Exam M Additional Sample Questions

1. For a fully discrete whole life insurance of 1000 on (40), you are given:

(i) Death and withdrawal are the only decrements.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

(iv) The probabilities of withdrawal are:

$$q_{40+k}^{(w)} = \begin{cases} 0.2, & k = 0 \\ 0, & k > 0 \end{cases}$$

(v) Withdrawals occur at the end of the year.

(vi) The following expenses are payable at the beginning of the year:

	Percent of Premium	Per 1000 Insurance
All Years	10%	1.50

(vii) ${}_kCV_{40} = \frac{1000k}{3}\,{}_kV_{40}, \quad k \le 3$

(viii) ${}_2AS = 24$

Calculate the gross premium, G.

(A) 15.4

(B) 15.8

(C) 16.3

(D) 16.7

(E) 17.2

1. (solution)

$${}_{1}V_{40} = 1 - \frac{\ddot{a}_{41}}{\ddot{a}_{40}} = 1 - \frac{14.6864}{14.8166} = 0.00879$$

$${}_{1}CV_{40} = \frac{(1000)(1)}{3}(0.00879) = 2.93$$

$${}_{1}AS = \frac{\left(G - 0.1G - (1.50)(1)\right)(1.06) - 1000q_{40}^{(d)} - {}_{1}CV_{40} \times q_{40}^{(w)}}{1 - q_{40}^{(d)} - q_{40}^{(w)}}$$

$$= \frac{(0.9G - 1.50)(1.06) - (1000)(0.00278) - (2.93)(0.2)}{1 - 0.00278 - 0.2}$$

$$= \frac{0.954G - 1.59 - 2.78 - 0.59}{0.79722}$$

$$= 1.197G - 6.22$$

$${}_{2}AS = \frac{\left({}_{1}AS + G - 0.1G - (1.50)(1)\right)(1.06) - 1000q_{41}^{(d)} - {}_{2}CV_{40} \times q_{41}^{(w)}}{1 - q_{41}^{(d)} - q_{41}^{(w)}}$$

$$= \frac{(1.197G - 6.22 + G - 0.1G - 1.50)(1.06) - (1000)(0.00298) - {}_{2}CV_{40} \times 0}{1 - 0.00298 - 0}$$

$$= \frac{(2.097G - 7.72)(1.06) - 2.98}{0.99702}$$

$$= 2.229G - 11.20$$

$$2.229G - 11.20 = 24$$

$$G = 15.8$$

2. For a fully discrete insurance of 1000 on (x), you are given:

(i) ${}_4AS = 396.63$

(ii) ${}_5AS = 694.50$

(iii) $G = 281.77$

(iv) ${}_5CV = 572.12$

(v) $c_4 = 0.05$ is the fraction of the gross premium paid at time 4 for expenses.

(vi) $e_4 = 7.0$ is the amount of per policy expenses paid at time 4.

(vii) $q_{x+4}^{(1)} = 0.09$ is the probability of decrement by death.

(viii) $q_{x+4}^{(2)} = 0.26$ is the probability of decrement by withdrawal.

Calculate i.

(A) 0.050

(B) 0.055

(C) 0.060

(D) 0.065

(E) 0.070

2. **(solution)**

$$ {}_5AS = \frac{\left({}_4AS + G(1-c_4) - e_4\right)(1+i) - 1000q_{x+4}^{(1)} - {}_5CV \times q_{x+4}^{(2)}}{1 - q_{x+4}^{(1)} - q_{x+4}^{(2)}} $$

$$ = \frac{\left(396.63 + 281.77(1-0.05) - 7\right)(1+i) - 90 - 572.12 \times 0.26}{1 - 0.09 - 0.26} $$

$$ = \frac{(657.31)(1+i) - 90 - 148.75}{0.65} $$

$$ = 694.50 $$

$$ (657.31)(1+i) = 90 + 148.75 + (0.65)(694.50) $$

$$ 1+i = \frac{690.18}{657.31} = 1.05 $$

$$ i = 0.05 $$

3 – 5. *Use the following information for questions* 3 – 5.

