SOCIETY OF ACTUARIES

Advanced Long-Term Actuarial Mathematics Exam

Exam ALTAM

Date: April 22, 2025

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 6 questions numbered 1 through 6 with a total of 60 points. The points for each question are indicated at the beginning of the question.

2. **Question 1 is to be answered in the Excel workbook. For this question only the work in the Excel workbook will be graded.**

3. Questions 2-6 are to be answered in pen in the Yellow Answer Booklet provided. For these questions graders will only look at the work in the Yellow Answer Booklet. Excel may be used for calculations, for referencing tables, or for statistical functions, but any work in the Excel booklet will not be graded.

4. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Excel Answer Instructions

1. For Question 1, you should answer directly in the Excel Question worksheet. The question will indicate where to record your answers.
2. You should generally use formulas in Excel rather than entering solutions as hard coded numbers. This will aid graders in assigning appropriate credit for your work.
3. Graders for Excel questions will not have access to any comments or calculations provided in the Yellow Answer Booklet.
4. For Question 1, you may add notes to the Excel Question worksheet if you feel that might help graders. However, these should be entered directly into the Excel Question worksheet. Graders may not be able to read notes entered as comments.
5. When you finish, save your Excel workbook with a filename in the format xxxxx_ALTAM where xxxxx is your candidate number. Your name must not appear in the filename.
6. Record your candidate number in the indicated cell in the Excel Question worksheet.

Pen and Paper Answer Instructions

1. Write your candidate number and the number of the question you are answering at the top of each sheet. Your name must not appear.

2. Start each question on a fresh sheet. You do not need to start each sub-part of a question on a new sheet.

3. Write in pen on the lined side of the answer sheet.

4. The answer should be confined to the question as set.

5. When you are asked to calculate, show all your work including any applicable formulas in the Yellow Answer Booklet.

6. If you use Excel for calculations for pen and paper answers, you should include as much information in the Yellow Answer Booklet as if you had used a calculator, including formulas and intermediate calculations where relevant. Written answers without sufficient support may not receive full or partial credit.

7. When you finish, hand in <u>all</u> your written answer sheets to the Prometric Center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

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****BEGINNING OF EXAMINATION****
****ADVANCED LONG-TERM ACTUARIAL MATHEMATICS****

Provide the response for Question 1 in the Excel Question worksheet

1.

(*11 points*) You are tasked with conducting a profit test of a Type B Universal Life policy issued to (50).

- CoI rates are given in the table below and the worksheet.
- The Additional Death Benefit (ADB) is 100,000.
- Assume that the policyholder pays an annual premium of P = 2,500.
- The annual credited interest rate is 5%.
- The interest rate used to discount the CoI is 4%.
- The expense charges (EC) consist of:
    - Percent of premium charge is 2%; and
    - Annual flat fee (charged at the start of each year) is 50.
- Surrender charges, as a percent of account value, are given in the table below and the worksheet.

1. Continued

Year	CoI Rates	Surrender Charge	Year	CoI Rates	Surrender Charge
1	0.00127	100%	26	0.01935	0%
2	0.00140	90%	27	0.02170	0%
3	0.00154	80%	28	0.02433	0%
4	0.00170	70%	29	0.02728	0%
5	0.00189	60%	30	0.03059	0%
6	0.00209	50%	31	0.03429	0%
7	0.00232	40%	32	0.03844	0%
8	0.00258	30%	33	0.04308	0%
9	0.00287	20%	34	0.04827	0%
10	0.00320	10%	35	0.05407	0%
11	0.00357	0%	36	0.06055	0%
12	0.00398	0%	37	0.06778	0%
13	0.00445	0%	38	0.07585	0%
14	0.00497	0%	39	0.08484	0%
15	0.00555	0%	40	0.09484	0%
16	0.00621	0%	41	0.10596	0%
17	0.00695	0%	42	0.11831	0%
18	0.00778	0%	43	0.13199	0%
19	0.00871	0%	44	0.14713	0%
20	0.00976	0%	45	0.16385	0%
21	0.01093	0%	46	0.18228	0%
22	0.01225	0%	47	0.20253	0%
23	0.01373	0%	48	0.22473	0%
24	0.01540	0%	49	0.24899	0%
25	0.01726	0%	50	0.27541	0%

(a) (*4 points*)

(i) Complete the table in the worksheet.

