.0075))^{(31/12)} = 1.01423$

Premium trend factor =  $1.01345 \times 1.01423 = 1.02787$ .

## GIRR Spring 2022 Question 16 (LOs 5b, 5c, 5d, 5e)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 27.

### Commentary on Question:

*This question tests the candidate's understanding of premium trend and adjusting premiums for trend for ratemaking purposes.*

### Solution:

- (a) Calculate the percentage increase in premiums that occurred from the rating differentials change on July 1, 2021.

Weighted average differentials using rates prior to July 1, 2021 = 1.0000125

Weighted average differentials using rates effective July 1, 2021 = 1.0025387  
(i.e., weighted averages use 2021 earned exposures)

Estimated percent premium change from differential change:  
 $1.0025387 / 1.0000125 - 1 = 0.25\%$

- (b) Recommend the annual premium trend rate to use for ratemaking for this line of business. Justify your recommendation.

	2017	2018	2019	2020	2021
Weighted average differential (using July 1, 2021 differentials)	0.9881932	0.9935602	0.9965078	0.9994956	1.0025387
Year-to-year change e.g., $0.9935602 / 0.9881932 - 1 = 0.54\%$		0.54%	0.30%	0.30%	0.30%

Recommended annual trend: 0.30%

Justification: Annual change has stabilized at 0.3% over the last 3 years, so it is reasonable to assume that trend will continue into the future rating period.

(c) Calculate the calendar year 2021 earned premiums to use for ratemaking.

Average earned date in future rating period for 12-month policies:	Oct. 1, 2023
Trending period (months) for 12-month policies: July 1, 2021 to Oct. 1, 2023:	27
Average earned date in future rating period for 6-month policies:	Jul. 1, 2023
Trending period (months) for 12-month policies: July 1, 2021 to Jul. 1, 2023:	24
Trending period (months) weighted by policy term ( $27 \times 2/3 + 24 \times 1/3$ )	26
 Trend factor = $(1 + 0.003)^{(26/12)} =$	 1.00651834
Trended premium for ratemaking = $25,256,000 \times 1.00651834 =$	25,420,627

### GIRR Spring 2022 Question 17 (LOs 5b, 5e, 6d, 6e, 6g, 6h)

#### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6d) Calculate loadings for catastrophes and large claims.
- (6e) Apply loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6h) Demonstrate the use of credibility in ratemaking.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27, 31, and 32.

#### Commentary on Question:

*This question tests the candidate's understanding of basic ratemaking, including the application of a loading for wildfire claims in ratemaking.*

#### Solution:

- (a) Calculate the ultimate pure premium for wildfire claims to be used as a loading in the homeowners premiums.

Average accident date in future rating period: Sep. 1, 2023

	(1)	(2)	(3)	(4)	(5)
		Wildfire - Ultimate			
Accident Year	Earned Exposures	Counts	Claims	Trending Period (years)	Severity Trend @3%
2015	11,200	0	0	8.167	1.2730
2016	11,850	0	0	7.167	1.2359
2017	12,500	1	1,500,000	6.167	1.1999
2018	13,750	0	0	5.167	1.1650
2019	15,000	1	1,120,000	4.167	1.1311
2020	16,250	0	0	3.167	1.0981
2021	17,500	1	500,000	2.167	1.0661
Total	98,050	3	3,120,000		

$$(6) = (3)(5) \quad (7) = (2)/(1) \quad (8) = (6)/(2) \quad (9) = (6)/(1)$$

Trended Ultimate Wildfire

Accident Year	Trended Ultimate Claims	Frequency	Severity	Pure Premium
2015	0	0.000000	0	0.00
2016	0	0.000000	0	0.00
2017	1,799,924	0.000080	1,799,924	143.99
2018	0	0.000000	0	0.00
2019	1,266,795	0.000067	1,266,795	84.45
2020	0	0.000000	0	0.00
2021	533,070	0.000057	533,070	30.46
Total	3,599,789	0.000031	1,199,930	36.71

- (b) Calculate the indicated total premium for the homeowners coverage, including a loading for wildfire claims.

	Credibility	Trended Ultimate Pure Premium
Insurer internal experience from part (a)	20%	36.71
Industry experience	80%	50.00
Credibility weighted wildfire claims experience (at Sept. 1, 2023 cost level): $0.2 \times 36.71 + 0.8 \times 50.00$		47.34
Non-wildfire claims per policy (PP) as of July 1, 2021: $21,507,500 \times 0.67 / 17,500 =$		823.43
Trended non-wildfire PP to future rating period = $823.43 \times [(1 + 0.04) / (1 + 0.025)]^{2.167} =$		849.76
Indicated premium = $(849.76 + 47.34 + 70) / (1 - 0.2 - 0.05) =$		1,289.47



## GIRR Fall 2022 Question 6 (LOs 3g, 3j, 6b, 6c, 6d)

### Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).
- (6b) Identify the different types of rate regulatory approaches for general insurance.
- (6c) Describe the purpose of base rates and rating factors and explain how they are used to determine an insured's premium.
- (6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 17, 18, 19, 21, 23, and 27.

### Commentary on Question:

*This question tests the candidate's understanding of estimating ultimate claims using the frequency severity method, the expected method and the Bornhuetter Ferguson method. This question also tests the candidate's ability to estimate reported claims with an adjustment for case outstanding strengthening.*

### Solution:

- (a) Calculate ultimate claims using the development-based frequency-severity method.

	(1)	(2)	(3) = (2)/(1)	(4)	(5) = (3)(4)	(6)
Accident Year	Earned Exposures	Ultimate Counts	Reported Frequency	Frequency Trend @ 1%	Trended Frequency	Calculated Ultimate Counts
2015	11,090	1,230	0.11091	1.06152	0.11773	1,234
2016	11,250	1,270	0.11289	1.05101	0.11865	1,264
2017	11,460	1,305	0.11387	1.04060	0.11850	1,300
2018	11,770	1,349	0.11461	1.03030	0.11809	1,349
2019	12,070	1,381	0.11442	1.02010	0.11672	1,397
2020	12,360	1,447	0.11707	1.01000	0.11824	1,445
2021	12,480	1,480	0.11859	1.00000	0.11859	1,474
Average (all years)					0.11807	

Selected frequency: 0.11807

Rationale: no outliers and no significant trend, so simple average is reasonable.

(6) =  $0.11807 \times (1)/(4)$

	(7)	(8)	(9) = (7)(8)	(10)	(11) = (6)(10)
Accident Year	Ultimate Severity	Severity Trend @ 6.5%	Trended Severity	Calculated Ultimate Severity	Projected Ultimate Claims
2015	4,349	1.45914	6,345.81	4,502	5,552,843
2016	4,666	1.37009	6,392.82	4,794	6,059,090
2017	5,002	1.28647	6,434.90	5,106	6,639,119
2018	5,358	1.20795	6,472.19	5,438	7,334,547
2019	5,881	1.13423	6,670.38	5,791	8,090,495
2020	6,314	1.06500	6,724.41	6,167	8,911,632
2021	6,540	1.00000	6,540.00	6,568	9,678,863
Average (all years)			6,511.50		52,266,590
Average (latest 5 years)			6,568.38		

Selected severity: 6,568.38

Rationale: there has been an increase in the more recent years, so use average of latest 5 years.

$$(10) = 6,568.38 / (8)$$

- (b) Construct the reported claims triangle adjusted for the change in case adequacy.

Accident Year	Adjusted Average Case Estimates						
	12	24	36	48	60	72	84
2015	3,019.21	4,711.60	6,331.08	7,611.32	8,629.94	9,217.78	9,584.81
2016	3,215.46	5,017.85	6,742.60	8,106.05	9,190.89	9,816.94	
2017	3,424.46	5,344.01	7,180.87	8,632.95	9,788.30		
2018	3,647.05	5,691.37	7,647.62	9,194.09			
2019	3,884.11	6,061.31	8,144.72				
2020	4,136.58	6,455.30					
2021	4,405.45						

e.g., AY2021 at 12 months:  $4,405.45 = (3,175,077 - 1,082,487) / (875 - 400)$   
 AY2019 at 24 months:  $6,061.31 = 6,455.30 / 1.065$

Accident Year	Adjusted Reported Claims						
	12	24	36	48	60	72	84
2015	1,930,388	2,761,294	3,589,678	4,284,121	4,884,010	5,284,288	5,274,875
2016	2,073,457	3,013,099	3,948,018	4,735,629	5,294,541	5,763,708	
2017	2,251,286	3,199,812	4,277,015	5,120,705	5,759,272		
2018	2,489,201	3,627,479	4,653,380	5,558,325			
2019	2,692,962	3,900,733	5,107,412				
2020	2,908,798	4,364,690					
2021	3,175,077						

e.g., AY2019 at 24 months:  $3,900,733 = 6,061.31 \times (975 - 618) + 1,736,844$

- (c) Recommend the revised annual claim severity trend. Justify your recommendation.

**Commentary on Question:**

*Other selections are acceptable as long as the justification matches the data.*

Accident Year	Ultimate Reported Severities	Year-to- Year Change
2015	4,316.59	
2016	4,561.67	5.68%
2017	4,813.61	5.52%
2018	5,066.25	5.25%
2019	5,441.62	7.41%
2020	5,802.31	6.63%
2021	5,990.39	3.24%

Average all years: 5.62%

Average excluding high & low: 5.77%

Average excluding last year: 6.10%

Recommended: 5.77%

Justification: select average excluding high & low to eliminate the variability.

- (d) Explain why you might expect the answer to part (c) to be lower than the original annual severity trend of 6.5%.

Due to the increase in the average case in the most recent diagonal, this will tend to overstate the annual severity trend. By adjusting the historical case estimates for the change, this will increase those values, which will tend to decrease the indicated annual reported severity trend.

- (e) Calculate ultimate claims using the ultimate counts provided and ultimate reported severities adjusted for the change in case adequacy.

Accident Year	Ultimate Counts	Ultimate Reported Severities	Ultimate Claims
2015	1,230	4,316.59	5,309,406
2016	1,270	4,561.67	5,793,321
2017	1,305	4,813.61	6,281,761
2018	1,349	5,066.25	6,834,371
2019	1,381	5,441.62	7,514,877
2020	1,447	5,802.31	8,395,943
2021	1,480	5,990.39	8,865,777

e.g., AY2019:  $7,514,877 = 1,381 \times 5,441.62$

- (f) Calculate expected claims for all accident years using the expected method and your recommended annual claim severity trend from part (c). Justify any selections.

Annual claim trend =  $(1 + 0.01)(1 + 0.0577) - 1 = 6.83\%$

Accident Year	Claim Trend @6.83%	Trended Pure Premiums Based on Reported	Expected Claims
2015	1.48623	711.54	5,311,155
2016	1.39125	716.44	5,755,608
2017	1.30234	713.87	6,263,319
2018	1.21911	707.89	6,871,912
2019	1.14120	710.52	7,528,174
2020	1.06827	725.66	8,235,350
2021	1.00000	710.40	8,882,995
Average all years (excl. 2021)		714.32	48,848,513
Average (excluding 2020)		711.78	
Selected 2021 level pure premium		711.78	

Justification: 2020 appears to be an outlier, so use average of all years excluding 2020.

e.g., AY2019:

$$710.52 = 7,514,877 \times 1.14120 / 12,070$$

$$7,528,174 = 711.78 \times 12,070 / 1.14120$$

- (g) Calculate ultimate claims for all accident years using the Bornhuetter Ferguson method.

	(12)	(13): part (e)	(14) = (13)/(12)	(15): part (f)	(16) = (12) + (15)[1 – 1/(14)]
Accident Year	Reported Claims	Ultimate Claims	Age-to-Ult Factor	Expected Claims	BF Estimate Ultimate Claims
2015	5,274,875	5,309,406	1.00655	5,311,155	5,309,417
2016	5,763,708	5,793,321	1.00514	5,755,608	5,793,128
2017	5,759,272	6,281,761	1.09072	6,263,319	6,280,227
2018	5,558,325	6,834,371	1.22957	6,871,912	6,841,381
2019	5,107,412	7,514,877	1.47137	7,528,174	7,519,137
2020	4,364,690	8,395,943	1.92361	8,235,350	8,318,835
2021	3,175,077	8,865,777	2.79230	8,882,995	8,876,829
					48,938,953

- (h) Recommend the selected ultimate claims for accident year 2021 for this line of business. Justify your recommendation.

Recommend using average of part (e), part (f) and part (g) estimates = 8,875,200

Justification:

- Development method (9,678,673) and part (a) estimate (9,678,863) do not adjust for the change in case outstanding, so both are inappropriate.
- Parts (e), (f) and (g) estimates all adjust for the change in case outstanding so are all reasonable methods.
- Recommend the average of all 3 since they are all close in value.

## GIRR Fall 2022 Question 14 (LOs 5b, 5c, 5d, 5e, 6e, 6g, 6h)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6e) Apply loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6h) Demonstrate the use of credibility in ratemaking.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26, 31, and 32.

### Commentary on Question:

*This question tests the candidate's understanding of loadings for large claims as well as basic ratemaking.*

### Solution:

- (a) Describe one way that large claims are differentiated from catastrophe claims when insurers are estimating loadings for ratemaking purposes.

Any one of the following are acceptable:

- Catastrophes typically result in GI claims for multiple insurers providing coverage in an affected area. Whereas large losses are limited to a few claims for an individual insurer.
- Catastrophes are associated with an event which is infrequent and results in unusually large aggregate losses.
- Catastrophes typically result in a significant number of GI claims for multiple insurers providing coverage in the area affected by the event. Large claims do not typically affect the entire GI industry, or even all GI companies operating in a specific area.

- (b) Recommend the annual pure premium trend for weather claims. Justify your recommendation.

Accident Year	Pure Premium per 100 EHY	Year-to-Year Change
2010	5,280	
2011	5,770	9.3%
2012	6,330	9.7%
2013	6,200	-2.1%

Accident Year	Pure Premium per 100 EHY	Year-to-Year Change
2014	6,920	11.6%
2015	7,140	3.2%
2016	7,560	5.9%
2017	8,300	9.8%
2018	8,460	1.9%
2019	8,850	4.6%
2020	9,400	6.2%
2021	9,940	5.7%
Average - all years		6.0%
Average - latest 5 years		5.7%
Average - all years excl. high & low		6.3%
Recommendation		6.3%

Justification: Include more years due to significant volatility. Excluding high & low eliminates outliers.

- (c) Recommend the trended ultimate pure premium for weather claims per 100 EHY to use in ratemaking. Justify your recommendation.

Accident Year	Trending Period (months)	Pure Premium Trend Factor	Trended Ultimate Pure Premium for Non-Hurricane Weather excluding Hail per 100 EHY
2010	169	2.3514	12,415
2011	157	2.2129	12,768
2012	145	2.0825	13,183
2013	133	1.9599	12,151
2014	121	1.8444	12,763
2015	109	1.7358	12,393
2016	97	1.6335	12,350
2017	85	1.5373	12,760
2018	73	1.4468	12,240
2019	61	1.3615	12,050
2020	49	1.2813	12,045
2021	37	1.2059	11,986
Average (all years)			12,425
Average (latest 5 years)			12,216

Average (latest 3 years) 12,027

Recommendation 12,027

Justification: Decreasing values in latest years so more weight to more recent data. Therefore, recommend average of latest 3 years.

- (d) Calculate the indicated rate level change, including a loading for weather claims.