For a semicontinuous 20-year endowment insurance of 25,000 on (x), you are given:

(i) The following expenses are payable at the beginning of the year:

	Percent of Premium	Per 1000 Insurance	Per Policy
First Year	25%	2.00	15.00
Renewal	5%	0.50	3.00

(ii) Deaths are uniformly distributed over each year of age.

(iii) $\bar{A}_{x:\overline{20|}} = 0.4058$

(iv) $A_{x:\overline{20|}}^{\;\;1} = 0.3195$

(v) $\ddot{a}_{x:\overline{20|}} = 12.522$

(vi) $i = 0.05$

(vii) Premiums are determined using the equivalence principle.

3. Calculate the expense-loaded first-year premium including policy fee assuming that per-policy expenses are matched separately by first-year and renewal policy fees.

(A) 884

(B) 899

(C) 904

(D) 909

(E) 924

3. **(solution)**

Excluding per policy expenses, policy fee, and expenses associated with policy fee.
APV (actuarial present value) of benefits $= 25,000\,\bar{A}_{x:\overline{20}|} = (25,000)(0.4058) = 10,145$

Let G denote the expense-loaded premium, excluding policy fee.

$$\begin{aligned}\text{APV of expenses} &= (0.25-0.05)G + 0.05G\,\ddot{a}_{x:\overline{20}|} + \left[(2.00-0.50)+0.50\,\ddot{a}_{x:\overline{20}|}\right](25,000/1000)\\ &= \left[0.20+(0.05)(12.522)\right]G + \left[1.50+(0.50)(12.522)\right]25\\ &= 0.8261G + 194.025\end{aligned}$$

APV of premiums $= G\,\ddot{a}_{x:\overline{20}|} = 12.522G$

Equivalence principle:

APV premium = APV benefits + APV expenses
$12.522G = 10,145 + 0.8261G + 194.025$

$$G = \frac{10,339.025}{(12.522-0.8261)} = 883.99$$

This G is the premium excluding policy fee.

Now consider only year 1 per policy expenses, the year one policy fee (call it F_1), and expenses associated with F_1.

APV benefits = 0
APV premium = F_1

Equivalence principle

$$F_1 = 15 + 0.25F_1$$

$$F_1 = \frac{15}{0.75} = 20$$

Total year one premium = $G + F_1$
= 884+20
= 904

3 – 5. *(Repeated for convenience). Use the following information for questions* 3 – 5.

For a semicontinuous 20-year endowment insurance of 25,000 on (x), you are given:

(i) The following expenses are payable at the beginning of the year:

	Percent of Premium	Per 1000 Insurance	Per Policy
First Year	25%	2.00	15.00
Renewal	5%	0.50	3.00

(ii) Deaths are uniformly distributed over each year of age.

(iii) $\bar{A}_{x:\overline{20|}} = 0.4058$

(iv) $A_{\overset{1}{x}:\overline{20|}} = 0.3195$

(v) $\ddot{a}_{x:\overline{20|}} = 12.522$

(vi) $i = 0.05$

(vii) Premiums are determined using the equivalence principle.

4. Calculate the expense-loaded renewal premiums including policy fee assuming that per-policy expenses are matched separately by first-year and renewal policy fees.

(A) 884

(B) 887

(C) 899

(D) 909

(E) 912

4. (solution)

Get G as in problem 3; $G = 884$

Now consider renewal per policy expenses, renewal policy fees (here called F_R) and expenses associated with F_R.

APV benefits = 0

$$\begin{aligned}\text{APV expenses} &= \left(3+0.05F_R\right)a_{x:\overline{19|}}\\ &= \left(3+0.05F_R\right)\left(12.522-1\right)\\ &= 34.566+0.5761F_R\end{aligned}$$

$$\begin{aligned}\text{APV premiums} &= a_{x:\overline{19|}}F_R\\ &= \left(12.522-i\right)F_R\\ &= 11.522F_R\end{aligned}$$

Equivalence principle:

$$11.522F_R = 34.566+0.5761F_R$$

$$F_R = \frac{34.566}{11.522-0.5761} = 3.158$$

$$\begin{aligned}\text{Total renewal premium} &= G+F_R\\ &= 884+3.16\\ &= 887\end{aligned}$$

Since all the renewal expenses are level, you could reason that at the start of every renewal year, you collect F_R and pay expenses of $3+0.05F_R$, thus $F_R = \dfrac{3}{1-0.05} = 3.16$

Such reasoning is valid, but only in the case the policy fee and all expenses in the policy fee calculation are level.

