COURSE STRATEGY GUIDE

INV 201 – Quantitative Finance

Nov 2025/Mar 2026/Jul 2026

This Course Strategy Guide (Guide) is to provide an overview of course INV 201, to help the candidate understand the essence of this course and to provide a guide for how to prepare for the final assessment. However, while this Guide can be a valuable aid in preparation, the content of this Guide will not be tested.

I. Purpose of this Course

The Investment (INV) practice area was born out of a vision to create a body of study for actuaries who desire to thrive in the field of investments. The vision is to train investment actuaries so that they may excel in situations where investment decisions are being made in the context of significant and complex liabilities.

Course INV 201 helps candidates understand:

- Key types of derivatives
- The principles and techniques for the valuation of derivatives
- Various applications and risks of derivatives

Many candidates take INV 201 after completing INV 101, which provides candidates with the skills to understand portfolio management objectives, the nature and the variety of asset classes that can be used in constructing a portfolio, portfolio construction and assessment, and best practices of investment risk management. Although material from INV 101 will not be directly assessed on the INV 201 assessment, candidates may benefit from a familiarity of concepts and terminology introduced there.

As a follow-up to these two foundational courses, candidates may want to consider course CP 351 Asset Liability Management, and, depending on their work context, CP 341 Advanced Life Reinsurance, or the 101 courses from the Retirement, the Individual Life and Annuities, or the General Insurance practice area.

II. Recommended Approach in Preparing for the Course Assessment

Our recommended study approach is to first read the descriptions of Learning Objectives and Learning Outcome Statements in the syllabus; then read the syllabus study materials in the order presented for each course topic section. The recommended order is purposeful. During this first reading make notes or flash cards of key points. Then proceed to a second, faster reading with stops to do practice problems or past exam problems to ensure the concepts have been understood accurately.

Keep in mind that each exam question is created by starting first with one or a combination of the Learning Objectives and Learning Outcome Statements. Each question will consider a context that entails a business situation or conflict. The insights or lessons learned from the syllabus study materials are there to help the candidate develop and apply a solution that best fits within the context of the exam question.

Note that a solution to a given problem presented within the study materials is appropriate for the context used within those study materials, but not necessarily appropriate for the context of the exam question (often the context is that of a case study company and its business strategies and management practices). Since the exam is focused on the demonstration of critical thinking, the candidate must learn how to take the learnings from one situation and apply them to a different situation. In creating such questions this exam seeks to emulate real-world situations which, most of the time, do not have solutions that conveniently appear within any textbook. Candidates are expected to apply the techniques or insights that they learn from the syllabus study materials to new real-world problems, using the study material as a tool to gain insights about the Learning Objectives and Learning Outcome Statements. These insights expressed in a solution to an exam question demonstrate critical thinking and true mastery of the topic.

III. Exam Syllabus Learning Objectives and Learning Outcomes

This course provides candidates with the skills to thrive in functions such as hedging, portfolio management, asset-liability management, and variable annuity product management.

Course 201 focuses on the following three topics:

- Key types of derivatives;
- Valuation of derivatives;
- Applications and risks of derivatives.

Details of the Learning Objectives, Learning Outcome Statements and syllabus study materials associated with the Learning Outcome Statements for each of the three topic sections are summarized in the accompanying syllabus document.

<u>Topic 1: Key Types of Derivatives</u>

In the last 40 years, derivatives have become increasingly important in finance. Futures and options are actively traded on many security exchanges throughout the world. Many different types of forward contracts, swaps, options, and other derivatives are entered into by financial institutions, fund managers, and corporate treasurers in the over-the-counter market.

Derivatives are added to bond issues, used in executive compensation plans, embedded in capital investment opportunities, used to transfer risks in mortgages from the original lenders to investors, and so on. We have now reached the stage where those who work in finance, and many who work outside finance, need to understand how derivatives work, how they are used, and how they are priced.

Seven chapters (Ch. 1, 2, 7, 12, 17, 26, 29) of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021** are included as these chapters, in combination, provide candidates with an understanding of basic derivative payoffs, differences between futures and forwards, differences among European, American, Asian and Bermudan options, examples of exotic options, mechanics of derivative trading, and basic derivative strategies. They align with the Learning Outcome Statements 1a through 1e.

Topic 2: Valuation of Derivatives

Although single-period models cannot give a realistic representation of a complex, dynamically changing stock market, we use such models to illustrate many important economic principles.

Chapters 5 and 13 of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021** explain the nature of the no-arbitrage arguments that are used for valuing options and the one-period binomial tree numerical procedure that is widely used for valuing American options and other derivatives.

INV201-100-25: Chapter 5 of Financial Mathematics – A Comprehensive Treatment, Campolieti makes a multinomial generalization of the one-period binomial market model, explaining the Arrow-Debreau security and the distinction between complete and incomplete markets. They align with the Learning Outcome Statements 2a and 2b.