Accident Year	Trended Earned Premiums at Current Rate Level	Trended Ultimate Claims	Claim Ratio	Accident Year Weights
2019	12,545,160	7,130,200	56.84%	25%
2020	12,777,120	7,449,200	58.30%	30%
2021	12,613,560	6,824,400	54.10%	45%
Weighted average trended claim ratio			56.05%	

- |      |  |            |
|------|--|------------|
| (1)  | Selected non-hurricane weather excluding hail pure premium per 100 EHY:                      | 12,027     |
| (2)  | CY2021 earned house years  | 16,860     |
| (3)  | CY2021 trended earned premiums at current rate level   | 12,613,560 |
| (4)  | Loading for non-hurricane weather expressed as a claim ratio =<br>$((1)/100) \times (2)/(3)$ | 16.08%     |
| (5)  | ULAE to claim ratio  | 12%        |
| (6)  | Total claim ratio including ULAE = $(56.05\% + (4))(1 + (5))$                                | 80.78%     |
| (7)  | Credibility of experience period = $\text{squareroot}(49,500 / 80,000)$                      | 78.66%     |
| (8)  | Countrywide trended, adjusted ultimate claim, including ULAE, ratio                          | 70%        |
| (9)  | Credibility-weighted experience claim, including ULAE, ratio<br>$= (6)(7) + [1 - (7)](8)$    | 78.48%     |
| (10) | Selected fixed expenses to premiums ratio  | 5%         |
| (11) | Selected variable expenses to premiums ratio   | 15%        |
| (12) | Selected profit and contingencies to premiums ratio  | 4%         |
| (13) | Indicated rate level change = $[(9) + (10)]/[1 - (11) - (12)] - 1$                           | 3.06%      |



## GIRR Fall 2022 Question 16 (LOs 5b, 5c, 5d, 5e)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 27.

### Commentary on Question:

*This question tests premium trending for ratemaking purposes.*

### Solution:

- (a) Calculate the quarterly change in average written premiums using:
  - (i) Change in quarter-to-quarter averages
  - (ii) Change in rolling 4-quarter volume-weighted averages

Experience Period Calendar Quarter Ending	Average On-Level Written Premiums		Quarterly Change in Average Written Premiums	
	Quarter Average	Rolling 4-Quarter Volume- Weighted Average	Quarter Average	Rolling 4-Quarter Volume- Weighted Average
		Average		Average
2018-1	516.48			
2018-2	526.28		1.90%	
2018-3	531.30		0.95%	
2018-4	533.12	527.01	0.34%	
2019-1	545.32	534.25	2.29%	1.37%
2019-2	541.82	538.05	-0.64%	0.71%
2019-3	556.50	544.46	2.71%	1.19%
2019-4	556.54	550.24	0.01%	1.06%
2020-1	558.31	553.50	0.32%	0.59%
2020-2	564.92	559.17	1.18%	1.03%
2020-3	578.59	564.88	2.42%	1.02%
2020-4	576.75	569.88	-0.32%	0.88%
2021-1	589.45	577.69	2.20%	1.37%
2021-2	596.74	585.63	1.24%	1.38%

Experience Period Calendar Quarter Ending	Average On-Level Written Premiums		Quarterly Change in Average Written Premiums	
	Quarter Average	Rolling 4-Quarter Volume- Weighted Average	Quarter Average	Rolling 4-Quarter Volume- Weighted Average
		Average		Average
2021-3	599.16	590.81	0.41%	0.88%
2021-4	605.94	598.01	1.13%	1.22%
2022-1	610.41	603.24	0.74%	0.88%
2022-2	621.06	609.43	1.74%	1.03%

e.g., 2018-4:

$$533.12 = 3,067,577 / 5,754$$

$$527.01 = (2,443,276 + 2,549,138 + 2,676,306 + 2,775,206) / (5,229 + 5,354 + 5,568 + 5,754)$$

- (b) Recommend the annual premium trend. Justify your recommendation.

	Quarter Average	Rolling 4-Quarter Volume-Weighted Average
Average all quarters	1.10%	1.04%
Average all quarters excl. high & low	1.10%	1.05%
Average latest 6 quarters	1.24%	1.13%
Recommended quarterly:		1.05%
Annual:		4.28%

Justification: Use the rolling values as it smooths out the variability. Recommend average excluding high & low.

- (c) Calculate the first quarter 2022 on-level earned premiums trended to the future rating period.

Average earned date in 2022-1:	15-Feb-22
Average earned date in future rating period:	01-Apr-24
Trend period (years):	2.125

$$\text{Trended on-level earned premiums: } 5,136,000 \times (1.0428)^{2.125} = 5,614,523$$

## GIRR Spring 2023 Question 5 (LOs 5b, 5c, 5d, 5e, 6g)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27 and 32.

### Commentary on Question:

*This question tests the candidate's understanding of trending premiums and indicated rate changes using claim ratios.*

### Solution:

- (a) Recommend the annual premium trend due to the shift in policy limits to use for ratemaking. Justify your recommendation.

### Commentary on Question:

*The year-to-year change in **average** increased limit factor (ILF) needs to be analyzed for the trend due to shift in policy limits.*

Experience Period	Weighted Average ILF	Annual Trend Due to Shift in ILF
2015	1.00018	
2016	1.00270	0.25%
2017	1.00603	0.33%
2018	1.00877	0.27%
2019	1.01202	0.32%
2020	1.01500	0.29%
2021	1.01769	0.27%
2022	1.01924	0.15%
Average:		0.27%
Average excluding high & low:		0.28%

Recommended trend: 0.28%

Justification: average excluding high and low removes the outliers, especially 2022.

- (b) Calculate the indicated rate level change for this line of business using a claims ratio approach. Justify any selection(s).

Average earned premium dates in 2022:	Jul. 1, 2022	
Effective date of new rates:	Sep. 1, 2023	# of months
Average earned premium dates in future rating period:		<u>trending period</u>
for 12-month policies	Sep. 1, 2024	26
for 6-month policies	Jun. 1, 2024	23
Average:		25.25

Annual premium trend =  $(1 + 0.28\%)(1 + -0.1\%) - 1 = 0.180\%$

Annual pure premium trend =  $(1 + 6\%)(1 + -1.2\%) - 1 = 4.728\%$

	(1)	(2)	(3)	(4) = (1.0018) <sup>[(2)/12]</sup>	(5) = (1)(3)(4) Earned Premiums Trended at Current Rates
Accident Year	Earned Premiums	Trending Period (months)	Premium On-Level Factors	Premium Trend Factors	
2018	15,804,847	73.25	1.064	1.01102	17,001,688
2019	15,333,428	61.25	1.106	1.00921	17,114,913
2020	15,526,085	49.25	1.104	1.00740	17,267,582
2021	16,625,910	37.25	1.049	1.00559	17,538,061
2022	17,102,494	25.25	1.026	1.00379	17,613,581

	(6)	(7) = (1.04728) <sup>[(2)/12]</sup>	(8) = (6)(7) Trended Ultimate Claims	(9) = (8) / (6) Claim Ratio
Accident Year	Ultimate Claims	Claim Trend Factors		
2018	8,703,669	1.32577	11,539,025	67.87%
2019	9,184,011	1.26591	11,626,161	67.93%
2020	9,602,493	1.20876	11,607,137	67.22%
2021	10,401,614	1.15419	12,005,466	68.45%
2022	11,309,041	1.10209	12,463,536	70.76%

Average: 68.45%

Average latest 3 years: 68.81%

Selected claim ratio = 68.81%

Justification for selected claim ratio: Increasing in most recent years, so give more weight to more recent 3 years.

Indicated rate change:  $[0.6881(1 + 0.07) + 0.05] / (1 - 0.23 - 0.04) = 7.71\%$

- (c) Describe one reason why an indicated rate change using a pure premium approach may not result in the same result as part (b).

The premium on-level factors are an approximation used to restate historical earned premiums as if they were at the current rate level for the forecast period.

- (d) Calculate the profit and contingencies to premium ratio implied by a 3% rate increase using your colleague's indicated rate change.

Claim ratio implied by the 6% rate indication:

$$(\text{Claim ratio} + 0.05)/(1 - 0.23 - 0.04) - 1 = 0.06 \rightarrow \text{claim ratio} = 72.38\%$$

Profit margin implied by a 3% rate change:

$$(0.7238 + 0.05)/(1 - 0.23 - Q) - 1 = 0.03$$

$$\rightarrow Q = 1.87\%$$

- (e) State two actions the company can take that could help achieve the target profit, given the 3% rate increase.

**Commentary on Question:**

*Other actions are possible.*

- decrease expenses
- decrease claims (e.g., changing mix of business, better risk selection)

## GIRR Spring 2023 Question 8 (LOs 5a, 5b, 5e, 6c, 6d)

### Learning Outcomes:

- (5a) Identify and describe the influences of portfolio changes on claim frequency and severity.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6c) Explain the requirements for loadings for catastrophes and large claims in ratemaking.
- (6d) Calculate loadings for catastrophes and large claims.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26, 27, and 31.

### Commentary on Question:

*This question tests the candidate's understanding of the loading for catastrophes in ratemaking.*

### Solution:

- (a) Explain why two trend adjustments must be made to the modeled expected earthquake claims to calculate the catastrophe loading for ratemaking.

Past adjustment: modeled catastrophe claims must be trended from February 1, 2022 to July 1, 2022 for exposure trend to reflect in-force exposures as of July 1, 2022

Future adjustment: modeled catastrophe claims must be trended from July 1, 2022 to mid-point of future rating period for severity trend to reflect the cost level in the future rating period

- (b) Calculate the catastrophe loading to be used for ratemaking, as a claim ratio.

Midpoint of future rating period:	October 1, 2024
Exposure trend period (months): February 1, 2022 to July 1, 2022	5
Exposure trend = $1.01^{(5/12)}$ =	1.00415
Severity trend period (months): July 1, 2022 to October 1, 2024	27
Severity trend = $1.06^{(27/12)}$ =	1.14009
Trended modeled catastrophe claims = $450,000 \times 1.00415 \times 1.14009$ =	515,170.85
Catastrophe loading = $515,170.85 / 15,450,000$ =	3.33%

- (c) Describe an additional step or approach that would increase your confidence in the estimate of expected earthquake claims.

Running alternative catastrophe models would increase confidence in the estimate of expected earthquake claims.

- (d) Describe how you would consider the effect of a demand surge in the calculation of the catastrophe loading for ratemaking.

Demand surge can result in a trend rate that is higher post-catastrophe than pre-catastrophe. Therefore, could recognize a demand surge by selecting a higher post event claim severity trend rate.

## GIRR Spring 2023 Question 14 (LOs 3g, 5c, 5d)

### Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 26.

### Commentary on Question:

*This question tests the development-based frequency-severity method for estimating ultimate claims.*

### Solution:

- (a) Recommend an annual claim frequency trend to use for the development-based frequency-severity method. Justify your recommendation.

	(1)	(2)	(3) = (2) / (1)	(4) = (3) <sub>i</sub> / (3) <sub>i-1</sub> - 1
	Ultimate Counts Based on			
Accident Year	Earned Exposures	Development Method	Indicated Frequency	Year-to-Year Change
2017	11,434	1,235	10.80%	
2018	11,635	1,247	10.72%	-0.773%
2019	11,681	1,249	10.69%	-0.234%
2020	11,821	1,260	10.66%	-0.314%
2021	12,044	1,256	10.43%	-2.163%
2022	12,240	1,301	10.63%	1.924%
Average:				-0.312%
Selected frequency trend:				-0.312%

Justification: The year-to-year changes are quite erratic, with an overall decrease over the period. The average of all years provides a reasonable measure of the overall trend.



- (b) Estimate ultimate claims for all accident years using the development-based frequency-severity method.

	(5)	(6) = (3)(5)	(7) = 10.59%×(1)/(5)
Accident Year	Frequency Trend @ -0.312%	Trended Frequency	Calculated Ultimate Counts
2017	0.98450	10.63%	1,230
2018	0.98758	10.58%	1,248
2019	0.99067	10.59%	1,249
2020	0.99377	10.59%	1,260
2021	0.99688	10.40%	1,279
2022	1.00000	10.63%	1,296

Average excluding 2022

- all years	10.56%
- latest 3 years	10.53%
- excl. hi-lo	10.59%

Selected freq. at 2022 cost level 10.59%

Justification: Excluding high and low values excludes the outlier value in 2021.

	(8)	(9)	(10) = (8)(9)	(11) = 5,900.79/(9)	(12) = (7)(11)
Accident Year	Ultimate Severity	Severity Trend @ 7.5%	Trended Severity	Calculated Ultimate Severity	Projected Ultimate Claims
2017	4,104	1.43563	5,891.82	4,110.25	5,055,292
2018	4,384	1.33547	5,854.70	4,418.52	5,512,721
2019	4,751	1.24230	5,902.15	4,749.90	5,931,044
2020	5,066	1.15563	5,854.40	5,106.15	6,432,161
2021	5,531	1.07500	5,945.83	5,489.11	7,023,037
2022	5,897	1.00000	5,897.00	5,900.79	7,648,692

Average excluding 2022

- all years	5,889.78
- latest 3 years	5,900.79
- excl. hi-lo	5,882.89

Selected severity at 2022 cost level 5,900.79

Justification: Latest 3 years gives more consideration to the increasing more recent experience.

- (c) Describe two scenarios when projections from the frequency-severity method are preferred.

Any two of the following are acceptable:

- For immature periods (i.e., most recent accident years)
- Following the introduction of new GI products when limited or no historical experience is available
- Following entry into a new geographical area for which limited or no historical data exists
- If there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future

### GIRR Fall 2023 Question 3 (LOs 5b, 5e, 6d)

#### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6d) Calculate loadings for catastrophes and large claims.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26 and 31.

#### Commentary on Question:

*This question tests the candidate's understanding of a loading for large claims that is used in ratemaking.*

#### Solution:

- (a) State two reasons for using a large claim loading approach when estimating ultimate claims at total limits for ratemaking.

Any two of the following are acceptable:

- The loading factor smooths the influence of large claims over time
- The actuary can introduce a greater volume of experience
- The claims at a limited value are more reliable

- (b) Calculate the large claim loadings at 500,000 limit, adjusted to the cost level for each accident year.

Average earned date in rating period: July 1, 2024 (i.e., 9 months after effective date)  
July 1, 2022 to July 1, 2024 = 24 months

Accident Year	Trending Period (months)	(1) Severity	(2) Trend Factor at:	(3) = (2)/(1) Trend Factors for Loading for Large Claims	(4) = 1.28 / (3) Loadings for Large Claims Adjusted to Cost Level of AY
		5.0%	7.0%		
2019	60	1.276	1.403	1.099	1.165
2020	48	1.216	1.311	1.078	1.187
2021	36	1.158	1.225	1.058	1.210
2022	24	1.103	1.145	1.038	1.233

- (c) Calculate ultimate claims at total limits for each accident year using selected ultimate claims at a 500,000 limit and the large claim loadings from part (b).

	(4)	(5)	(6) = (4)(5)
Accident Year	Selected Ultimate Claims at 500,000 Limit	Loadings for Large Claims Adjusted to Cost Level of AY	Indicated Ultimate Claims at Total Limits based on Projections at 500,000 Limits
2019	9,850,000	1.165	11,472,916
2020	10,365,000	1.187	12,302,726
2021	11,275,000	1.210	13,637,761
2022	12,385,000	1.233	15,265,711

- (d) Describe how the calculations in part (b) are affected when the experience is less than fully credible.

**Commentary on Question:**

*This question asks to describe how the calculations in part (b) are affected when the experience is less than fully credible. Providing an explanation of what credibility is does not answer the question.*

The calculations are affected in two ways:

- Need to develop credibility-weighted trend rates
- Need to develop credibility-weighted loadings

**GIRR Fall 2023 Question 6 (LOs 5c)****Learning Outcomes:**

- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 27.

**Commentary on Question:**

*This question tests the candidate's understanding of premium trend.*

**Solution:**

- (a) Describe why you would adjust actual historical premiums to current rate levels before analyzing premium trend.

Using unadjusted (actual historical) premiums could result in estimates of trend that were actually due to rate changes.

- (b) Describe an advantage of using written premiums instead of earned premiums for a premium trend analysis.

Written premiums reflect shifts in the mix of exposures more quickly than earned premiums.

- (c) Describe why an adjustment for inflation is required if premiums are based on inflation-sensitive exposures.

Without such adjustment, the premium trend could double-count what is in fact change due to inflation.

- (d) Describe why an increasing proportion of insureds replacing their old vehicles with new vehicles might affect premium trend factors.

Newer vehicles would have higher rate group factors, leading to increased premiums and therefore increasing premium trend.

- (e) Describe how a premium trend analysis for an insurer's book of business is different from a premium trend analysis for a self-insurer.