3 - 5. *(Repeated for convenience). Use the following information for questions* 3 – 5.

For a semicontinuous 20-year endowment insurance of 25,000 on (x), you are given:

(i) The following expenses are payable at the beginning of the year:

	Percent of Premium	Per 1000 Insurance	Per Policy
First Year	25%	2.00	15.00
Renewal	5%	0.50	3.00

(ii) Deaths are uniformly distributed over each year of age.

(iii) $\bar{A}_{x:\overline{20|}} = 0.4058$

(iv) $A_{x:\overline{20|}}^{\;\;1} = 0.3195$

(v) $\ddot{a}_{x:\overline{20|}} = 12.522$

(vi) $i = 0.05$

(vii) Premiums are determined using the equivalence principle.

5. Calculate the level annual expense-loaded premium.

(A) 884

(B) 888

(C) 893

(D) 909

(E) 913

5. **(solution)**

Let P denote the expense-loaded premium

From problem 3, APV of benefits = 10,145
From calculation exactly like problem 3,
APV of premiums = 12.522 P

APV of expenses = $\left(0.25-0.05\right)P+0.05\,P\,\ddot{a}_{x:\overline{20}|}+\left[\left(2.00-0.50\right)+0.50\,\ddot{a}_{x:\overline{20}|}\right]\left(25000/1000\right)$

$$+\left(15-3\right)+3\,\ddot{a}_{x:\overline{20}|}$$

$$=0.20P+\left(0.05P\right)\left(12.522\right)+\left(1.50+\left(0.50\right)\left(12.522\right)\right)\left(25\right)+12+\left(3\right)\left(12.522\right)$$

$$=0.8261P+243.59$$

Equivalence principle:

$$12.522\,P=10{,}145+0.8261P+244$$

$$P=\frac{10{,}389}{12.522-0.8261}$$

$$=888$$

6. For a 10-payment 20-year endowment insurance of 1000 on (40), you are given:

(i) The following expenses:

	First Year		Subsequent Years	
	Percent of Premium	Per Policy	Percent of Premium	Per Policy
Taxes	4%	0	4%	0
Sales Commission	25%	0	5%	0
Policy Maintenance	0	10	0	5

(ii) Expenses are paid at the beginning of each policy year.

(iii) Death benefits are payable at the moment of death.

(iv) The expense-loaded premium is determined using the equivalence principle.

Which of the following is a correct expression for the expense-loaded premium?

(A) $\left(1000\bar{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{9}|}\right)/\left(0.96\ddot{a}_{40:\overline{10}|} - 0.25 - 0.05\ddot{a}_{40:\overline{9}|}\right)$

(B) $\left(1000\bar{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{9}|}\right)/\left(0.91\ddot{a}_{40:\overline{10}|} - 0.2\right)$

(C) $\left(1000\bar{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{19}|}\right)/\left(0.96\ddot{a}_{40:\overline{10}|} - 0.25 - 0.05\ddot{a}_{40:\overline{9}|}\right)$

(D) $\left(1000\bar{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{19}|}\right)/\left(0.91\ddot{a}_{40:\overline{10}|} - 0.2\right)$

(E) $\left(1000\bar{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{9}|}\right)/\left(0.95\ddot{a}_{40:\overline{10}|} - 0.2 - 0.04\ddot{a}_{40:\overline{20}|}\right)$

6. **(solution)**

Let G denote the expense-loaded premium.
Actuarial present value (APV) of benefits = $1000\bar{A}_{40:\overline{20|}}$

APV of premiums = $G\ddot{a}_{40:\overline{10|}}$

$$\begin{aligned}\text{APV of expenses } &= (0.04+0.25)G+10+(0.04+0.05)G\,a_{40:\overline{9|}}+5a_{40:\overline{19|}}\\ &= 0.29G+10+0.09G\,a_{40:\overline{9|}}+5a_{40:\overline{19|}}\\ &= 0.2G+10+0.09G\,\ddot{a}_{40:\overline{10|}}+5a_{40:\overline{19|}}\end{aligned}$$

(The above step is getting an $\ddot{a}_{40:\overline{10|}}$ term since all the answer choices have one. It could equally well have been done later on).