(ii) Calculate the Account Value (AV) for the policy at the end of the 5^{th} year. You should find the answer is 13,000 to the nearest 100.

(iii) Calculate the Cash Value (CV) for the policy at the end of the 5^{th} year. You should find the answer is 5,200 to the nearest 100.

1. Continued

(b) (*1 point*) Briefly describe the main purpose of the insurer including a surrender charge in a Universal Life policy.

You are given the following information regarding the profit test basis:

- The hurdle rate is 10%.
- The precontract expense is 1,200.
- The yearly maintenance expense incurred at the start of each year including the first is 180.
- The percent of premium expense is 3%.
- The earned asset rate is 6.5%.
- Expected mortality rates are given in the table below and the worksheet.
- Death benefits are paid at the end of the year of death, based on the account value at the end of the year of death.
- Expected surrender rates, per policyholder in force at year end, are given in the table below and the worksheet.
- All policyholders surviving to age 100 are assumed to surrender their policies at age 100.
- The insurer holds the account value as the policy's reserve.

1. **Continued**

Year	Mortality Rates	Lapse Rates	Year	Mortality Rates	Lapse Rates
1	0.00114	0%	26	0.01742	5%
2	0.00126	0%	27	0.01953	5%
3	0.00139	0%	28	0.02190	5%
4	0.00153	0%	29	0.02456	5%
5	0.00170	0%	30	0.02753	5%
6	0.00188	1%	31	0.03086	5%
7	0.00209	2%	32	0.03459	5%
8	0.00232	3%	33	0.03877	5%
9	0.00259	4%	34	0.04344	5%
10	0.00288	5%	35	0.04866	5%
11	0.00321	5%	36	0.05449	5%
12	0.00358	5%	37	0.06100	5%
13	0.00400	5%	38	0.06826	5%
14	0.00447	5%	39	0.07635	5%
15	0.00500	5%	40	0.08536	5%
16	0.00559	5%	41	0.09537	5%
17	0.00625	5%	42	0.10648	5%
18	0.00700	5%	43	0.11879	5%
19	0.00784	5%	44	0.13242	5%
20	0.00878	5%	45	0.14747	5%
21	0.00984	5%	46	0.16405	5%
22	0.01103	5%	47	0.18228	5%
23	0.01236	5%	48	0.20226	5%
24	0.01386	5%	49	0.22409	5%
25	0.01554	5%	50	0.24787	100%

(c) (*4 points*)

(i) Calculate the NPV for this policy. You may add columns to the right of the table for additional work. You should find that the answer is 950 to the nearest 10.

(ii) Determine the breakeven year for this policy.

(iii) Calculate the profit margin for this policy.

1. Continued

(d) (*2 points*) Your supervisor has asked you to explore ways to increase the profit margin of this product to at least 5%.

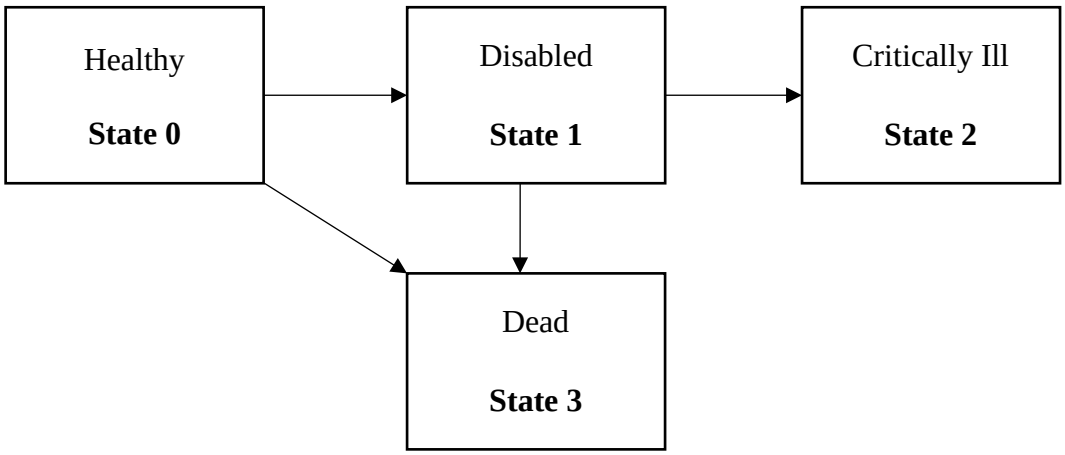
(i) Use Goal Seek to determine the minimum percent of premium charge required to obtain a profit margin of 5%, holding everything else constant. You should show the parameters used for the Goal Seek in the Table in the worksheet.