**Commentary on Question:**

*Some candidates explained that self-insurers would typically use pure premiums instead of historical earned premiums. This does not explain how a premium trend analysis for an insurer's book of business is different from a premium trend analysis for a self-insurer.*

The difference is that a self-insurer is essentially a single policy, not a series of policies written over the period. Therefore, the average written dates would reflect the actual date the policy is written.

**GIRR Fall 2023 Question 8 (LOs 3g, 5c, 5d, 5e)****Learning Outcomes:**

- (3g) Estimate ultimate values using the methods cited in (3e).
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

**Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 26.

**Commentary on Question:**

*This question tests the development-based frequency-severity method for estimating ultimate claims.*

**Solution:**

- (a) Explain why a linear trend model may not be appropriate when trend is decreasing.

If the trend is decreasing, as frequency trends often are, then eventually the application of a linear trend will result in a negative value, which cannot occur for GI frequency, severity, or pure premium.

- (b) Recommend an annual claim frequency trend to use for this line of business. Justify your recommendation.

Recommended trend:  $-1.11\%$

Justification: the increase for 2022 might be an anomaly, so exclude that year from the average.

- (c) Calculate projected ultimate claims using the development-based frequency-severity method and your recommended annual claim frequency trend.

Accident Year	Indicated Frequency	Frequency Trend	Trended Frequency	Calculated Ultimate Counts
2016	9.170%	0.935221	8.576%	1,472.57
2017	9.000%	0.945718	8.511%	1,482.03
2018	8.960%	0.956334	8.569%	1,476.93
2019	8.900%	0.967068	8.607%	1,465.38
2020	8.720%	0.977923	8.527%	1,468.39
2021	8.650%	0.988900	8.554%	1,486.51
2022	8.760%	1.000000	8.760%	1,462.54

Average, excluding 2022

All years 8.557%

Latest 3 years 8.563%

Selected frequency at 2022 cost level 8.684%

Accident Year	Ultimate Severity	Severity Trend	Trended Ultimate Severity	Calculated Ultimate Severity	Ultimate Claims
2016	3,750.00	1.418519	5,319.45	3,764.58	5,543,602
2017	3,993.00	1.338226	5,343.53	3,990.45	5,913,955
2018	4,230.00	1.262477	5,340.28	4,229.88	6,247,220
2019	4,489.00	1.191016	5,346.47	4,483.67	6,570,290
2020	4,679.00	1.123600	5,257.32	4,752.69	6,978,783
2021	5,048.00	1.060000	5,350.88	5,037.85	7,488,816
2022	5,409.00	1.000000	5,409.00	5,340.12	7,810,150

Average, excluding 2022

All years 5,326.32

Latest 3 years 5,318.23

Selected frequency at 2022 cost level 5,340.12

46,552,817



## GIRR Fall 2023 Question 11 (LOs 5b, 5c, 5d, 5e, 6a)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27 and 30.

### Commentary on Question:

*This question tests the candidate's understanding of expenses used for ratemaking, including trending of fixed expenses.*

### Solution:

- (a) Identify why a separate trending procedure for fixed expenses may not be required when analyzed on a per-exposure basis.

### Commentary on Question:

*Some candidates misunderstood this question and answered assuming no trend would be needed for fixed expenses as opposed to asking why a separate trending procedure for fixed expenses may not be required.*

When the forces affecting changes in expenses (i.e., the expense trend) are similar to those driving changes in premiums, a separate trend adjustment for fixed expenses may not be necessary.

- (b) Recommend an annual fixed expense trend. Justify your recommendation.

Calendar Year	Fixed Expense to Earned Premiums at Current Rates Ratio	Year-to- Year Change
2016	6.84%	
2017	7.03%	2.77%
2018	7.13%	1.52%
2019	7.29%	2.24%
2020	7.51%	2.96%
2021	7.68%	2.26%

2022	7.92%	3.13%
Average - All years		2.48%
Average - excl hi-lo		2.56%
Recommended fixed expense trend:		2.56%

Justification: 2018 appears to be an anomaly. Exclude highest and lowest to smooth out the variation.

- (c) Recommend a fixed expense ratio to be used in ratemaking. Justify your recommendation.

Average earned premium date in 2022	1-Jul-22	
Average earned premium dates in future rating period:		# months:
for 12-month policies	1-Nov-24	28

Calendar Year	Trending Period (months)	Trending Period (years)	Expense Trend at 2.56%	Trended Fixed Expenses	Trended Fixed Expense Ratio
2016	100	8.33	1.2343	569,624	8.44%
2017	88	7.33	1.2035	594,138	8.46%
2018	76	6.33	1.1735	622,353	8.37%
2019	64	5.33	1.1442	653,791	8.34%
2020	52	4.33	1.1157	694,861	8.38%
2021	40	3.33	1.0878	723,949	8.35%
2022	28	2.33	1.0607	769,701	8.40%

Average - all years 8.39%

Recommended trended fixed expense ratio: 8.39%

Justification: No significant outliers and no significant trend, so all years average is reasonable.

## GIRR Fall 2023 Question 12 (LOs 5b, 5c, 5d, 5e, 6g, 6h)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6h) Demonstrate the use of credibility in ratemaking.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26 and 32.

Actuarial Standards of Practice, Actuarial Standards Board of the American Academy of Actuaries, No. 25, Credibility Procedures, 2013.

### Commentary on Question:

*This question tests basic ratemaking using a pure premium approach, including the application of credibility.*

### Solution:

- (a) Calculate the trended pure premiums for each accident year.

Past pure premium trend:  $(1.06)(1 - 0.01) - 1 = 4.94\%$

Future pure premium trend:  $(1.06)(1.01) - 1 = 7.06\%$

Past trend period: For AY2022, from average accident date in AY2022 (July 1, 2022) to December 31, 2022 = 0.5 years

Future pure premium trend period: From average accident date in 2022 to average accident date in future rating period: 1/1/2023 to 3/1/2025 = 26 months, or 2.167 years

Accident Year	Earned Exposures	Ultimate Claims	Pure Premium (PP)	Past Trending Period (yrs.)	Future Trending Period (yrs.)
2018	10,146	13,085,953	1,289.76	4.5	2.167
2019	10,127	14,011,147	1,383.54	3.5	2.167
2020	10,298	14,968,858	1,453.57	2.5	2.167
2021	10,291	15,499,745	1,506.15	1.5	2.167
2022	10,573	18,068,228	1,708.90	0.5	2.167

Accident Year	Future Trending Period (yrs.)	Past PP Trend	Future PP Trend	Total PP Trend	Trended Ultimate Pure Premium
2018	2.167	1.2423	1.1593	1.4402	1,857.54
2019	2.167	1.1838	1.1593	1.3724	1,898.80
2020	2.167	1.1281	1.1593	1.3078	1,900.99
2021	2.167	1.0750	1.1593	1.2462	1,877.03
2022	2.167	1.0244	1.1593	1.1876	2,029.46

- (b) Recommend a trended pure premium. Justify your recommendation.

**Commentary on Question:**

*Other weights are possible.*

AY	Trended Ultimate Pure Premium	Weights
2018	1,857.54	22.50%
2019	1,898.80	22.50%
2020	1,900.99	22.50%
2021	1,877.03	22.50%
2022	2,029.46	10.00%

**Averages**

- all years straight	1,912.76
- weighted	1,898.18
Recommended:	1,898.18

Justification: AY2022 is possibly an anomaly so less weight for that year. Include all years due to credibility (i.e., all years is 4,341 ultimate counts, so still not fully credible even using all years).

- (c) Calculate the pure premium to use for the complement of credibility.

Pure premium trend (future, since future trend started Jan. 1, 2023)	7.06%
Average accident date of prior filing	Jul. 1, 2023
Average accident date of forecast period	Mar. 1, 2025
Trending period in months	20
Pure premium used for complement of credibility: [1,700(1.076) <sup>(20/12)</sup> ] =	1,904.70

- (d) Calculate the credibility-weighted indicated rate.

**Commentary on Question:**

*The number of claims for credibility need to match the years that were included in the part (b) selection. For example, if only the most recent three years were included in the selection in part (b), then the number of claims to use for credibility in this part should be:  $2,610 = 875 + 852 + 883$ .*

Weighted average pure premium (from part (b)):	1,898.18
Number of claims to use for credibility:	4,341
Credibility: $(4,341 / 4,654)^{0.5}$	96.58%
Credibility-weighted pure premium: $1,898.18 \times 96.58\% + 1,904.70(1 - 96.58\%)$	1,898.40
Indicated rate: $(1,898.40(1.04) + 125) / (1 - 0.18 - 0.05)$	2,726.41

- (e) Identify one adjustment that is necessary when relying on a complement of credibility that is a pure premium based on industry experience.

Either of the following is acceptable:

- adjusted to reflect the insurer's mix of business
- adjusted to the cost level of the forecast period

### GIRR Spring 2024 Question 3 (LOs 5b, 5c, 5e)

#### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5e) Calculate trend factors for claims and exposures.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 27.

#### Commentary on Question:

*This question tests the candidate's understanding of premium trend analysis, particularly when the trend rate changes.*

#### Solution:

- (a) Explain the purpose of quantifying the effect of shifts in the mix of exposures and rating characteristics on the premium during the experience period.

The purpose is so that the historical premiums can be adjusted to reflect the average premium level that is expected during the forecast period.

- (b) Calculate the 2020 premium trend factor to be used to adjust 2020 earned premiums for the ratemaking exercise.

All policies written between July 1, 2019 and December 31, 2020 contribute toward 2020 earned premium. Average written date = Apr. 1, 2020

Past trend period: Apr. 1, 2020 to Jan. 1, 2024 = 45 months, or 3.75 years

New policies effective: Oct. 1, 2024 for 1 year

Average written date in future rating period: Apr. 1, 2025

Future trend period: Jan. 1, 2024 to Apr. 1, 2025 = 15 months, or 1.25 years

$$2020 \text{ premium trend factor} = (1 + 1.5\%)^{3.75}(1 + 3.0\%)^{1.25} = 1.097221$$

- (c) Explain how the premium trend factors would be affected by the following:
  - (i) An increasing proportion of insureds choosing a lower policy limit at the beginning of 2024
  - (ii) An increasing proportion of insureds choosing a higher deductible at the beginning of 2024

- (i) The decreased insured value would decrease the premiums, so the premium trend factors would decrease.
  - (ii) The higher deductible would decrease the premiums, so the premium trend factor would decrease.
- (d) Describe why the trending periods would be different in the part (b) calculation if this trending analysis is done for a self-insurer.

A self-insurer is essentially one policy and not a series of policies written over the period. Therefore, the average written dates would be based on the self-insurer's fiscal year (e.g., fiscal year running from May 1 through April 30 would have an average date of November 1).

## GIRR Spring 2024 Question 5 (LOs 2d, 5b, 5e, 6f, 6g, 6h, 6j, 6k)

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6f) Explain the requirements for loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate loadings for catastrophes and large claims.
- (6h) Apply loadings for catastrophes and large claims in ratemaking.
- (6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6k) Demonstrate the use of credibility in ratemaking.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 26, 31, and 32.

### Commentary on Question:

*This question tests the candidate's ability to calculate the indicated average rate, while considering adjustments to earned premium and a loading for non-hurricane weather claims.*

### Solution:

- (a) Calculate the trended ultimate non-hurricane weather excluding hail pure premium per 100 EHY for all years.

Average accident date in future rating period: June 1, 2025 (9 months after start date).

# months from 2023 average accident date to June 1, 2025: 23

Accident Year	Ultimate		Trending Period (months)	Trend Factors	
	Frequency per 100 EHY	Severity		Frequency @ -1.0%	Severity @ 5.0%
2014	2.02	4,100	131	0.8961	1.7034
2015	0.39	3,500	119	0.9051	1.6223
2016	1.99	2,900	107	0.9143	1.5450
2017	0.1	4,400	95	0.9235	1.4715
2018	1.99	2,800	83	0.9328	1.4014
2019	0.8	4,200	71	0.9423	1.3347
2020	0.63	2,600	59	0.9518	1.2711
2021	2.73	3,600	47	0.9614	1.2106
2022	0.56	2,100	35	0.9711	1.1529
2023	1.69	3,100	23	0.9809	1.0980



Accident Year	Frequency per 100 EHY	<u>Trended Ultimate</u>	
		Severity	Pure Premium per 100 EHY
2014	1.810	6,983.94	12,642
2015	0.353	5,678.00	2,004
2016	1.819	4,480.60	8,152
2017	0.092	6,474.43	598
2018	1.856	3,923.89	7,284
2019	0.754	5,605.56	4,226
2020	0.600	3,304.87	1,982
2021	2.625	4,358.07	11,438
2022	0.544	2,421.15	1,317
2023	1.658	3,403.88	5,643
Average:			
-all years	1.211	4,663.44	5,529

- (b) Recommend the trended ultimate non-hurricane weather excluding hail pure premium per 100 EHY to use in determining a weather loading. Justify your recommendation.

Recommend all years average: 5,529

Justification: should use more years to smooth out fluctuations; no significant trend.

- (c) Calculate the non-hurricane weather excluding hail loading percentage to use for ratemaking.

Selected state S PP per 100 EHY 5,529

Credibility-Weighted Pure Premium per 100 EHY 5,069.96

Expected Non-Hurricane Weather Claims 909,095.18

Weather loading as a claim ratio =  $909,095 / 13,089,711 = 6.95\%$

- (d) Identify two considerations when choosing the number of years and/or the weights to assign to each of the years.

Any 2 of the following are acceptable:

- professional judgment
- assessment of the relevance and reliability of the insurer's historical experience
- whether there are regulation requirements
- balance between stability and responsiveness
- management input

- credibility consideration - want enough years for full credibility, if possible
- also acceptable to note that give more weight to recent experience to account for recent changes

- (e) Recommend the number of years to include when estimating the weighted average trended claim ratio for the indicated rate change. Justify your recommendation.

AY	Ultimate Counts	Running Total
		Ultimate Counts
2019	1,070	5,447
2020	1,075	<b>4,377</b>
2021	1,074	3,302
2022	1,141	2,228
2023	1,087	1,087

Recommend 4 years.

Justification: Full credibility (3,654) is met by including at least the most recent 4 years.

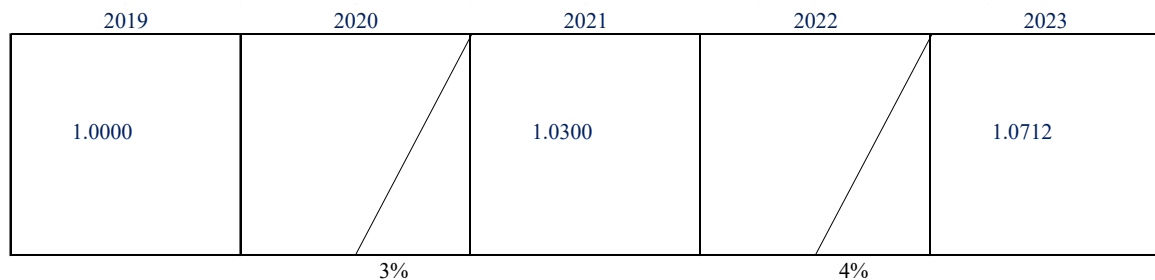
- (f) Recommend the weights to assign to each year when estimating the weighted average trended claim ratio for the indicated rate change. Justify your recommendation.

**Commentary on Question:**

*Candidates can also select weights judgmentally, as long as the number of years used matches the number of years recommended in part (e).*

AY	Earned Exposures	AY Weights		
		Initial	Limited	Balanced
2020	19,937	27.3%	23.4%	24.4%
2021	17,061	23.4%	23.4%	24.4%
2022	17,992	24.7%	24.6%	25.6%
2023	17,931	24.6%	24.6%	25.6%
Total	72,921		96.0%	

(g) Calculate the indicated rate change for this line of business.