Equivalence principle:

$$G\ddot{a}_{40:\overline{10|}}=1000\bar{A}_{40:\overline{20|}}+0.2G+10+0.09G\,\ddot{a}_{40:\overline{10|}}+5\,a_{40:\overline{19|}}$$

$$G\left(\ddot{a}_{40:\overline{10|}}-0.2-0.09\,\ddot{a}_{40:\overline{10|}}\right)=1000\bar{A}_{40:\overline{20|}}+10+5\,a_{40:\overline{19|}}$$

$$G=\frac{1000\bar{A}_{40:\overline{20|}}+10+5\,a_{40:\overline{19|}}}{0.91\ddot{a}_{40:\overline{10|}}-0.2}$$

7. For a fully discrete whole life insurance of 100,000 on (x), you are given:

(i) Expenses, paid at the beginning of the year, are as follows:

Year	Percentage of Premium Expenses	Per 1000 Expenses	Per Policy Expenses
1	50%	2.0	150
2+	4%	0.5	25

(ii) $i = 0.04$

(iii) $\ddot{a}_x = 10.8$

(iv) Per policy expenses are matched by a level policy fee to be paid in each year.

Calculate the expense-loaded premium using the equivalence principle.

(A) 5800

(B) 5930

(C) 6010

(D) 6120

(E) 6270

7. (solution)

Let G denote the expense-loaded premium excluding policy fee.
Actuarial Present Value (APV) of benefits $= 1000A_x$

$$= 100{,}000(1 - d\,\ddot{a}_x)$$

$$= 100{,}000\left(1 - \left(\frac{0.04}{1.04}\right)(10.8)\right)$$

$$= 58{,}462$$

APV of premiums $= G\,\ddot{a}_x = 10.8G$

Excluding per policy expenses and expenses on the policy fee,
APV(expenses) $= 0.5G + (2.0)(100) + (0.04G + (0.5)(100))a_x$

$$= 0.5G + 200 + (0.04G + 50)(9.8)$$

$$= 0.892G + 690$$

Equivalence principle:

$$10.8G = 58{,}462 + 0.892G + 690$$

$$G = \frac{59{,}152}{9.908} = 5970.13$$

Let F denote the policy fee.
APV of benefits = 0
APV of premiums = $F\,\ddot{a}_x = 10.8F$
APV of expenses = $150 + 25a_x + 0.5F + 0.04F\,a_x$

$$= 150 + 25(9.8) + 0.5F + 0.04F(9.8)$$

$$= 395 + 0.892F$$

Equivalence principle:

$$10.8F = 395 + 0.892F$$

$$F = \frac{395}{10.8 - 0.892}$$

$$= 39.87$$

Total premium $= G + F$

$$= 5970.13 + 39.87$$

$$= 6010$$

Note: Because both the total expense-loaded premium and the policy fee are level, it was not necessary to calculate the policy fee separately. Let P be the combined expense-loaded premium.

7. **(continued)**

APV benefits = 58,462
APV premiums = 10.8P
APV expenses = $0.892P+690+150+(25)(9.8)$
$= 0.892P+1085$

where 0.892P + 690 is comparable to the expenses in G above, now including all percent of premium expense.

Equivalence principle:

$$10.8P = 58,462+0.892P+1085$$

$$P = \frac{59547}{10.8-0.892}$$

$$= 6010$$

This (not calculating the policy fee separately, even though there is one) only works with level premiums and level policy fees.