(ii) Your colleague notes that higher premium levels will result in increased profitability, and hence suggests obtaining the required profit margin by changing the annual premium from 2500 to 3000. Critique your colleague's suggestion.

Note: Before submitting your worksheet, you should reset the premium expense charge to 2%.

2.

(*10 points*) You are analyzing an insurance policy with disability income benefits and critical illness benefits. You use the following multiple state model:



You are given the following information:

(i) The policy is issued to Healthy individuals only.

(ii) The level premium is paid continuously while the policyholder is Healthy.

(iii) A disability income annuity of 10,000 per year is paid continuously while the policyholder is Disabled.

(iv) A critical illness benefit of 120,000 is paid immediately on becoming Critically Ill.

(v) A death benefit of 100,000 is paid immediately on Death, provided that the policyholder has not already received a critical illness benefit.

(vi) The force of interest is $\delta = 0.05$.

(vii) Transition intensities for all ages x are:

$$\mu_x^{01} = 0.02, \quad \mu_x^{12} = 0.04, \quad \mu_x^{03} = 0.05, \quad \mu_x^{13} = 0.10.$$

(a) (*1 point*) Explain whether this model satisfies the Markov property.

2. Continued

(b) (*3 points*)

(i) Write down the Kolmogorov forward differential equations for ${}_t p_x^{00}$ and ${}_t p_x^{01}$ with the associated boundary conditions.

(ii) Show that ${}_t p_x^{01} = \frac{2}{7}\left(e^{-0.07t} - e^{-0.14t}\right)$.

(c) (*2 points*)

(i) Show that $\bar{a}_x^{01} = 0.88$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(ii) Show that $\bar{A}_x^{02} = 0.04\bar{a}_x^{01}$.

(d) (*2 points*) You are given that the expected present value of the death benefit for this policy is 50,438.60.

Calculate the annual level premium of this policy using the equivalence principle.

(e) (*1 point*) State the Thiele differential equation for ${}_tV^{(1)}$, the State 1 policy value at time *t*.

(f) (*1 point*) The insurer decides to include an additional death benefit of 50,000 in this policy, payable when the policyholder dies while Critically Ill.

Sketch a diagram of a revised model incorporating this change.

3.

(*10 points*) A fully discrete 20-year last survivor endowment insurance policy is issued to Kim (60) and Alex (70). You are given:

(i) A death benefit of 10,000 is paid at the end of the year of the second death if it occurs within 20 years of the issue date.

(ii) A survival benefit of 10,000 is paid at time 20 if at least one of Kim or Alex is alive at that time.

(iii) Level annual premiums are payable while the policy is in force.

(iv) Mortality follows the Standard Ultimate Mortality Model.

(v) $i = 5\%$

(vi) The future lifetimes of Kim and Alex are independent.

(a) (*1 point*) Calculate the probability that the survival benefit will be paid.

(b) (*3 points*)

(i) Show that the expected present value of the benefits under this policy is 3,865 to the nearest 5. You should calculate the value to the nearest 0.1.

(ii) Show that the net premium for this policy is 300 to the nearest 10. You should calculate the value to the nearest 0.1.

(c) (*3 points*) You are given that $\ddot{a}_{\overline{70:80:10|}} = 7.9879$.