Rate Index	Area in CY				
	2019	2020	2021	2022	2023
1.0000	100%	75%	0%	0%	0%
1.0300	0%	25%	100%	75%	0%
1.0712	0%	0%	0%	25%	100%
Average rate level:	1.0000	1.0075	1.0300	1.0403	1.0712
On-level factor:	1.0712	1.0632	1.0400	1.0297	1.0000

Claim Ratio Trend:  $(1 + -1.0\%)(1 + 5\%) - 1 = 3.95\%$

AY	Earned Premiums	On-Level Factor	On-Level Earned Premiums	Ultimate Claims
2019	13,510,549	1.07120	14,472,500	8,709,600
2020	13,268,660	1.06323	14,107,582	8,673,608
2021	11,739,370	1.04000	12,208,945	7,919,295
2022	12,638,750	1.02970	13,014,158	8,605,528
2023	13,089,711	1.00000	13,089,711	9,489,317

AY	Claim Trend Period (yrs)	Claim Trend Factor	Trended Ult. Claims	Claim Ratio	Weights
2019	5.9167	1.25761	10,953,253	75.68%	0.0%
2020	4.9167	1.20982	10,493,496	74.38%	24.4%
2021	3.9167	1.16385	9,216,849	75.49%	24.4%
2022	2.9167	1.11962	9,634,939	74.03%	25.6%
2023	1.9167	1.07708	10,220,730	78.08%	25.6%
Weighted:				75.51%	

Weighted Average Trended Claim Ratio (including non-hurricane weather loading): 82.46%

Ratio of ULAE to Claims 5.00%

Weighted Average Trended Claim Ratio including ULAE =  $0.8245 \times (1 + 6.7598) = 86.58\%$

Fixed Expenses as Ratio to Premiums at Current Rate Level 3.00%

Variable Expenses – Ratio to Premiums	12.00%
Profit and Contingencies Ratio to Premiums	4.00%
Permissible Claim Ratio = $(1 - 0.12 - 0.04) / (1 + 0.03/0.8658) =$	81.19%
Indicated Rate Change = $0.8658 / 0.8119 - 1 =$	6.64%

## GIRR Spring 2024 Question 12 (LOs 3g, 5c, 5d)

### Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 26.

### Commentary on Question:

*This question tests the development-based frequency-severity method for estimating ultimate claims.*

### Solution:

- (a) Describe two options to consider when experience is not fully credible for trending.

Any two of the following are acceptable:

- Rely on industry data for a similar line of business in a similar jurisdiction.
- Combine the insurer's experience in specific states or provinces with the experience of a larger region.
- Combine the insurer's experience with that of other insurers in a group under common ownership.

- (b) Recommend the annual claim frequency trend to use for this line of business. Justify your recommendation.

Accident Year	Earned Exposures	Ultimate Counts	Indicated Frequency	Annual Change in Frequency
2018	16,451	1,485	9.027%	
2019	16,557	1,492	9.011%	-0.172%
2020	16,815	1,499	8.915%	-1.072%
2021	16,915	1,503	8.886%	-0.326%
2022	17,147	1,474	8.596%	-3.256%
2023	17,461	1,491	8.539%	-0.666%
Average:				-1.098%
Exponential fitted:				-1.200%
Selected:				-1.200%

Justification: use all years due to erratic changes.

- (c) Calculate the ultimate counts using the development-based frequency-severity method with your selected frequency trend from part (b). Justify any selections.

Accident Year	Freq trend @- 1.2%	Trended Frequency	F-S Ultimate Counts
2018	0.941431	8.498%	1,498
2019	0.952864	8.587%	1,489
2020	0.964436	8.598%	1,494
2021	0.976148	8.674%	1,485
2022	0.988002	8.493%	1,487
2023	1.000000	8.539%	1,496

Average trended frequency at 2023 cost level excluding 2023

all years 8.570%

excluding hi-lo 8.561%

Selected frequency @ 2023 level: 8.570%

Justification for selected frequency: No significant trend; no significant outliers

- (d) State one other influence that the trend rate should also recognize.

Social influences, (i.e., the impact on insurance costs of societal changes such as changes in claim consciousness, court practices, and legal precedents, as well as in other noneconomic factors).

- (e) Calculate the ultimate claims using the development-based frequency-severity method. Justify any selections.

Accident Year	Severity Trend @5.0%	Trended Reported Severity	F-S Ultimate Severity	F-S Ultimate Claims
2018	1.276282	6,022.77	4,966.93	7,438,122
2019	1.215506	6,493.23	5,215.28	7,766,041
2020	1.157625	6,503.54	5,476.05	8,182,046
2021	1.102500	6,457.34	5,749.85	8,538,549
2022	1.050000	6,219.15	6,037.34	8,979,399
2023	1.000000	6,168.00	6,339.21	9,485,828

Average trended severity at 2023 cost level excluding 2023

all years 6,339.21

excluding hi-lo 6,389.91

Selected severity @ 2023 level: 6,339.21

Justification for selected severity: No significant trend; no significant outliers.

## GIRR Fall 2024 Question 8 (LOs 5e)

### Learning Outcomes:

(5e) Calculate trend factors for claims and exposures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 26.

### Commentary on Question:

*This question tests the calculation of pure premium trend, as well as considerations when selecting data points to include in trending procedures.*

### Solution:

- (a) Describe one reason for relying on a longer period of time when trending a long-tailed line of business.

One reason for using more data points is to account for the greater uncertainty inherent in the projection of ultimate claims for long-tail coverages, particularly for the most recent years in the experience period.

- (b) Provide an example where a longer period of time may not be appropriate for trending a long-tailed line of business.

Due to potential changes in coverage as well as in the economic, regulatory, and legal environments over time.

- (c) State two considerations when selecting which data points to include in trending procedures.

Any two of the following are acceptable:

- Balance the need for stability with the need for responsiveness to the most recent experience.
- Assign greater weight to the most recent experience for short-tail lines of business.
- Have a sufficient number of data points in the experience period to determine a pattern for the annual change.
- Consider both long-term and short-term trend indications for long-tail lines of business.
- Consider the effect of changes in coverage, economic, regulatory and legal environments over time.
- The experience of the most recent data points may be too immature for long-tail lines of business.
- Consider excluding outliers.

- (d) Calculate the pure premium trend factor for accident year 2022.

# months trending period:

12-month policies (given): 45

6-month policies: 42

Weighted average # months:  $43.2 (0.4 \times 45 + 0.6 \times 42)$

Exponent =  $43.2 / 6 = 7.2$

Pure premium trend factor =  $e^{(0.045 \times 7.2)} \cdot e^{(-0.007 \times 7.2)} = 1.31469$



## GIRR Fall 2024 Question 11 (LOs 2d, 5b, 5e)

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13 and 27.

### Commentary on Question:

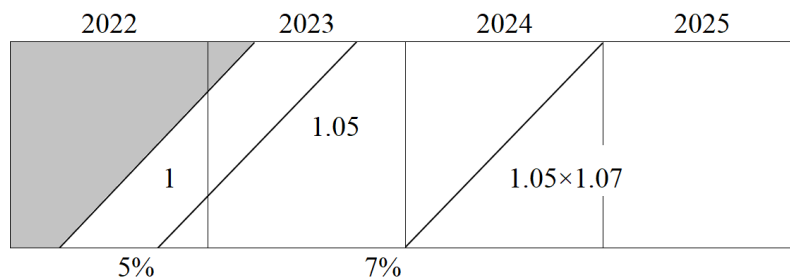
*This question tests the candidate's ability to adjust premium to current rate levels and adjust premiums for trend for ratemaking purposes.*

### Solution:

- (a) Calculate the on-level premium factors for calendar year 2022 and 2023.

### Commentary on Question:

*Since the company started writing a new line of business on March 1, 2022, the shaded area in the diagram below has no earned premiums and should not be included in estimating the percent of premiums earned in each calendar year.*

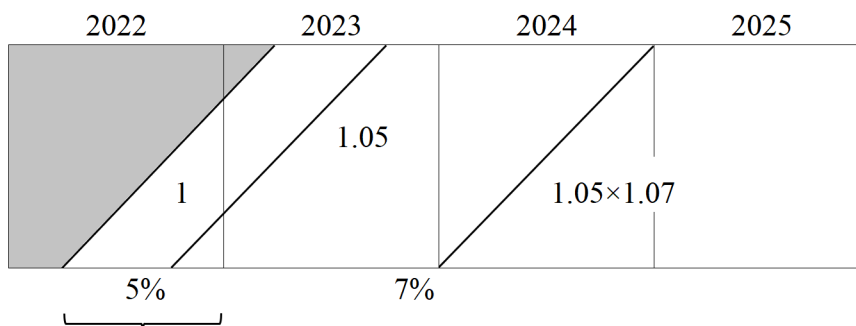


Rate Change History			Percent Premium Earned in Each Calendar Year (CY) at Rate Level	
Effective Date of Rate Change	Rate Change %	Rate Level Index	2022	2023
Prior to Mar 1/22			65.28%	1.39%
		1.00000	29.17%	20.83%
1-Sep-22	5%	1.05000	5.56%	77.78%
1-Jan-24	7%	1.12350	-	-
Total			34.72%	98.61%
Average Rate Level in each CY:			1.00800	1.03944
On-Level Factors:			1.1146	1.0809

Notes:

- $5.56\% = 0.5(4/12)^2$
- $29.17\% = 0.5(10/12)^2 - 5.56\%$
- Avg rate level in CY 2022 =  $(1.0000 \times 29.17\% + 1.0500 \times 5.56\%) / 34.72\% = 1.0080$
- $20.83\% = 0.5(8/12)^2 - 0.5(2/12)^2$
- $77.78\% = 1 - 0.5(8/12)^2$
- Avg rate level in CY 2023 =  $(1.0000 \times 20.83\% + 1.0500 \times 77.78\%) / 98.61\% = 1.0394$

(b) Calculate premium trend factors for calendar year 2022 and 2023.



for 2022: all policies written between Mar 1, 2022 & Dec 31, 2022 contribute toward 2022 EP

for 2023: all policies written between Mar 1, 2022 & Dec 31, 2023 contribute toward 2023 EP

Trend from average written date in experience period to average written date in future rating period.

Calendar Year	Average Written Date in Experience Period	Average Written Date Rating Period	Trending Period in Months	Trending Period in Years	Trend Factor
2022	1-Aug-22	1-Oct-25	38	3.167	0.98425
2023	1-Feb-23	1-Oct-25	32	2.667	0.98672

e.g.,  $0.98425 = (1 - 0.005)^{3.167}$

## GI 101 – LEARNING OBJECTIVE 6

### 6. Topic: Ratemaking

The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

## GIRR Fall 2020 Question 5 (LOs 6a)

### Learning Outcomes:

- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 30.

### Commentary on Question:

*This question tests the candidate's understanding of expense ratios used in ratemaking.*

### Solution:

- (a) Calculate the historical trend in fixed expenses.

Calendar Year	(1) Earned Premiums	(2) Earned Premiums at Current Rate Level	(3) Fixed Expenses
2014	4,526,480	5,850,000	172,580
2015	4,830,080	6,166,130	186,220
2016	5,279,580	6,451,780	200,650
2017	5,542,320	6,658,360	214,400
2018	6,139,740	6,901,520	231,200
2019	6,873,650	7,231,270	253,090

Calendar Year	(4) = (3)/(2) Fixed Expense per On-Level Earned Premium	(5) = (4) <sub>i</sub> /(4) <sub>i-1</sub> - 1 Change in Fixed Expense per On-Level Earned Premium
2014	2.95%	
2015	3.02%	2.37%
2016	3.11%	2.98%
2017	3.22%	3.54%
2018	3.35%	4.04%
2019	3.50%	4.48%

Average all years: 3.48%  
Average most recent 3 years: 4.02%

*{Note: averages not needed for part (a) but helpful for part (c)}*

- (b) Assess the reasonableness of using the publicly-available cost index for this line of business in comparison to using the historical trend in fixed expenses.

The company trend may have been similar to the publicly available cost index in older years, but the recent increases make that index not reasonable.

- (c) Recommend the annual fixed expense trend. Justify your recommendation.

**Commentary on Question:**

*Other recommendations are acceptable where the required justification matches the outcome of the trend analysis.*

Recommend an annual fixed expense trend of 4.0%.

Justification: There is a clear increasing trend rate so more weight should be given to more recent years.

- (d) Calculate the fixed expense ratio to be used in ratemaking, using a simple average from calendar years 2017, 2018 and 2019.

Rates effective: April 1, 2021

Average incurred date in rating period: April 1, 2022 (i.e., 12 months following the effective date as policies are annual and in effect for 12 months)

Calendar Year	<u>Average Incurred Date</u>		Trend Period (months)	Trend Factors	Trended Fixed Expenses	Fixed Expense per On-Level Earned Premium
	Experience Period	Forecast Period				
2017	2017-07-01	2022-04-01	57	1.2048	258,305	3.88%
2018	2018-07-01	2022-04-01	45	1.1584	267,832	3.88%
2019	2019-07-01	2022-04-01	33	1.1139	281,914	3.90%
					Average:	3.89%

e.g., for CY 2018:

Trend factor:  $1.1584 = 1.04^{(45/12)}$

Trended fixed expenses:  $267,832 = 231,200 \times 1.1584$

Fixed expense per on-level earned premium:  $3.88\% = 267,832 / 6,901,520$

### GIRR Fall 2020 Question 13 (LOs 6c, 6d)

#### Learning Outcomes:

- (6c) Explain the requirements for loadings for catastrophes and large claims in ratemaking.
- (6d) Calculate loadings for catastrophes and large claims.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

#### Commentary on Question:

*This question tests the candidate's understanding of loadings for large claims for ratemaking.*

#### Solution:

- (a) Explain why actuaries typically conduct separate analyses of property and liability claims for homeowners insurance when determining a loading for large claims.

There are very different forces influencing the claim development, severities, frequencies, and the trends applicable to property and liability coverages.

- (b) Calculate the loadings for 500,000 to total limits for each accident year.

Average accident date in each experience year = July 1  
Average accident date in forecast period = April 1, 2022

Severity trend for 1,000,000 limit =  $4.00\% \times 0.60 + 5.00\% \times 0.40 = 4.40\%$

Severity trend for total limit =  $5.00\% \times 0.50 + 6.00\% \times 0.50 = 5.50\%$

Accident Year	(1) Trend Period (months)	(2) <u>Severity Trend at:</u> 4.40%	(3) 5.50%	(4) Trended Claims at 1,000,000 Limit	(5) Total Limit
2016	69	1.281	1.361	9,505	10,816
2017	57	1.227	1.290	9,570	10,510
2018	45	1.175	1.222	9,990	10,622
2019	33	1.126	1.159	10,300	10,798

Notes: (4) = (Selected ultimate claims at 1,000,000 limit)(2)

(5) = (Selected ultimate claims at total limit)(3)

	(6) = (5) / (4)	(7)	(8) = (6)(7)
Accident Year	Loading for 1,000,000 to Total Limit	Loadings for 500,000 to 1,000,000 Limit	Loadings for 500,000 to Total Limit
2016	1.138	1.182	1.345
2017	1.098	1.185	1.301
2018	1.063	1.270	1.350
2019	1.048	1.285	1.347

- (c) Recommend a loading for 500,000 to total limits for ratemaking purposes. Justify your recommendation.

Average of all years excluding 2017 = 1.348

Justification: 2017 appears to be an outlier so the average of all other years is reasonable.

- (d) Explain why severity trend is used for the part (b) calculation instead of pure premium trend.

Limiting claims to remove the effect of large claims does not affect the frequency of claims on a given portfolio; capping only affects the severities. Therefore, using pure premium trend would have overstated the large claim loading.

### GIRR Fall 2020 Question 16 (LOs 2d, 5b, 5e, 6g)

#### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 26, 27, and 32.