8. For a fully discrete whole life insurance of 10,000 on (x), you are given:

(i) ${}_{10}AS = 1600$

(ii) $G = 200$

(iii) ${}_{11}CV = 1700$

(iv) $c_{10} = 0.04$ is the fraction of gross premium paid at time 10 for expenses.

(v) $e_{10} = 70$ is the amount of per policy expense paid at time 10.

(vi) Death and withdrawal are the only decrements.

(vii) $q_{x+10}^{(d)} = 0.02$

(viii) $q_{x+10}^{(w)} = 0.18$

(ix) $i = 0.05$

Calculate ${}_{11}AS$.

(A) 1302

(B) 1520

(C) 1628

(D) 1720

(E) 1878

8. **(solution)**

$$ {}_{11}AS = \frac{\left({}_{10}AS + G - c_{10}\,G - e_{10}\right)(1+i) - 10{,}000\,q_{x+10}^{(d)} - {}_{11}CV\,q_{x+10}^{(w)}}{1 - q_{x+10}^{(d)} - q_{x+10}^{(w)}} $$

$$ = \frac{\left(1600 + 200 - (0.04)(200) - 70\right)(1.05) - (10{,}000)(0.02) - (1700)(0.18)}{1 - 0.02 - 0.18} $$

$$ = \frac{1302.1}{0.8} $$

$$ = 1627.63 $$

9. For a fully discrete 10-year endowment insurance of 1000 on (35), you are given:

(i) Expenses are paid at the beginning of each year.

(ii) Annual per policy renewal expenses are 5.

(iii) Percent of premium renewal expenses are 10% of the expense-loaded premium.

(iv) $1000P_{35:\overline{10}|} = 76.87$

(v) The expense reserve at the end of year 9 is negative 1.67.

(vi) Expense-loaded premiums were calculated using the equivalence principle.

Calculate the expense-loaded premium for this insurance.

(A) 80.20

(B) 83.54

(C) 86.27

(D) 89.11

(E) 92.82

9. (solution)

Let G denote the expense-loaded premium.
G = benefit premium plus level premium (e) for expenses.
Expense reserve = Actuarial Present Value (APV) of future expenses – APV of future expense premiums.

At duration 9, there is only one future year's expenses and due future premium, both payable at the start of year 10.

Expense reserve = APV of expenses – APV of expense premiums

$$= 0.10G + 5 - e$$
$$= 0.10\left(1000P_{35:\overline{10}|} + e\right) + 5 - e$$
$$= (0.10)(76.87) + 5 - 0.9e$$
$$= 12.687 - 0.9e$$

$$12.687 - 0.9e = -1.67$$
$$e = 15.95$$

$$G = 1000P_{35:\overline{10}|} + e$$
$$= 76.87 + 15.95$$
$$= 92.82$$

(See Table 15.2.4 of Actuarial Mathematics for an example of expense reserve calculations).

10. For a fully discrete whole life insurance of 1000 on (x), you are given:

(i) $G = 30$

(ii) $e_k = 5, \quad k = 1,2,3,...$

(iii) $c_k = 0.02, \quad k = 1,2,3,...$

(iv) $i = 0.05$

(v) ${}_4CV = 75$

(vi) $q_{x+3}^{(d)} = 0.013$

(vii) $q_{x+3}^{(w)} = 0.05$

(viii) ${}_3AS = 25.22$

If withdrawals and all expenses for year 3 are each 120% of the values shown above, by how much does ${}_4AS$ decrease?

(A) 1.59

(B) 1.64

(C) 1.67

(D) 1.93

(E) 2.03

10. **(solution)**

$$ {}_4AS = \frac{\left({}_3AS + G - c_4 G - e_4\right)(1+i) - 1000 q_{x+3}^{(d)} - {}_4CV q_{x+3}^{(w)}}{1 - q_{x+3}^{(d)} - q_{x+3}^{(w)}} $$