(i) Calculate the net premium policy value at the end of 10 years if both Kim and Alex are alive.

(ii) Calculate the net premium policy value at the end of 10 years if only Kim is alive.

(d) (*3 points*) At the issue date, policyholders may select a waiver of premium rider. In this case, the policyholders would pay a higher premium while both lives survive but would pay no premiums after the first death.

(i) Calculate the additional net premium for the rider.

(ii) Describe two factors that Kim and Alex might take into consideration when deciding whether to purchase the rider.

4.

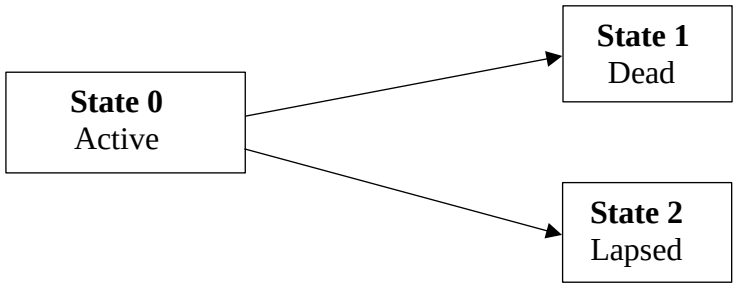
(*10 points*) You are pricing a special fully discrete whole life insurance on a life age 60. The death benefit is 100,000 during the first 20 years and 200,000 thereafter.

You are given:

- Mortality follows the Standard Ultimate Mortality Model (SUMM).
- The only expenses are commissions, which are 50% of premium in year 1, and 5% of premiums thereafter.
- $i = 0.05$
- Level gross premiums are determined using the equivalence principle.

(a) (*2 points*) Calculate the annual gross premium.

The insurer decides to include lapses in the pricing model, as illustrated in the following multiple state model.



You are given:

- There are no benefits payable upon lapse.
- The force of mortality is unchanged, that is, $\mu_x^{01} = A + Bc^x$ where $A = 0.00022$, $B = 2.7 \times 10^{-6}$, and $c = 1.124$.

(b) (*2 points*)

(i) State the meaning of "lapse supported business".

(ii) State one disadvantage to the insurer of issuing lapse supported policies.

4. Continued

You have calculated these actuarial values incorporating a lapse assumption:

x	$\ddot{a}_x^{00}$	A_x^{01}	${}_{10}p_x^{00}$
60	8.8023	0.10330	0.49694
70	8.7967	0.24061	0.54578
80	6.9365	0.41889	0.37068

(c) (*3 points*)

(i) Show that the gross premium is 1,850 to the nearest 10. You should calculate the value to the nearest 0.1.

(ii) Show that the gross premium policy value at the end of year 10 is 22,650 to the nearest 50. You should calculate the value to the nearest 1.

(d) (*3 points*) You are given that $\mu_{70}^{02} = 0.04$ and $\mu_{70.5}^{02} = 0.045$.

(i) Use Euler's forward method with step size $h = 0.5$ to show that $p_{70}^{00} = 0.95$ to the nearest 0.01. You should calculate the value to the nearest 0.001.

(ii) Use Euler's forward method with step size $h = 0.5$ to show that $p_{70}^{01} = 0.01$ to the nearest 0.01. You should calculate the value to the nearest 0.001.

(iii) Using recursion, calculate the gross premium policy value at time 11.

5.

(*10 points*) Jo is a member of a final average salary Defined Benefit (DB) pension plan. The annual accrual rate is 2% and the final average salary is based on the average of the previous three years' salaries.

The pension benefit is a monthly whole life annuity-due, payable from age 65.
At the valuation date, December 31, 2025, Jo is age 50 and has 25 years of pensionable service.

Jo's salary will be 78,000 in 2026.

The valuation and funding basis is as follows:

- There are no decrements before age 65 other than deaths.
- Members surviving to age 65 retire immediately.
- Mortality before and after retirement follows the Standard Ultimate Mortality Model.
- There is no benefit payable on death.
- Monthly annuities are calculated using Woolhouse's 2-term formula.
- Salaries increase by 2% every January 1 starting in 2027.
- $i = 0.05$
- The Projected Unit Credit (PUC) funding method is used.