#### Commentary on Question:

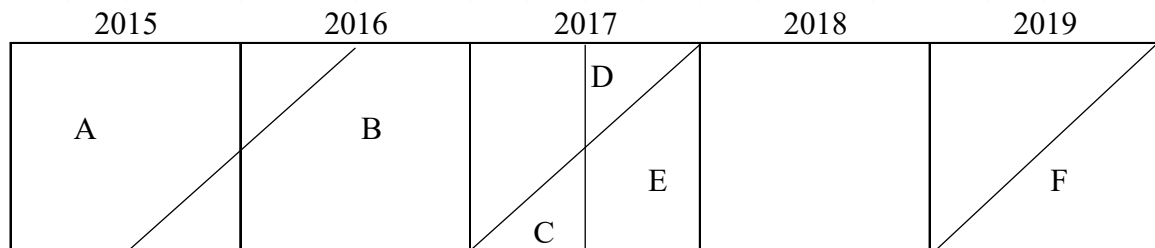
*This question tests basic ratemaking using a claim ratio approach. The candidate also needs to understand earned premiums adjusted to current rate level for ratemaking purposes.*

#### Solution:

- (a) Calculate premium on-level factors for accident years 2015-2019 to use for ratemaking purposes.

#### Commentary on Question:

*The diagram is helpful to solve the question but not required for credit.*





		<u>Percent Premium Earned in Each CY at Rate</u>				
	Rate Level	<u>Level</u>				
Level	Index	2015	2016	2017	2018	2019
A	1.00000	87.5%	12.5%			
B	1.08000	12.5%	87.5%	37.5%		
C	1.18800			12.5%		
D	0.86400			12.5%		
E	0.95040			37.5%	100.0%	50.0%
F	0.99792					50.0%
Total		100.0%	100.0%	100.0%	100.0%	100.0%

Average rate level in each CY:      1.01000   1.07000   1.01790   0.95040   0.97416

On-level factors for ratemaking:      0.9880   0.9326   0.9804   1.0500   1.0244

e.g.,     $0.97416 = 0.5 \times 0.95040 + 0.5 \times 0.99792$   
 $1.2044 = 0.99792 / 0.97416$

- (b) Calculate the trended on-level claim ratios for each accident year.

Trend from the average accident date in each AY (i.e., July 1) to the average accident date in future rating period.

Average accident date in future rating period: November 1, 2021

Accident Year	Trending Period in Years	Earned Premiums	<u>Premium Adj. Factors</u>		Trended Earned Prem. at Current Rate Level
			Trend at 1.00%	On-Level	
2015	6.333	11,755,570	1.0650	0.9880	12,370,486
2016	5.333	11,864,520	1.0545	0.9326	11,668,350
2017	4.333	12,406,530	1.0441	0.9804	12,698,923
2018	3.333	12,492,860	1.0337	1.0500	13,559,877
2019	2.333	12,394,530	1.0235	1.0244	12,995,072

e.g., for AY2019:     $1.0235 = 1.01^{2.333}$   
 $12,995,072 = 12,394,530 \times 1.0235 \times 1.0244$

Accident Year	Ultimate Claims	Pure Premium Trend Factor at 4.00%	Regulation Adjustment to Claims	Trended Claims	Trended Claim Ratio
2015	8,130,150	1.2820	0.80	8,338,086	67.40%
2016	7,970,110	1.2327	0.80	7,859,570	67.36%
2017	7,781,380	1.1853	0.90	8,300,615	65.36%
2018	8,001,680	1.1397	1.00	9,119,247	67.25%
2019	7,995,960	1.0958	1.00	8,762,239	67.43%

e.g., for AY 2019:  $1.0958 = 1.04^{2.333}$   
 $8,762,239 = 7,995,960 \times 1.0958 \times 1.00$   
 $67.43\% = 8,762,239 / 12,995,072$

- (c) Recommend a trended claim ratio to use for ratemaking. Justify your recommendation.

Accident Year	Trended Claim Ratio	Accident Year Weights
2015	67.40%	10%
2016	67.36%	15%
2017	65.36%	20%
2018	67.25%	25%
2019	67.43%	30%

Weighted average trended claim ratio = 66.96%

Justification: No significant outliers, so average of all years with more weight to more recent experience.

- (d) Calculate the indicated rate change.

Weighted average trended claim ratio	66.96%
Ratio of ULAE to claims	10.00%
Weighted average trended claim ratio including ULAE = $0.6696 \times (1 + 0.10) =$	73.65%
Fixed expenses as ratio to premiums at current rate level	6.00%
Variable expenses (ratio to premiums)	19.00%
Profit and contingencies ratio to premiums	5.00%
Permissible claim ratio = $(1 - 0.19 - 0.05) / (1 + 0.06/0.7365) =$	70.28%
Indicated rate change = $0.7365 / 0.7028 - 1 =$	4.81%

- (e) Explain why an indicated rate increase of 5% is not necessarily indicative of deteriorating experience.

We are told that rates were adequate at the time of the rate change. Therefore, if experience does not get better or worse after the change, then experience should change with expected net trend.

Net trend = (claim trend)/(premium trend) – 1 =  $(1 + 0.04) / (1 + 0.01) - 1 = 2.97\%$

Time from the change to the effective date of the new rates = 1.5 years

Therefore, experience should change with respect to net trend =  $(1 + 0.0297)^{1.5} - 1 = 4.5\%$

Since this is close to the rate change implemented at that time, this is as expected and does not indicate deteriorating experience.

## GIRR Spring 2021 Question 12 (LOs 5b, 5c, 5d, 5e, 6g, 6h)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6h) Demonstrate the use of credibility in ratemaking.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27 and 32.

### Commentary on Question:

*This question tests the candidate's understanding of premium trend. This question also tests basic ratemaking using a claim ratio approach incorporating the complement of credibility.*

### Solution:

- (b) Recommend the annual premium trend to use for ratemaking. Justify your recommendation.

### Commentary on Question:

*Change in annual written premium is needed to analyze the trend.*

Calendar Year	Average On-Level Written Premium (OLWP)	Year-to-Year Change in Average OLWP
2011	540.00	
2012	546.48	1.20%
2013	552.71	1.14%
2014	560.01	1.32%
2015	572.21	2.18%
2016	579.54	1.28%
2017	587.30	1.34%
2018	593.65	1.08%
2019	601.07	1.25%
2020	608.52	1.24%
Average all years		1.34%
Average excluding 2015 outlier		1.23%

Recommended annual trend = 1.23%.

Justification: Annual trend is reasonably stable except for 2015, which appears to be an outlier.

- (b) Calculate the trended claim ratio for each accident year.

Average earned premium date in the future rating period = 9 months after August 1, 2021 = May 1, 2022

Accident Year	<u>Average Earned Premium Date</u>		Trending Period (months)	Trended On-Level Earned Premium @1.23%	Trended Claim Ratios
	Experience Period	Forecast Period			
2016	2016-07-01	2022-05-01	70	9,065,912.50	75.42%
2017	2017-07-01	2022-05-01	58	8,888,948.54	72.76%
2018	2018-07-01	2022-05-01	46	8,419,705.00	69.45%
2019	2019-07-01	2022-05-01	34	8,166,989.29	70.21%
2020	2020-07-01	2022-05-01	22	8,435,636.28	67.27%
All year average:					71.02%
Excluding high/low:					70.81%
Average (2018-2020):					68.98%

- (c) Recommend a trended claim ratio to use for ratemaking. Justify your recommendation.

**Commentary on Question:**

*Other recommendations acceptable as long as the justification matches the data.*

Selected weighted average trended experience claim ratio: 70.81%.

Justification: Exclude high and low years to smooth the erratic values. No clear trend.

- (d) Calculate the claim ratio to use for the complement of credibility.

Indicated rate change for policies effective January 1, 2021 through June 30, 2021	4%
Approved rate change for policies effective January 1, 2021 through June 30, 2021	2%
Permissible claim ratio for policies effective January 1, 2021 through June 30, 2021	55%
Pure premium trend	5.0%
Premium trend	1.23%
Average accident date of prior filing	01-Apr-21
Average accident date of forecast period	01-May-22
Trending period in months	13
Complement of credibility claim ratio = $1.04/1.02 \times 0.55 \times (1.05/1.0123)^{(13/12)}$	58.34%

(e) Calculate the indicated rate change.

Selected trended claim ratio	70.81%
Credibility assigned to the experience claim ratio	77.00%
Complement of credibility	58.34%
Credibility weighed claim ratio	67.94%
<hr/>	
Indicated rate change = $(67.94\% + 15\%)/(1 - 11\% - 4\%) - 1 =$	-2.42%

## GIRR Spring 2021 Question 16 (LOs 6c, 6d)

### Learning Outcomes:

- (6c) Explain the requirements for loadings for catastrophes and large claims in ratemaking.
- (6d) Calculate loadings for catastrophes and large claims.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26 and 31.

### Commentary on Question:

*This question tests the candidate's understanding of loadings for large claims for ratemaking.*

### Solution:

- (a) Demonstrate that the all-years simple average of the loadings for large claims were calculated correctly in the table above.

Average earned date in rating period is 12 months following the effective date of the rates, or February 1, 2023.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Selected Ultimate Claims at Alternative Limits (000)			Trend Period (years)	Severity Trend at		
Accident Year	250,000	500,000	Total Limits		4.5%	5.0%	5.7%
2013	3,990	4,560	4,560	115	1.525	1.596	1.701
2014	3,988	3,988	3,988	103	1.459	1.520	1.609
2015	3,846	5,198	5,370	91	1.396	1.448	1.523
2016	4,301	6,367	6,829	79	1.336	1.379	1.440
2017	4,545	6,489	6,489	67	1.279	1.313	1.363
2018	4,256	4,256	4,256	55	1.224	1.251	1.289
2019	4,840	7,164	7,779	43	1.171	1.191	1.220
2020	5,038	7,349	7,349	31	1.120	1.134	1.154

Notes: (4) = average earned date in each year (i.e., July 1), to February 1, 2023.

(5): e.g.,  $1.171 = 1.045^{(43/12)}$

	(8) = (1)/(5)	(9) = (2)/(6)	(10) = (3)/(7)	(11) = (9)/(8)	(12) = (10)/(8)	(13) = (10)/(9)
	Trended Ultimate Claims at Limit			Loading for Large Claims		
Accident Year	250,000	500,000	Total Limits	250,000 to 500,000	250,000 to Total Limits	500,000 to Total Limits
2013	6,084	7,278	7,757	1.196	1.275	1.066
2014	5,819	6,062	6,418	1.042	1.103	1.059
2015	5,370	7,525	8,176	1.401	1.523	1.086
2016	5,747	8,779	9,837	1.528	1.712	1.121
2017	5,811	8,521	8,843	1.466	1.522	1.038
2018	5,207	5,323	5,487	1.022	1.054	1.031
2019	5,667	8,533	9,488	1.506	1.674	1.112
2020	5,645	8,336	8,481	1.477	1.502	1.017
Average				1.330	1.421	1.066

Therefore, the loadings provided were not calculated correctly.

- (b) Calculate the ultimate claims at total limits for each accident year from 2016 to 2020, using selected ultimate claims at the following limits:
- (i) 250,000
  - (ii) 500,000

**Commentary on Question:**

*Candidates can use either the loadings for large claims provided or the correct loadings calculated in part (a). The model solution shown here uses the loadings as provided in the question. Both solutions are shown in the Excel file.*

	Loading for Large Claims		
	250,000 to 500,000	250,000 to Total Limits	500,000 to Total Limits
Loadings for large claims	1.323	1.404	1.059
Countrywide	1.530		1.050
State X credibility	50.0%		20.0%
Credibility-weighted loading for large claims	1.42650	1.50039	1.05180

e.g.,  $1.05180 = 1.059 \times 0.2 + 1.05 \times 0.8$   
 $1.50039 = 1.42650 \times 1.05180$



	(14) = (7)/(5)		(15) = (7)/(6)		(16) = 1.50039/(14)	(17) = 1.0518/(15)	(18) = (1)(16)	(19) = (2)(17)
	Trend Factor for Large <u>Claim Loading</u>		Large Claim Loading <u>adjusted for cost level</u>		Indicated Ultimate Claims at Total Limits based on projections at <u>Alternative Limits (000)</u>			
	250,000 to	500,000 to	250,000 to	500,000 to				
Accident Year	Total Limits	Total Limits	Total Limits	Total Limits	250,000	500,000		
2016	1.078	1.045	1.392	1.007	5,986	6,410		
2017	1.066	1.038	1.408	1.013	6,398	6,577		
2018	1.054	1.031	1.424	1.020	6,060	4,342		
2019	1.042	1.024	1.440	1.027	6,971	7,358		
2020	1.030	1.017	1.457	1.034	7,339	7,598		
Total					32,754	32,285		

- (c) Explain why a loading for catastrophe claims might still be appropriate for the State X property business ratemaking despite including a loading for large claims.

Large claims and catastrophe claims are different. A large claim typically affects one policyholder for one insurer, whereas a catastrophe involves numerous claims involving many insurers. Ratemaking data may not include any catastrophe claims but exposure does exist and should be accounted for.

## GIRR Spring 2021 Question 18 (LOs 6a)

### Learning Outcomes:

- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

### Commentary on Question:

*This question tests the candidate's understanding of expenses used in ratemaking.*

### Solution:

- (a) Recommend a fixed and a variable expense ratio to use for ratemaking. Justify your recommendation.

### Commentary on Question:

*Justification for recommendations is required for full credit.*

Calendar Year	General and Other Acquisition Expenses Ratio	Commission Expenses	Premium Taxes and Licenses
2017	9.8%	12.0%	2.8%
2018	10.1%	12.0%	2.8%
2019	9.7%	12.0%	2.8%
2020	9.2%	12.0%	2.8%
Budgeted ratio:	10.0%		
Average:		12.0%	2.8%

Notes: General and other acquisition expenses are a percent of earned premiums  
Commission expenses and premium taxes and licenses are a percent of written premiums.

Recommended general and other acquisition expense ratio is 10%.

Justification: Budget is similar to all other prior years except 2020, so budget appears to be a reasonable ratio. Also, 2020 may be an outlier due to premium growth in excess premium growth exhibited in prior years.

Fixed expense ratio =  $10\% \times 30\% = 3\%$

Variable expense ratio for general and other acquisition expenses =  $10\% \times 70\% = 7\%$

Total variable expense ratio =  $7.0\% + 12.0\% + 2.8\% = 21.8\%$

- (b) Identify a potential distortion to a ratemaking analysis when selecting a fixed expense percentage that is applied to a projected average premium.

Any one of the following is acceptable:

1. Recent rate changes can result in differences in the relationship between the fixed expenses and premium during the experience period.
  2. Differences between the average premiums of the experience period and the forecast period that arise because of shifts in the mix of business may lead to inadequate or excessive expenses.
  3. A premium-based fixed expense ratio analysis may be distorted if countrywide expense ratios are used to project fixed expenses for a specific jurisdiction.
- (c) Recommend a solution to the potential distortion identified in part (b).

**Commentary on Question:**

*The recommended solution must match the distortion identified in part (b).*

1. Use premiums adjusted to on level.
2. Trend premiums.
3. Track fixed expenses by state and calculate fixed expense ratios for each state.

### GIRR Fall 2021 Question 4 (LOs 5b, 5e, 6d, 6e, 6g)

#### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6d) Calculate loadings for catastrophes and large claims.
- (6e) Apply loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27, 31, and 32.

#### Commentary on Question:

*This question tests the candidate's understanding of basic ratemaking, including the application of a loading for catastrophes in ratemaking.*

#### Solution:

- (a) Calculate the pure premium for the earthquake endorsement.

Midpoint of future rating period:	July 1, 2023
Exposure trend period (months): July 1, 2020 to October 1, 2020	3
Exposure trend = $(1.035^{(3/12)}) =$	1.00864
Severity trend period (months): October 1, 2020 to July 1, 2023	33
Severity trend = $(1.07^{(33/12)}) =$	1.20450
Trended modeled catastrophe claims = $225,000 \times 1.009 \times 1.204 =$	273,352.46
Trended exposures = $15,000 \times (1 + 0.035)^{((3 + 33)/12)} =$	16,630.77
Pure premium = $273,352.46 / 16,630.77 =$	16.44

- (b) Calculate the premium for the earthquake endorsement.

$$\text{Endorsement premium} = (16.44 + 5) / (1 - 0.1 - 0.25) = 30.77$$

- (c) Calculate the indicated rate for the basic homeowners coverage. Justify any selections.