Next, we use a number of different arbitrage arguments to explore the relationships among European option prices, American option prices, and the underlying stock price. The most important of these relationships is put-call parity, which is the relationship among the price of a European call option, the price of a European put option, and the underlying stock price. This is illustrated in Chapter 11 of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021**. It aligns with Learning Outcome Statement 2c.

Brownian motion/Wiener process is a keystone in the foundation of mathematical finance. Brownian motion is a continuous-time stochastic process but it can be constructed as a limiting case of symmetric random walks.

Any variable whose value changes over time in an uncertain way is said to follow a stochastic process. Stochastic processes can be classified as of discrete-time or continuous-time. A discrete-time stochastic process is one where the value of the variable can change only at certain fixed points in time, whereas a continuous-time stochastic process is one where changes can take place at any time. Stochastic processes can also be classified as of continuous-variable or discrete-variable. In a continuous-variable process, the underlying variable can take any value within a certain range, whereas in a discrete-variable process only are certain discrete values possible.

Chapter 14 of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021** develops a continuous-variable, continuous-time stochastic process for stock prices. A similar process is often assumed for the prices of other assets. Learning about this process is the first step to understanding the pricing of options and other more complicated derivatives. It should be noted that, in practice, we do not observe stock prices following continuous-variable, continuous-time processes. Stock prices are restricted to discrete values and changes can be observed only when the exchange is open for trading. Nevertheless, the continuous-variable, continuous-time process proves to be a useful model for many purposes.

Chapters 28 and 31 of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021** explain the theoretical underpinnings of risk-neutral valuation when interest rates are stochastic and show that there are many different risk-neutral worlds that can be assumed in any given situation. We first define a parameter known as the market price of risk and show that the excess return over the risk-free interest rate earned by any derivative in a short period of time is linearly related to the market prices of risk of the underlying stochastic variables. What we will refer to as the traditional risk-neutral world assumes that all market prices of risk are zero, but we will find that other assumptions about the market price of risk are useful in some situations.

Martingales and measures are critical to a full understanding of risk-neutral valuation. A martingale is a zero-drift stochastic process. A measure is the unit in which we value security prices. A key result in chapter 28 will be the equivalent martingale measure result. This states that if we use the price of a traded security as the unit of measurement then there is a market price of risk for which all security prices follow martingales.

The paper by Steve Strommen, **Understanding the Connection Between Real Real-World and Risk Risk-Neutral Generators, SOA Research, Aug 2022**, presents the difference and relationship between real-world calibration and risk-neutral calibration for various models given their distinct purposes. Along with chapters 14, 28, and 31 of John Hull, these readings align with Learning Outcome Statements 2d and 2e.

Chapter 15 of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021** covers Merton's approach to deriving the Black–Scholes–Merton model. It explains how volatility can be either estimated from historical data or implied from option prices using the model. It shows how the risk-neutral valuation argument introduced in chapter 13 can be used. It also shows how the Black–Scholes–Merton model can be extended to deal with European call and put options on dividend-paying stocks and presents some results on the pricing of American call options on dividend paying stocks. This aligns with the Learning Outcome Statements 2e and 2f.

The martingale approach to pricing and hedging is then presented in **INV201-101-25: Chapters 6 of Introduction to Stochastic Finance with Market Examples by Privault**. It complements the Partial Differential Equation approach by recovering the Black-Scholes formula via a probabilistic argument. This aligns with the Learning Outcome Statement 2f.

INV201-102-25: How to Use the Holes in Black-Scholes presents the limitations of the Black-Scholes-Merton model. This aligns with the Learning Outcome Statement 2g.

INV201-103-25: Calibrating Interest Rate Models helps candidates understand and apply numerical discretization methods to price options including Euler-Maruyama discretization and transition density methods, as well as calibrate a model to observed prices of traded securities including fitting to a given yield curve. This aligns with Learning Outcome Statements 2g, 2h, and 2i.

Chapters 8 and 10 of ***The Volatility Smile,*** **Derman, Emanuel and Miller, Michael, 2016** and chapters 20 and 27 of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021** help candidate define and explain the concept of volatility smiles and describe several approaches for modeling smiles, including stochastic volatility, local volatility, and jump-diffusions. This aligns with Learning Outcome Statement 2j.

Finally, the selected pages from ***Problems and Solutions in Mathematical Finance: Stochastic Calculus,*** **Chin, Eric, Nel, Dian and Olafsson, Sverrir, 2014** provide lots of examples illustrating the important concepts of stochastic calculus theory and technique used in pricing derivatives. They align with Learning Outcome Statements 2a, 2b, 2d, and 2f.