(a) (*3 points*) Show that the Actuarial Liability of Jo's pension benefits at the valuation date is 304,900 to the nearest 100. You should calculate the value to the nearest 1.

(b) (*2 points*)

(i) Calculate the projected monthly pension that Jo can expect to receive on retirement at age 65, including future service.

(ii) Calculate Jo's projected replacement ratio at age 65.

5. Continued

Jo's employer will replace the existing DB Pension with a Defined Contribution (DC) pension plan effective January 1, 2026. The key features of the new plan are as follows:

- All members must transfer to the new DC plan.
- 12% of members' salaries are contributed into their DC accounts at the end of each year.
- In addition to the new DC plan benefits, existing members must choose one of the following options:

  Option A: Retain their past service pension as a deferred pension payable from age 65, based on their final average salary at age 65; or

  Option B: Have a lump sum deposited into their DC account on January 1, 2026. The lump sum would be equal to the Actuarial Liability of their past service benefits under the DB plan, based on the December 31, 2025 valuation, as calculated in (a).

(c) (*3 points*) You are given the following assumptions about Jo's future salary increases and investment returns.

- Jo will receive a 2% salary increase every January 1 until age 65.
- The DC account will earn interest at 6% per year.
- At age 65, Jo will immediately convert the DC account assets into a level whole life annuity-due. The assets will be converted to income using the equivalence principle, and using the valuation assumptions above.

(i) Calculate the total projected monthly pension payable to Jo including the DC part under Option A.

(ii) Calculate the total projected monthly pension payable to Jo under Option B.

(d) (*2 points*) Describe one advantage and one disadvantage for Jo of Option A compared with Option B.

6.

(*9 points*) An insurer issues 10-year Variable Annuity policies, each with a single premium of $P = 100,000,$ to lives age 50 at issue.

After an initial expense charge of 3% of the premium, the remainder is invested in a separate account called the policyholder's fund.

A management charge of $m = 2\%$ per year is deducted from the policyholder's fund continuously while the policy is in force.

The policy offers a Guaranteed Minimum Maturity Benefit (GMMB) of 100% of the premium. The insurer uses the Black-Scholes model to value the GMMB.

You are given the following valuation assumptions:

- There are no lapses or surrenders.
- Mortality follows the Standard Ultimate Mortality Model.
- The asset volatility is $\sigma = 0.20$.
- The risk-free rate is $r = 0.04$, compounded continuously.

(a) (*3 points*) Show that the value of the GMMB option at issue is $\pi(0) = 12{,}360$ to the nearest 10. You should calculate the value to the nearest 1.

At time 6, the policyholder's fund value is $F_6 = 140,000$.

(b) (*3 points*) Calculate the value at time 6 of the GMMB, assuming no changes to the valuation assumptions.

(c) (*1 point*) The policy also offers a Guaranteed Minimum Income Benefit (GMIB) at maturity. The GMIB is calculated using a Benefit Base, and a guaranteed annuitization rate of $\gamma = 8\%$. The GMIB Benefit Base is equal to the maximum of the initial premium *P* accumulated at 3% per year, with a step-up guarantee every 3 years.

You are given that the policyholder's fund value at time 3 was $F_3 = 103,000$.

Calculate the guaranteed minimum annual income under the GMIB, based on the information available at time 6.

6. Continued

(d) (*1 point*) At time 6, the insurer offers the opportunity to exchange the policy's existing GMIB for a Guaranteed Minimum Withdrawal Benefit (GMWB). The GMWB allows lifetime withdrawals of 6% of the GMWB Benefit Base, starting at time 10.

The GMWB Benefit Base is the greater of:

- 105% of the policyholder's fund value at time 6; and
- the highest year-end value of the fund during the first 4 years after the exchange.

Calculate the minimum guaranteed withdrawal amount payable from age 60, based on the information at the time of the exchange offer.

(e) (*1 point*) Describe one advantage and one disadvantage of the GMWB compared with the GMIB, from the perspective of the policyholder.

****END OF EXAMINATION****