	(1)	(2)	(3)	(4)	(5)	(6) = (5)/(4)
Accident Year	On Level Earned Premium (OLEP)	Ultimate Claims	Trend Period (years)	Trended OLEP	Trended Ultimate Claims	Claim Ratio
2018	15,500,000	9,000,000	5	17,113,252	12,622,966	0.7376
2019	16,250,000	8,000,000	4	17,589,523	10,486,368	0.5962
2020	17,000,000	8,200,000	3	18,040,536	10,045,353	0.5568

Notes: (3) For 2020: July 1, 2020 to July 1, 2023 = 3 years

(4) For 2020:  $18,040,536 = 17,000,000 \times 1.02^3$

(5) For 2020:  $10,045,353 = 8,200,000 \times 1.07^3$

Selected claim ratio = 0.5765 (average of 2019 and 2020 is used as 2018 is an outlier)

Indicated rate change =  $0.5765 / 0.57 - 1 = 0.0114$

Indicated rate =  $1,050 \times 1.0114 = 1,061.97$

- (d) State whether you agree with management's proposal. Justify your response.

**Commentary on Question:**

*Other justification responses are possible.*

Do not agree.

Justification: There is additional administrative cost related to this optional add-on, such as the mid-term addition or cancellation.

## GIRR Fall 2021 Question 15 (LOs 6g)

### Learning Outcomes:

- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 32.

### Commentary on Question:

*This question tests the candidate's understanding of basic ratemaking as well as forecasting profit.*

### Solution:

- (a) Demonstrate that the indicated rate change using the pure premium approach is 5.9%.

#### Commentary on Question:

*The pure premium approach is required for this part.*

Average earned premium at current rate level	
$= 8,100,000 \times 1.030 \times 1.007 / 11,000 =$	763.76
Trended ultimate claims $= 0.78 \times 11,000 \times 763.76 =$	6,553,093
Trended pure premium $= 6,553,093 / 11,000 =$	595.74
Total fixed expenses $= 0.05 \times 763.76 =$	38.19
Indicated rate $= (595.74 \times (1 + 0.09) + 38.19) / (1 - 0.10 - 0.05) =$	808.87
Indicated rate change $= 808.87 / 763.76 - 1 =$	5.91%

- (b) Calculate the forecasted profit per policy for policies written in 2022, 2023, 2024 and 2025.

#### Commentary on Question:

*Use the equation: Premiums = Claims + Expenses + Profit and Contingencies, to compare the per policy charged premium to the per policy expenses plus claims each year to solve for the profit per policy.*

	(1)	(2) = (1)×0.05	(3) = (1)×0.1	(4)	(5)	(6)
Calendar Year	Required Premium	Fixed Expenses	Variable Expenses	PP with ULAE	Charged Premium	Profit
2022	808.87	40.44	80.89	649.35	794.31	23.63
2023	812.11	40.61	81.21	662.39	794.31	10.11
2024	815.35	40.77	81.54	675.69	794.31	−3.67
2025	818.62	40.93	81.86	689.25	794.31	−17.73

Notes:

- (1) Required premium for 2023 =  $808.87 \times 1.004$  (increase with premium trend each year)
- (4) PP with ULAE for 2023 =  $595.74 \times 1.09 \times 1.015 \times 1.005$  (increase with frequency and severity trend each year)
- (5)  $763.76 \times 1.04$
- (6) = (5) − (2) − (3) − (4)

## GIRR Fall 2021 Question 20 (LOs 6a)

### Learning Outcomes:

- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

### Commentary on Question:

*This question tests the candidate's understanding of expenses used for ratemaking.*

### Solution:

- (a) Calculate the total variable expense ratio for each calendar year.

Calendar Year	(1) General Expenses	(2) As a % of Premiums	(3) Commission and Premium Tax Expense Ratio	(4) = (2) + (3) Total Variable Expense Ratio
	Variable			
2018	1,016,250	11.68%	15.91%	27.59%
2019	1,087,500	11.57%	15.83%	27.40%
2020	1,117,500	11.29%	15.88%	27.17%

Notes: (1) = 75% × (General Expenses)

(2) = (1) / (Direct Earned Premium)

(3) = (Total Commission Expenses and Premium Taxes) / (Direct Written Premium)

- (b) Recommend the total variable expense ratio to use in ratemaking. Justify your recommendation.

Calendar Year	Variable General Expense Ratio	Commission and Premium Tax Expense Ratio	Total Variable Expense Ratio
2018	11.68%	15.91%	27.59%
2019	11.57%	15.83%	27.40%
2020	11.29%	15.88%	27.17%
Average	11.51%	15.87%	27.39%
Selection:	11.29%	15.87%	27.16%

Justification:



- Variable general expense ratio is decreasing, so recommend the latest year of 11.29% to reflect the decrease
- Commission and premium tax expense ratio is steady so recommend the average of all 3 years, or 15.87%
- Recommended total variable expense ratio =  $11.29\% + 15.87\% = 27.16\%$

(c) Recommend the fixed expense per exposure to use in ratemaking. Justify your recommendation.

	(5)	(6)
Calendar Year	Fixed General Expense (000)	Fixed General Expense Per Exposure
2018	338,750.0	10.42
2019	362,500.0	10.76
2020	372,500.0	10.61
Average		10.60

Notes: (5) =  $25\% \times (\text{General Expenses})$

(6) = (5) / (Earned Exposures)

Recommended fixed general expense per exposure = 10.60 (no significant trend so average of all 3 years is reasonable)

Provision for new system =  $1,200,000 / 37,000 / 5 = 6.49$  (amortize over 5 years)

Recommended fixed expense per exposure to use in ratemaking =  $10.60 + 6.49 = 17.09$

## **GIRR Spring 2022 Question 10 (LOs 6a)**

### **Learning Outcomes:**

- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

### **Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

### **Commentary on Question:**

*This question tests the candidate's understanding of expenses used in ratemaking.*

### **Solution:**

- (a) Describe the five major categories of expenses that are considered in a ratemaking analysis as defined by U.S. Standards.

### **Commentary on Question:**

*Candidates need to describe each expense in addition to simply listing the expenses.*

- Loss adjustment expenses: expenses associated with investigating adjusting administering and settling claims
  - Commission and brokerage fees: the compensation paid to agents and brokers for generating business
  - Other acquisition expenses: all costs other than commissions and brokerage fees associated with the acquisition of business
  - General administrative expenses: operational and administrative expenses (other than investment expenses)
  - Taxes, licenses and fees: all taxes and miscellaneous fees except federal and foreign income taxes
- (b) Describe two different ways for an insurer to incorporate non-proportional reinsurance in a ratemaking analysis.
1. Conduct the ratemaking analysis net of reinsurance excluding ceded premiums and ceded claims.
  2. Conduct the ratemaking analysis on a gross of reinsurance basis and include the net cost of reinsurance as an expense.
- (c) Describe the purpose of a residual market mechanism.

A residual market mechanism provides a means of obtaining coverage for individuals or organizations who are unable to secure insurance protection in the open market.

- (d) Describe each of the following as used in U.S. workers compensation ratemaking:
- (i) An expense constant
  - (ii) A premium discount plan
  - (i) A fixed/flat expense per policy for administrative costs that do not vary with premium.
  - (ii) A premium discount to recognize the administrative cost savings associated with larger insureds with higher premiums.

### GIRR Spring 2022 Question 17 (LOs 5b, 5e, 6d, 6e, 6g, 6h)

#### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6d) Calculate loadings for catastrophes and large claims.
- (6e) Apply loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6h) Demonstrate the use of credibility in ratemaking.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27, 31, and 32.

#### Commentary on Question:

*This question tests the candidate's understanding of basic ratemaking, including the application of a loading for wildfire claims in ratemaking.*

#### Solution:

- (a) Calculate the ultimate pure premium for wildfire claims to be used as a loading in the homeowners premiums.

Average accident date in future rating period: Sep. 1, 2023

	(1)	(2)	(3)	(4)	(5)
		Wildfire - Ultimate			
Accident Year	Earned Exposures	Counts	Claims	Trending Period (years)	Severity Trend @3%
2015	11,200	0	0	8.167	1.2730
2016	11,850	0	0	7.167	1.2359
2017	12,500	1	1,500,000	6.167	1.1999
2018	13,750	0	0	5.167	1.1650
2019	15,000	1	1,120,000	4.167	1.1311
2020	16,250	0	0	3.167	1.0981
2021	17,500	1	500,000	2.167	1.0661
Total	98,050	3	3,120,000		

$$(6) = (3)(5) \quad (7) = (2)/(1) \quad (8) = (6)/(2) \quad (9) = (6)/(1)$$

Trended Ultimate Wildfire

Accident Year	Trended Ultimate Claims	Frequency	Severity	Pure Premium
2015	0	0.000000	0	0.00
2016	0	0.000000	0	0.00
2017	1,799,924	0.000080	1,799,924	143.99
2018	0	0.000000	0	0.00
2019	1,266,795	0.000067	1,266,795	84.45
2020	0	0.000000	0	0.00
2021	533,070	0.000057	533,070	30.46
Total	3,599,789	0.000031	1,199,930	36.71

- (b) Calculate the indicated total premium for the homeowners coverage, including a loading for wildfire claims.

	Credibility	Trended Ultimate Pure Premium
Insurer internal experience from part (a)	20%	36.71
Industry experience	80%	50.00
Credibility weighted wildfire claims experience (at Sept. 1, 2023 cost level): $0.2 \times 36.71 + 0.8 \times 50.00$		47.34
Non-wildfire claims per policy (PP) as of July 1, 2021: $21,507,500 \times 0.67 / 17,500 =$		823.43
Trended non-wildfire PP to future rating period = $823.43 \times [(1 + 0.04) / (1 + 0.025)]^{2.167} =$		849.76
Indicated premium = $(849.76 + 47.34 + 70) / (1 - 0.2 - 0.05) =$		1,289.47

## GIRR Fall 2022 Question 5 (LOs 6a)

### Learning Outcomes:

- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

### Commentary on Question:

*This question tests the candidate's understanding of expenses used in ratemaking.*

### Solution:

- (a) Calculate the historical annual trend in fixed expenses.

Calendar Year	Fixed Expense per On-Level Earned Premium	Change in Fixed Expense per On- Level Earned Premium
2016	3.110%	
2017	3.216%	3.420%
2018	3.320%	3.220%
2019	3.423%	3.120%
2020	3.526%	2.981%
2021	3.632%	3.020%

e.g., for 2019:

- $3.423\% = 684,470 / 19,993,320$
- $3.120\% = 3.423\% / 3.320\% - 1$

- (b) Recommend the annual fixed expense trend. Justify your recommendation.

### Commentary on Question:

*Other recommendations are acceptable with appropriate justification.*

Average all years:	3.15%
Average most recent 3 years:	3.04%
Recommendation:	3.04%

Justification: There is a clear decreasing trend rate so give more weight to more recent years and select the average of the latest 3 years.

- (c) Calculate the fixed expense ratio to be used in ratemaking, using a simple average from calendar years 2019, 2020 and 2021.

Average incurred date in rating period: June 1, 2024 (i.e., 12 months after effective date)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	<u>Average Incurred Date</u>						Fixed
Calendar Year	Experience Period	Forecast Period	Trend Period (months)	Expense Trend Factors	Trended Fixed Expenses	Trended On-Level Earned Premiums	Expense per On-Level Earned Premiums
2019	July 1, 2019	June 1, 2024	59	1.1586	793,049	20,995,763	3.78%
2020	July 1, 2020	June 1, 2024	47	1.1244	825,626	21,654,218	3.81%
2021	July 1, 2021	June 1, 2024	35	1.0913	864,682	22,458,417	3.85%
Average							3.81%

Notes:

(3) = number of months from (1) to (2)

(4) =  $1.0304^{(3)/12}$

(5) = (4) × (Trended Fixed Expenses)

(6) = (Earned Premiums at Current Rate Level) ×  $1.01^{(3)/12}$

(7) = (5) / (6)

## GIRR Fall 2022 Question 14 (LOs 5b, 5c, 5d, 5e, 6e, 6g, 6h)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6e) Apply loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6h) Demonstrate the use of credibility in ratemaking.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26, 31, and 32.

### Commentary on Question:

*This question tests the candidate's understanding of loadings for large claims as well as basic ratemaking.*

### Solution:

- (a) Describe one way that large claims are differentiated from catastrophe claims when insurers are estimating loadings for ratemaking purposes.

Any one of the following are acceptable:

- Catastrophes typically result in GI claims for multiple insurers providing coverage in an affected area. Whereas large losses are limited to a few claims for an individual insurer.
- Catastrophes are associated with an event which is infrequent and results in unusually large aggregate losses.
- Catastrophes typically result in a significant number of GI claims for multiple insurers providing coverage in the area affected by the event. Large claims do not typically affect the entire GI industry, or even all GI companies operating in a specific area.

- (b) Recommend the annual pure premium trend for weather claims. Justify your recommendation.

Accident Year	Pure Premium per 100 EHY	Year-to-Year Change
2010	5,280	
2011	5,770	9.3%
2012	6,330	9.7%
2013	6,200	-2.1%



Accident Year	Pure Premium per 100 EHY	Year-to-Year Change
2014	6,920	11.6%
2015	7,140	3.2%
2016	7,560	5.9%
2017	8,300	9.8%
2018	8,460	1.9%
2019	8,850	4.6%
2020	9,400	6.2%
2021	9,940	5.7%
Average - all years		6.0%
Average - latest 5 years		5.7%
Average - all years excl. high & low		6.3%
Recommendation		6.3%

Justification: Include more years due to significant volatility. Excluding high & low eliminates outliers.

- (c) Recommend the trended ultimate pure premium for weather claims per 100 EHY to use in ratemaking. Justify your recommendation.

Accident Year	Trending Period (months)	Pure Premium Trend Factor	Trended Ultimate Pure Premium for Non-Hurricane Weather excluding Hail per 100 EHY
2010	169	2.3514	12,415
2011	157	2.2129	12,768
2012	145	2.0825	13,183
2013	133	1.9599	12,151
2014	121	1.8444	12,763
2015	109	1.7358	12,393
2016	97	1.6335	12,350
2017	85	1.5373	12,760
2018	73	1.4468	12,240
2019	61	1.3615	12,050
2020	49	1.2813	12,045
2021	37	1.2059	11,986
Average (all years)			12,425
Average (latest 5 years)			12,216

Average (latest 3 years) 12,027

Recommendation 12,027

Justification: Decreasing values in latest years so more weight to more recent data. Therefore, recommend average of latest 3 years.

- (d) Calculate the indicated rate level change, including a loading for weather claims.

Accident Year	Trended Earned Premiums at Current Rate Level	Trended Ultimate Claims	Claim Ratio	Accident Year Weights
2019	12,545,160	7,130,200	56.84%	25%
2020	12,777,120	7,449,200	58.30%	30%
2021	12,613,560	6,824,400	54.10%	45%
Weighted average trended claim ratio			56.05%	

- |      |  |            |
|------|--|------------|
| (1)  | Selected non-hurricane weather excluding hail pure premium per 100<br>EHY:                   | 12,027     |
| (2)  | CY2021 earned house years  | 16,860     |
| (3)  | CY2021 trended earned premiums at current rate level   | 12,613,560 |
| (4)  | Loading for non-hurricane weather expressed as a claim ratio =<br>$((1)/100) \times (2)/(3)$ | 16.08%     |
| (5)  | ULAE to claim ratio  | 12%        |
| (6)  | Total claim ratio including ULAE = $(56.05\% + (4))(1 + (5))$                                | 80.78%     |
| (7)  | Credibility of experience period = $\text{squareroot}(49,500 / 80,000)$                      | 78.66%     |
| (8)  | Countrywide trended, adjusted ultimate claim, including ULAE, ratio                          | 70%        |
| (9)  | Credibility-weighted experience claim, including ULAE, ratio<br>= $(6)(7) + [1 - (7)](8)$    | 78.48%     |
| (10) | Selected fixed expenses to premiums ratio  | 5%         |
| (11) | Selected variable expenses to premiums ratio   | 15%        |
| (12) | Selected profit and contingencies to premiums ratio  | 4%         |
| (13) | Indicated rate level change = $[(9) + (10)]/[1 - (11) - (12)] - 1$                           | 3.06%      |

## GIRR Spring 2023 Question 4 (LOs 6a)

### Learning Outcomes:

- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

### Commentary on Question:

*This question tests the candidate's understanding of different types of expenses required for ratemaking.*

### Solution:

- (a) Recommend the total variable expense ratio to use in ratemaking. Justify your recommendation.