Topic 3: Applications and Risks of Derivatives

A financial institution that sells an option to a client in the over-the-counter markets is faced with the problem of managing its risk. If the option happens to be the same as one that is traded actively on an exchange or in the over-the-counter market, the financial institution can neutralize its exposure by buying the same option as it has sold. But when the option has been tailored to the needs of a client and does not correspond to the products traded on any exchanges, hedging the exposure is far more difficult.

Chapter 19 of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021** covers what are commonly referred to as the "Greek letters", or simply the "Greeks". Each Greek letter measures a different dimension to the risk in an option position, and the aim of a trader is to manage the Greeks so that all risks are acceptable. The analysis presented in this chapter is applicable to market makers in options on an exchange as well as to traders working in the over-the-counter market for financial institutions.

Toward the end of the chapter, the text explains the creation of options synthetically. This turns out to be very closely related to the hedging of options. Creating an option position synthetically is essentially the same task as hedging the opposite option position. For example, creating a long call option synthetically is the same as hedging a short position in the call option. This aligns with Learning Outcome Statement 3a.

Derivatives such as European and American call and put options are what are termed plain vanilla products. They have standard well-defined properties and are traded actively. Their prices or implied volatilities are quoted by exchanges or by interdealer brokers on a regular basis. One of the exciting aspects of the over-the-counter derivatives market is the number of non-standard products that have been created by financial engineers. These products are termed exotic options, or simply exotics.

Section 26.17 of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021** describes how to replicate static options. This aligns with Learning Outcome Statement 3b.

A call option and its underlying stock can be combined to form an instantaneously riskless portfolio. By using the Black-Scholes-Merton equation one can hedge the risk of an option in a variety of ways. The profit and loss (P&L) from hedging an option depends on which volatility one uses to hedge. In real life, one can rebalance the hedge only a finite number of times. And the P&L picks up a random component. The more often one hedges, the smaller the deviation from perfection. Transaction costs affect things, too.

Chapters 3, 5, 6, and 7 of ***The Volatility Smile***, **Derman, Emanuel and Miller, Michael, 2016, INV201-104-25: Which Free Lunch Would You Like Today, Sir?** and Chapter 37 of ***Options, Futures, and Other Derivatives,*** **Hull, John C, Pearson, 11th Edition, 2021** help the candidate understand some of the nuances of delta hedging, the interplay between hedging assumptions and hedging outcomes, the concepts of realized versus implied volatility, and derivatives mishaps. This aligns with Learning Outcome Statements 3c, 3d, and 3e.

Traditionally, Variable Annuity (VA) contracts have offered guaranteed minimum death benefits but they have not contained any guarantees relating to principal or a minimum rate of return. However, although not at the same level of Fixed Indexed Annuity (FIA) guarantees, newer VA products frequently contain one or more guaranteed living benefits (GLB). One popular GLB is the guaranteed minimum accumulation benefit (GMAB). Specifically, no minimum guarantees apply to interim cash (accumulation) values prior to the maturity of the GMAB in a VA contract. In contrast, FIA accumulation values never fall below their guaranteed amounts and interest credits, once locked-in, cannot be lost or forfeited.

INV201-105-25: An Introduction to Computational Risk Management of Equity-Linked Insurance, Feng, 2018, **INV201-106-25: Variable Annuity Volatility Management: An Era of Risk-Control, INV201-108-25: Mitigating Interest Rate Risk in Variable Annuities: An Analysis of Hedging Effectiveness under Model Risk,** and **INV201-107-25: It's RILA time: An introduction to registered index-linked annuities** identify and evaluate embedded options in liabilities and demonstrate techniques for hedging embedded options. This aligns with Learning Outcome Statements 3f and 3g.

Over the last several years extreme volatility levels have been observed in global equity markets. Studying the recent market crashes in detail (e.g., the market crashes of 2002 and 2008), one notices that dramatic market downturns often happen in scenarios with extreme market volatility. Practitioners often recognize rising volatility as a good indicator for a falling market.

Many modern investment instruments with participation in equities include embedded financial derivatives in their design. High volatility levels dramatically increase prices of derivatives linked to these equities or equity indices. As a result, investment instruments with embedded equity derivatives become less attractive to investors. These days, many investors are looking for simple allocation mechanisms to protect their portfolio from significant losses due to market crashes.

In order to address these needs, recently several investment banks and asset management companies have developed a new investment mechanism called a volatility target mechanism which became very popular among practitioners. The volatility target mechanism is used in order to create and dynamically re-balance an investment portfolio with a specific level of volatility.

For a portfolio consisting of a risky asset (e.g., an equity index) and a risk-less asset (e.g., zero-coupon bond), the volatility of the risky asset over the most recent time period is estimated. According to this estimated volatility level and the pre-specified volatility target level, the assets are reallocated in the portfolio so that the overall portfolio volatility level is kept under control. The target volatility concept is illustrated in **INV201-109-25: Investment Instruments with Volatility Target Mechanism.** This aligns with Learning Outcome Statement 3g.