	(1)	(2)	(3)	(4)	(5)
Calendar Year	Earned Exposures	Direct Written Premium	Direct Earned Premiums	Total Commission Expenses and Premium Taxes	General Expenses
2019	8,700	7,447,430	7,377,050	670,269	243,420
2020	9,150	7,895,360	7,846,640	710,582	253,065
2021	9,340	8,112,390	8,090,270	730,115	260,640
2022	9,240	8,097,340	8,083,570	728,761	268,436
2023 Budget	9,120	8,050,000	8,048,900	724,500	285,000

	(6) = (5) × 75%	(7) = (6) / (3)	(8) = (4) / (2)
Calendar Year	General Expenses	As a % of Earned Premiums	Commission and Premium Tax Expense Ratio
2019	182,565	2.47%	9.00%
2020	189,799	2.42%	9.00%
2021	195,480	2.42%	9.00%
2022	201,327	2.49%	9.00%
2023 Budget	213,750	2.66%	9.00%
Recommended		2.66%	9.00%

Total variable expense ratio = 2.66% + 9.00% = 11.66%

Justification: There is a significant increase expected from budget, so give more consideration to the budget.

- (b) Recommend the fixed expense per exposure to use in ratemaking. Justify your recommendation.

$$(10) = (5) \times 25\% \quad (11) = (10) / (1)$$

Calendar Year	Fixed General Expense (000)	Fixed General Expense Per Exposure
2019	60,855	6.99
2020	63,266	6.91
2021	65,160	6.98
2022	67,109	7.26
2023 Budget	71,250	7.81
Selection:		7.81

Justification: There is a significant increase expected from budget, so give more consideration to the budget.

Provision for new system =  $2,500,000 / 9,120 / 4 = 68.53$  (amortized over 4 years)

Total:  $7.81 + 68.53 = 76.34$

### GIRR Spring 2023 Question 5 (LOs 5b, 5c, 5d, 5e, 6g)

#### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27 and 32.

#### Commentary on Question:

*This question tests the candidate's understanding of trending premiums and indicated rate changes using claim ratios.*

#### Solution:

- (a) Recommend the annual premium trend due to the shift in policy limits to use for ratemaking. Justify your recommendation.

#### Commentary on Question:

*The year-to-year change in **average** increased limit factor (ILF) needs to be analyzed for the trend due to shift in policy limits.*

Experience Period	Weighted Average ILF	Annual Trend Due to Shift in ILF
2015	1.00018	
2016	1.00270	0.25%
2017	1.00603	0.33%
2018	1.00877	0.27%
2019	1.01202	0.32%
2020	1.01500	0.29%
2021	1.01769	0.27%
2022	1.01924	0.15%
Average:		0.27%
Average excluding high & low:		0.28%

Recommended trend: 0.28%

Justification: average excluding high and low removes the outliers, especially 2022.

- (b) Calculate the indicated rate level change for this line of business using a claims ratio approach. Justify any selection(s).

Average earned premium dates in 2022:	Jul. 1, 2022	
Effective date of new rates:	Sep. 1, 2023	# of months
Average earned premium dates in future rating period:		<u>trending period</u>
for 12-month policies	Sep. 1, 2024	26
for 6-month policies	Jun. 1, 2024	23
Average:		25.25

Annual premium trend =  $(1 + 0.28\%)(1 + -0.1\%) - 1 = 0.180\%$

Annual pure premium trend =  $(1 + 6\%)(1 + -1.2\%) - 1 = 4.728\%$

	(1)	(2)	(3)	(4) = (1.0018) <sup>[(2)/12]</sup>	(5) = (1)(3)(4) Earned Premiums Trended at Current Rates
Accident Year	Earned Premiums	Trending Period (months)	Premium On- Level Factors	Premium Trend Factors	
2018	15,804,847	73.25	1.064	1.01102	17,001,688
2019	15,333,428	61.25	1.106	1.00921	17,114,913
2020	15,526,085	49.25	1.104	1.00740	17,267,582
2021	16,625,910	37.25	1.049	1.00559	17,538,061
2022	17,102,494	25.25	1.026	1.00379	17,613,581

	(6)	(7) = (1.04728) <sup>[(2)/12]</sup>	(8) = (6)(7) Trended Ultimate Claims	(9) = (8) / (6) Claim Ratio
Accident Year	Ultimate Claims	Claim Trend Factors		
2018	8,703,669	1.32577	11,539,025	67.87%
2019	9,184,011	1.26591	11,626,161	67.93%
2020	9,602,493	1.20876	11,607,137	67.22%
2021	10,401,614	1.15419	12,005,466	68.45%
2022	11,309,041	1.10209	12,463,536	70.76%
		Average:		68.45%
		Average latest 3 years:		68.81%

Selected claim ratio = 68.81%

Justification for selected claim ratio: Increasing in most recent years, so give more weight to more recent 3 years.

Indicated rate change:  $[0.6881(1 + 0.07) + 0.05]/(1 - 0.23 - 0.04) = 7.71\%$

- (c) Describe one reason why an indicated rate change using a pure premium approach may not result in the same result as part (b).

The premium on-level factors are an approximation used to restate historical earned premiums as if they were at the current rate level for the forecast period.

- (d) Calculate the profit and contingencies to premium ratio implied by a 3% rate increase using your colleague's indicated rate change.

Claim ratio implied by the 6% rate indication:

$(\text{Claim ratio} + 0.05)/(1 - 0.23 - 0.04) - 1 = 0.06 \rightarrow \text{claim ratio} = 72.38\%$

Profit margin implied by a 3% rate change:

$(0.7238 + 0.05)/(1 - 0.23 - Q) - 1 = 0.03$

$\rightarrow Q = 1.87\%$

- (e) State two actions the company can take that could help achieve the target profit, given the 3% rate increase.

**Commentary on Question:**

*Other actions are possible.*

- decrease expenses
- decrease claims (e.g., changing mix of business, better risk selection)

## GIRR Spring 2023 Question 8 (LOs 5a, 5b, 5e, 6c, 6d)

### Learning Outcomes:

- (5a) Identify and describe the influences of portfolio changes on claim frequency and severity.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6c) Explain the requirements for loadings for catastrophes and large claims in ratemaking.
- (6d) Calculate loadings for catastrophes and large claims.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26, 27, and 31.

### Commentary on Question:

*This question tests the candidate's understanding of the loading for catastrophes in ratemaking.*

### Solution:

- (a) Explain why two trend adjustments must be made to the modeled expected earthquake claims to calculate the catastrophe loading for ratemaking.

Past adjustment: modeled catastrophe claims must be trended from February 1, 2022 to July 1, 2022 for exposure trend to reflect in-force exposures as of July 1, 2022

Future adjustment: modeled catastrophe claims must be trended from July 1, 2022 to mid-point of future rating period for severity trend to reflect the cost level in the future rating period

- (b) Calculate the catastrophe loading to be used for ratemaking, as a claim ratio.

Midpoint of future rating period:	October 1, 2024
Exposure trend period (months): February 1, 2022 to July 1, 2022	5
Exposure trend = $1.01^{(5/12)}$ =	1.00415
Severity trend period (months): July 1, 2022 to October 1, 2024	27
Severity trend = $1.06^{(27/12)}$ =	1.14009
Trended modeled catastrophe claims = $450,000 \times 1.00415 \times 1.14009$ =	515,170.85
Catastrophe loading = $515,170.85 / 15,450,000$ =	3.33%

- (c) Describe an additional step or approach that would increase your confidence in the estimate of expected earthquake claims.

Running alternative catastrophe models would increase confidence in the estimate of expected earthquake claims.



- (d) Describe how you would consider the effect of a demand surge in the calculation of the catastrophe loading for ratemaking.

Demand surge can result in a trend rate that is higher post-catastrophe than pre-catastrophe. Therefore, could recognize a demand surge by selecting a higher post event claim severity trend rate.

### GIRR Fall 2023 Question 3 (LOs 5b, 5e, 6d)

#### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6d) Calculate loadings for catastrophes and large claims.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26 and 31.

#### Commentary on Question:

*This question tests the candidate's understanding of a loading for large claims that is used in ratemaking.*

#### Solution:

- (a) State two reasons for using a large claim loading approach when estimating ultimate claims at total limits for ratemaking.

Any two of the following are acceptable:

- The loading factor smooths the influence of large claims over time
- The actuary can introduce a greater volume of experience
- The claims at a limited value are more reliable

- (b) Calculate the large claim loadings at 500,000 limit, adjusted to the cost level for each accident year.

Average earned date in rating period: July 1, 2024 (i.e., 9 months after effective date)  
July 1, 2022 to July 1, 2024 = 24 months

Accident Year	Trending Period (months)	(1) Severity	(2) Trend Factor at:	(3) = (2)/(1) Trend Factors for Loading for Large Claims	(4) = 1.28 / (3) Loadings for Large Claims Adjusted to Cost Level of AY
		5.0%	7.0%		
2019	60	1.276	1.403	1.099	1.165
2020	48	1.216	1.311	1.078	1.187
2021	36	1.158	1.225	1.058	1.210
2022	24	1.103	1.145	1.038	1.233

- (c) Calculate ultimate claims at total limits for each accident year using selected ultimate claims at a 500,000 limit and the large claim loadings from part (b).

	(4)	(5)	(6) = (4)(5)
Accident Year	Selected Ultimate Claims at 500,000 Limit	Loadings for Large Claims Adjusted to Cost Level of AY	Indicated Ultimate Claims at Total Limits based on Projections at 500,000 Limits
2019	9,850,000	1.165	11,472,916
2020	10,365,000	1.187	12,302,726
2021	11,275,000	1.210	13,637,761
2022	12,385,000	1.233	15,265,711

- (d) Describe how the calculations in part (b) are affected when the experience is less than fully credible.

**Commentary on Question:**

*This question asks to describe how the calculations in part (b) are affected when the experience is less than fully credible. Providing an explanation of what credibility is does not answer the question.*

The calculations are affected in two ways:

- Need to develop credibility-weighted trend rates
- Need to develop credibility-weighted loadings

## GIRR Fall 2023 Question 11 (LOs 5b, 5c, 5d, 5e, 6a)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27 and 30.

### Commentary on Question:

*This question tests the candidate's understanding of expenses used for ratemaking, including trending of fixed expenses.*

### Solution:

- (a) Identify why a separate trending procedure for fixed expenses may not be required when analyzed on a per-exposure basis.

### Commentary on Question:

*Some candidates misunderstood this question and answered assuming no trend would be needed for fixed expenses as opposed to asking why a separate trending procedure for fixed expenses may not be required.*

When the forces affecting changes in expenses (i.e., the expense trend) are similar to those driving changes in premiums, a separate trend adjustment for fixed expenses may not be necessary.

- (b) Recommend an annual fixed expense trend. Justify your recommendation.

Calendar Year	Fixed Expense to Earned Premiums at Current Rates Ratio	Year-to- Year Change
2016	6.84%	
2017	7.03%	2.77%
2018	7.13%	1.52%
2019	7.29%	2.24%
2020	7.51%	2.96%
2021	7.68%	2.26%

2022	7.92%	3.13%
Average - All years		2.48%
Average - excl hi-lo		2.56%
Recommended fixed expense trend:		2.56%

Justification: 2018 appears to be an anomaly. Exclude highest and lowest to smooth out the variation.

- (c) Recommend a fixed expense ratio to be used in ratemaking. Justify your recommendation.

Average earned premium date in 2022	1-Jul-22	
Average earned premium dates in future rating period:		# months:
for 12-month policies	1-Nov-24	28

Calendar Year	Trending Period (months)	Trending Period (years)	Expense Trend at 2.56%	Trended Fixed Expenses	Trended Fixed Expense Ratio
2016	100	8.33	1.2343	569,624	8.44%
2017	88	7.33	1.2035	594,138	8.46%
2018	76	6.33	1.1735	622,353	8.37%
2019	64	5.33	1.1442	653,791	8.34%
2020	52	4.33	1.1157	694,861	8.38%
2021	40	3.33	1.0878	723,949	8.35%
2022	28	2.33	1.0607	769,701	8.40%

Average - all years 8.39%

Recommended trended fixed expense ratio: 8.39%

Justification: No significant outliers and no significant trend, so all years average is reasonable.

## GIRR Fall 2023 Question 12 (LOs 5b, 5c, 5d, 5e, 6g, 6h)

### Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6h) Demonstrate the use of credibility in ratemaking.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26 and 32.

Actuarial Standards of Practice, Actuarial Standards Board of the American Academy of Actuaries, No. 25, Credibility Procedures, 2013.

### Commentary on Question:

*This question tests basic ratemaking using a pure premium approach, including the application of credibility.*

### Solution:

- (a) Calculate the trended pure premiums for each accident year.

Past pure premium trend:  $(1.06)(1 - 0.01) - 1 = 4.94\%$

Future pure premium trend:  $(1.06)(1.01) - 1 = 7.06\%$

Past trend period: For AY2022, from average accident date in AY2022 (July 1, 2022) to December 31, 2022 = 0.5 years

Future pure premium trend period: From average accident date in 2022 to average accident date in future rating period: 1/1/2023 to 3/1/2025 = 26 months, or 2.167 years

Accident Year	Earned Exposures	Ultimate Claims	Pure Premium (PP)	Past Trending Period (yrs.)	Future Trending Period (yrs.)
2018	10,146	13,085,953	1,289.76	4.5	2.167
2019	10,127	14,011,147	1,383.54	3.5	2.167
2020	10,298	14,968,858	1,453.57	2.5	2.167
2021	10,291	15,499,745	1,506.15	1.5	2.167
2022	10,573	18,068,228	1,708.90	0.5	2.167

Accident Year	Future Trending Period (yrs.)	Past PP Trend	Future PP Trend	Total PP Trend	Trended Ultimate Pure Premium
2018	2.167	1.2423	1.1593	1.4402	1,857.54
2019	2.167	1.1838	1.1593	1.3724	1,898.80
2020	2.167	1.1281	1.1593	1.3078	1,900.99
2021	2.167	1.0750	1.1593	1.2462	1,877.03
2022	2.167	1.0244	1.1593	1.1876	2,029.46

- (b) Recommend a trended pure premium. Justify your recommendation.

**Commentary on Question:**  
*Other weights are possible.*

AY	Trended Ultimate Pure Premium	Weights
2018	1,857.54	22.50%
2019	1,898.80	22.50%
2020	1,900.99	22.50%
2021	1,877.03	22.50%
2022	2,029.46	10.00%

Averages

- all years straight	1,912.76
- weighted	1,898.18
Recommended:	1,898.18

Justification: AY2022 is possibly an anomaly so less weight for that year. Include all years due to credibility (i.e., all years is 4,341 ultimate counts, so still not fully credible even using all years).

- (c) Calculate the pure premium to use for the complement of credibility.

Pure premium trend (future, since future trend started Jan. 1, 2023)	7.06%
Average accident date of prior filing	Jul. 1, 2023
Average accident date of forecast period	Mar. 1, 2025
Trending period in months	20
Pure premium used for complement of credibility: $[1,700(1.076)^{(20/12)}] =$	1,904.70

- (d) Calculate the credibility-weighted indicated rate.

**Commentary on Question:**

*The number of claims for credibility need to match the years that were included in the part (b) selection. For example, if only the most recent three years were included in the selection in part (b), then the number of claims to use for credibility in this part should be:  $2,610 = 875 + 852 + 883$ .*

Weighted average pure premium (from part (b)):	1,898.18
Number of claims to use for credibility:	4,341
Credibility: $(4,341 / 4,654)^{0.5}$	96.58%
Credibility-weighted pure premium: $1,898.18 \times 96.58\% + 1,904.70(1 - 96.58\%)$	1,898.40
Indicated rate: $(1,898.40(1.04) + 125) / (1 - 0.18 - 0.05)$	2,726.41

- (e) Identify one adjustment that is necessary when relying on a complement of credibility that is a pure premium based on industry experience.

Either of the following is acceptable:

- adjusted to reflect the insurer's mix of business
- adjusted to the cost level of the forecast period



## GIRR Spring 2024 Question 5 (LOs 2d, 5b, 5e, 6f, 6g, 6h, 6j, 6k)

### Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.
- (5b) Identify the time periods associated with trending procedures.
- (5e) Calculate trend factors for claims and exposures.
- (6f) Explain the requirements for loadings for catastrophes and large claims in ratemaking.
- (6g) Calculate loadings for catastrophes and large claims.
- (6h) Apply loadings for catastrophes and large claims in ratemaking.
- (6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6k) Demonstrate the use of credibility in ratemaking.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 26, 31, and 32.

### Commentary on Question:

*This question tests the candidate's ability to calculate the indicated average rate, while considering adjustments to earned premium and a loading for non-hurricane weather claims.*

### Solution:

- (a) Calculate the trended ultimate non-hurricane weather excluding hail pure premium per 100 EHY for all years.

Average accident date in future rating period: June 1, 2025 (9 months after start date).

# months from 2023 average accident date to June 1, 2025: 23

Accident Year	Ultimate		Trending Period (months)	Trend Factors	
	Frequency per 100 EHY	Severity		Frequency @ -1.0%	Severity @ 5.0%
2014	2.02	4,100	131	0.8961	1.7034
2015	0.39	3,500	119	0.9051	1.6223
2016	1.99	2,900	107	0.9143	1.5450
2017	0.1	4,400	95	0.9235	1.4715
2018	1.99	2,800	83	0.9328	1.4014
2019	0.8	4,200	71	0.9423	1.3347
2020	0.63	2,600	59	0.9518	1.2711
2021	2.73	3,600	47	0.9614	1.2106
2022	0.56	2,100	35	0.9711	1.1529
2023	1.69	3,100	23	0.9809	1.0980

Accident Year	Frequency per 100 EHY	<u>Trended Ultimate</u>	
		Severity	Pure Premium per 100 EHY
2014	1.810	6,983.94	12,642
2015	0.353	5,678.00	2,004
2016	1.819	4,480.60	8,152
2017	0.092	6,474.43	598
2018	1.856	3,923.89	7,284
2019	0.754	5,605.56	4,226
2020	0.600	3,304.87	1,982
2021	2.625	4,358.07	11,438
2022	0.544	2,421.15	1,317
2023	1.658	3,403.88	5,643
Average:			
-all years	1.211	4,663.44	5,529

- (b) Recommend the trended ultimate non-hurricane weather excluding hail pure premium per 100 EHY to use in determining a weather loading. Justify your recommendation.

Recommend all years average: 5,529

Justification: should use more years to smooth out fluctuations; no significant trend.

- (c) Calculate the non-hurricane weather excluding hail loading percentage to use for ratemaking.

Selected state S PP per 100 EHY 5,529

Credibility-Weighted Pure Premium per 100 EHY 5,069.96

Expected Non-Hurricane Weather Claims 909,095.18

Weather loading as a claim ratio =  $909,095 / 13,089,711 = 6.95\%$

- (d) Identify two considerations when choosing the number of years and/or the weights to assign to each of the years.

Any 2 of the following are acceptable:

- professional judgment
- assessment of the relevance and reliability of the insurer's historical experience
- whether there are regulation requirements
- balance between stability and responsiveness
- management input

- credibility consideration - want enough years for full credibility, if possible
- also acceptable to note that give more weight to recent experience to account for recent changes

- (e) Recommend the number of years to include when estimating the weighted average trended claim ratio for the indicated rate change. Justify your recommendation.

AY	Ultimate Counts	Running Total
		Ultimate Counts
2019	1,070	5,447
2020	1,075	<b>4,377</b>
2021	1,074	3,302
2022	1,141	2,228
2023	1,087	1,087

Recommend 4 years.

Justification: Full credibility (3,654) is met by including at least the most recent 4 years.

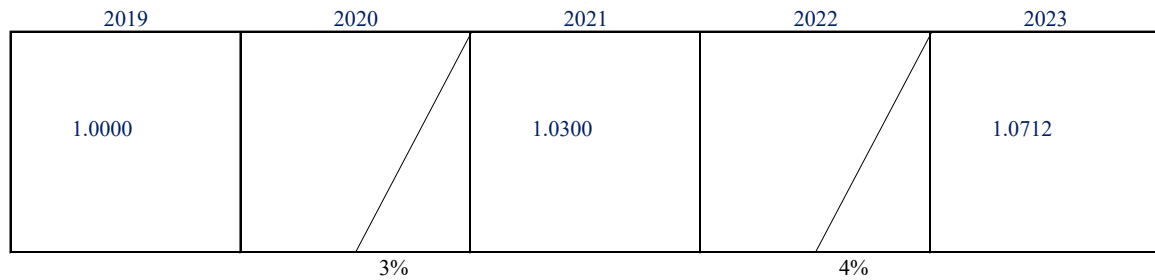
- (f) Recommend the weights to assign to each year when estimating the weighted average trended claim ratio for the indicated rate change. Justify your recommendation.

**Commentary on Question:**

*Candidates can also select weights judgmentally, as long as the number of years used matches the number of years recommended in part (e).*

AY	Earned Exposures	AY Weights		
		Initial	Limited	Balanced
2020	19,937	27.3%	23.4%	24.4%
2021	17,061	23.4%	23.4%	24.4%
2022	17,992	24.7%	24.6%	25.6%
2023	17,931	24.6%	24.6%	25.6%
Total	72,921		96.0%	

- (g) Calculate the indicated rate change for this line of business.



Rate Index	Area in CY				
	2019	2020	2021	2022	2023
1.0000	100%	75%	0%	0%	0%
1.0300	0%	25%	100%	75%	0%
1.0712	0%	0%	0%	25%	100%
Average rate level:	1.0000	1.0075	1.0300	1.0403	1.0712
On-level factor:	1.0712	1.0632	1.0400	1.0297	1.0000

Claim Ratio Trend:  $(1 + -1.0\%)(1 + 5\%) - 1 = 3.95\%$

AY	Earned Premiums	On-Level Factor	On-Level Earned Premiums	Ultimate Claims
2019	13,510,549	1.07120	14,472,500	8,709,600
2020	13,268,660	1.06323	14,107,582	8,673,608
2021	11,739,370	1.04000	12,208,945	7,919,295
2022	12,638,750	1.02970	13,014,158	8,605,528
2023	13,089,711	1.00000	13,089,711	9,489,317

AY	Claim Trend Period (yrs)	Claim Trend Factor	Trended Ult. Claims	Claim Ratio	Weights
2019	5.9167	1.25761	10,953,253	75.68%	0.0%
2020	4.9167	1.20982	10,493,496	74.38%	24.4%
2021	3.9167	1.16385	9,216,849	75.49%	24.4%
2022	2.9167	1.11962	9,634,939	74.03%	25.6%
2023	1.9167	1.07708	10,220,730	78.08%	25.6%
Weighted:				75.51%	

Weighted Average Trended Claim Ratio (including non-hurricane weather loading): 82.46%

Ratio of ULAE to Claims 5.00%

Weighted Average Trended Claim Ratio including ULAE =  $0.8245 \times (1 + 6.7598) =$  86.58%

Fixed Expenses as Ratio to Premiums at Current Rate Level 3.00%

Variable Expenses - Ratio to Premiums 12.00%

Profit and Contingencies Ratio to Premiums 4.00%

$$\begin{aligned}\text{Permissible Claim Ratio} &= (1 - 0.12 - 0.04) / (1 + 0.03/0.8658) = \\ \text{Indicated Rate Change} &= 0.8658 / 0.8119 - 1 =\end{aligned}$$

$$\begin{aligned}81.19\% \\ 6.64\%\end{aligned}$$

## **GIRR Spring 2024 Question 6 (LOs 6a)**

### **Learning Outcomes:**

- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

### **Source References:**

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

### **Commentary on Question:**

*This question tests the candidate's understanding of the expenses for ratemaking.*

### **Solution:**

- (a) Describe how you might account for a start-up cost expense.

An annual provision using an appropriate amortization period could be added.

- (b) Explain whether a residual market assessment would be considered a fixed or variable expense.

It depends on the assessment.

- If the assessment is a fixed amount (i.e., variable on policy counts), then it should be considered a fixed expense.
- If the assessment is variable on premium, then it should be considered a variable expense.

- (c) Describe a possible consequence to an insurer treating fixed expenses as variable expenses when determining rates.

Treating all expenses as variable can lead to inadequate expense provisions for insureds with low premium and excessive expense provisions for insureds with high premium.

- (d) Describe two situations where you might cap the percentage of variable expenses in a ratemaking analysis.

- Where regulations limit the amount of expenses
- Where there is an expense that is not expected in the future or expected to be lower in the future.

### GIRR Fall 2024 Question 3 (LOs 1l, 6d, 6e)

#### Learning Outcomes:

- (1l) Understand credibility as used for actuarial work.
- (6d) Calculate loadings for catastrophes and large claims.
- (6e) Apply loadings for catastrophes and large claims in ratemaking.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 6 and 31.

#### Commentary on Question:

*This question tests the candidate's understanding of loadings for large claims by analyzing claims at various limits.*

#### Solution:

- (a) Identify two other considerations in assigning credibility to an experience set of data.

#### Commentary on Question:

*This question is about assigning the credibility and not about what is considered for the complement of credibility.*

Any two of the following are acceptable:

- The number of years of claim data underlying the experience
- The stability or variability observed in claims from year to year
- The presence or absence of large or unusual claims
- Changes in the internal or external environment
- The age, relevance, and reliability of the experience
- The age, relevance, and reliability of other data to which the complement of credibility would be applied

- (b) Calculate the loadings for 500,000 to total limits for each accident year.

Severity trend for 1,000,000 limit =  $7.0\% \times 0.70 + 6.0\% \times 0.30 = 6.7\%$

Severity trend for total limit =  $8.6\% \times 0.50 + 7.0\% \times 0.50 = 7.8\%$

Accident Year	Trend Period	<u>Severity Trend at:</u>		Trended Claims at 1,000,000 Limit	Total Limit
		6.7%	7.8%		
2021	4.667	1.353	1.420	5,817,559	6,365,155
2022	3.667	1.268	1.317	5,541,683	6,068,833
2023	2.667	1.189	1.222	5,813,421	6,228,374

Accident Year	Loading for 1,000,000 to Total Limit	Loadings for 500,000 to 1,000,000 Limit	Loadings for 500,000 to Total Limit
2021	1.094	1.196	1.309
2022	1.095	1.165	1.276
2023	1.071	1.185	1.270

- (c) Recommend a loading for 500,000 to total limits for ratemaking purposes. Justify your recommendation.

Average of 2022 and 2023 = 1.273

Justification:

- Accident year 2021 loading is much higher than 2022 & 2023
- Therefore, use most recent 2 years as it is more stable, and it uses the most recent data.



## GIRR Fall 2024 Question 5 (LOs 6f, 6g)

### Learning Outcomes:

- (6f) Describe the claim ratio and pure premium methods of ratemaking.
- (6g) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 32.

### Commentary on Question:

*This question tests the candidate's ability to calculate the indicated average rate and the differences between the claim ratio and pure premium approaches to ratemaking.*

### Solution:

- (a) Demonstrate that the indicated rate change using the pure premium approach is similar to that using the claim ratio approach (i.e.,  $\pm 0.5\%$  of 5.91%).

Accident Year	Earned Exposures	Trended Ultimate Claims	Pure Premium
2019	18,640	10,866,820	582.98
2020	18,240	9,735,481	533.74
2021	17,061	9,235,310	541.31
2022	17,992	9,763,870	542.68
2023	17,931	10,191,450	568.37

Average pure premium 553.82

Ratio of ULAE to claims 8.00%

Fixed expenses per exposure =  $0.075 \times 13,878,594 / 17,931 =$  58.05

Indicated rate =  $(553.82 \times 1.08 + 58.05) / (1 - 0.15 - 0.05) =$  820.22

2023 trended earned premiums at current rate level 13,878,594

2023 earned exposures 17,931

Current average rate =  $13,878,594 / 17,931 =$  774.00

Indicated Rate Change =  $820.22 / 774.00 - 1 =$  5.97%

This is within 0.5% of 5.91%.

- (b) Describe one such reason.

The premium adjustment factors for trend and on-level factors are both approximations used to restate historical earned premiums as if they were at the current rate level and mix of exposures for the forecast period.

- (c) Calculate the profit and contingencies ratio implied by increasing the rates by 2%.

First, solve for CR:  $(CR + F/R_c) / (1 - V - Q) - 1 = 5.91\%$

CR = 77.23%

Solve for Q, where  $(CR + F/R_c) / (1 - V - Q) - 1 = 2\%$

$Q = 1 - V - (CR + F/R_c) / (1.02) = 1.93\%$

- (d) Explain how implementing a lower rate change than indicated will result in higher rate indications for the next rate review using the claim ratio approach.

Implementing a lower rate increase than indicated would mean charging lower premiums than needed to achieve the required profit. This will lead to higher claim ratios which will lead to higher rate indications for the next review than would have been had the full rate change been implemented.

### GIRR Fall 2024 Question 6 (LOs 6a)

#### Learning Outcomes:

- (6a) Quantify different types of expenses required for ratemaking including expense trending procedures.

#### Source References:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

#### Commentary on Question:

*This question tests the candidate's understanding of expenses used for ratemaking.*

#### Solution:

- (a) Calculate the total variable expense ratio for each of calendar years 2019 to 2023.

Calendar Year	(1) General Expenses	(2) As a % of Premiums	(3) Commission and Premium Tax Expense Ratio	(4) = (2) + (3) Total Variable Expense Ratio
	Variable			
2019	870,000	4.58%	13.0%	17.58%
2020	852,000	4.55%	13.0%	17.55%
2021	864,000	4.74%	12.5%	17.24%
2022	852,000	4.80%	12.0%	16.80%
2023	834,000	4.78%	12.0%	16.78%

Notes: (1) = 60% × (General Expenses)

(2) = (1) / (Direct Earned Premium)

(3) = (Total Commission Expenses and Premium Taxes) / (Direct Written Premium)

- (b) Recommend the total variable expense ratio to use in ratemaking. Justify your recommendation.

Calendar Year	Variable General Expense Ratio	Commission and Premium Tax Expense Ratio	Total Variable Expense Ratio
2019	4.58%	13.00%	17.58%
2020	4.55%	13.00%	17.55%
2021	4.74%	12.50%	17.24%
2022	4.80%	12.00%	16.80%
2023	4.78%	12.00%	16.78%
Average	4.69%	12.50%	17.19%
Selection:	4.77%	12.00%	16.77%

Justification:

- Latest 3 years average for variable general expense ratio due to the increase over the last 3 years.
- Select 12% for commission and premium tax expense ratio, as there has been a change to these ratios.

- (c) Recommend the fixed expense per exposure to use in ratemaking. Justify your recommendation.

Fixed expenses are incurred at the time each policy is written.

Therefore, need to trend from the average written date in each calendar year to the average written date in the future rating period.

Average written date in calendar year 2023: July 1, 2023

Average written date in future rating period: January 1, 2026

Therefore, trend period for 2023: 2.5 years

	(4)	(5)	(6)	(7)	(8)
Calendar Year	Fixed General Expense	Fixed General Expense Per Exposure	Trending Period	Fixed Expense Trend Factor	Trended Fixed Expenses
2019	580,000	22.48	6.5	1.13737	25.57
2020	568,000	23.18	5.5	1.11507	25.85
2021	576,000	24.94	4.5	1.09320	27.26
2022	568,000	25.94	3.5	1.07177	27.80
2023	556,000	26.80	2.5	1.05075	28.16
Selection:					27.74

Justification: Select the latest 3 years average due to the increase over the last 3 years.

Notes: (4) = 40%×(General Expenses)

(5) = (4) / (Earned Exposures)

(7) = 1.02<sup>(6)</sup>

(8) = (5)(7)