Plugging in the given values:

$$ {}_4AS = \frac{\left(25.22 + 30 - (0.02)(30) - 5\right)(1.05) - 1000(0.013) - 75(0.05)}{1 - 0.013 - 0.05} $$

$$ = \frac{35.351}{0.937} $$

$$ = 37.73 $$

With higher expenses and withdrawals:

$$ {}_4AS^{\text{ revised}} = \frac{25.22 + 30 - (1.2)\left((0.02)(30) + 5\right)(1.05) - 1000(0.013) - 75(1.2)(0.05)}{1 - 0.013 - (1.2)(0.05)} $$

$$ = \frac{(48.5)(1.05) - 13 - 4.5}{0.927} $$

$$ = \frac{33.425}{0.927} $$

$$ = 36.06 $$

$$ {}_4AS - {}_4AS^{\text{ revised}} = 37.73 - 36.06 $$
$$ = 1.67 $$

11. For a fully discrete 5-payment 10-year deferred 20-year term insurance of 1000 on (30), you are given:

(i) The following expenses:

	Year 1		Years 2-10	
	Percent of Premium	Per Policy	Percent of Premium	Per Policy
Taxes	5%	0	5%	0
Sales commission	25%	0	10%	0
Policy maintenance	0	20	0	10

(ii) Expenses are paid at the beginning of each policy year.

(iii) The expense-loaded premium is determined using the equivalence principle.

Which of the following is correct expression for the expense-loaded premium?

(A) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{19|}}\right)/\left(0.95\,\ddot{a}_{30:\overline{5|}} - 0.25 - 0.10\,\ddot{a}_{30:\overline{4|}}\right)$

(B) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{19|}}\right)/\left(0.85\,\ddot{a}_{30:\overline{5|}} - 0.15\right)$

(C) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{19|}}\right)/\left(0.95\,\ddot{a}_{30:\overline{5|}} - 0.25 - 0.10\,a_{30:\overline{4|}}\right)$

(D) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{9|}}\right)/\left(0.95\,\ddot{a}_{30:\overline{5|}} - 0.25 - 0.10\,\ddot{a}_{30:\overline{4|}}\right)$

(E) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{9|}}\right)/\left(0.85\,\ddot{a}_{30:\overline{5|}} - 0.15\right)$

11. **(solution)**

Let G denote the expense-loaded premium.
APV (actuarial present value) of benefits $= 1000\,{}_{10|20}A_{30}$.

APV of premiums $= G\ddot{a}_{30:\overline{5}|}$.

APV of expenses $= (0.05 + 0.25)G + 20$ first year

$+\left[(0.05+0.10)G + 10\right]a_{30:\overline{4}|}$ years 2-5

$+10\ {}_{5|}\ddot{a}_{35:\overline{4}|}$ years 6-10 (there is no premium)

$$= 0.30G + 0.15G\,a_{30:\overline{4}|} + 20 + 10\,a_{30:\overline{4}|} + 10\ {}_{5|}\ddot{a}_{30:\overline{5}|}$$

$$= 0.15G + 0.15G\ddot{a}_{30:\overline{5}|} + 20 + 10\,a_{30:\overline{9}|}$$

(The step above is motivated by the form of the answer. You could equally well put it that form later).

Equivalence principle:

$$G\ddot{a}_{30:\overline{5}|} = 1000\,{}_{10|20}A_{30} + 0.15G + 0.15G\,\ddot{a}_{30:\overline{5}|} + 20 + 10\,a_{30:\overline{9}|}$$

$$G = \frac{\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{9}|}\right)}{(1-0.15)\ddot{a}_{30:\overline{5}|} - 0.15}$$

$$= \frac{\left(1000\ {}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{9}|}\right)}{0.85\,\ddot{a}_{30:\overline{5}|} - 0.15}$$

12. For a special single premium 2-year endowment insurance on (x), you are given:

(i) Death benefits, payable at the end of the year of death, are:

$$b_1 = 3000$$
$$b_2 = 2000$$

(ii) The maturity benefit is 1000.

(iii) Expenses, payable at the beginning of the year:
  (a) Taxes are 2% of the expense-loaded premium.
  (b) Commissions are 3% of the expense-loaded premium.
  (c) Other expenses are 15 in the first year and 2 in the second year.

(iv) $i = 0.04$

(v) $p_x = 0.9$
$p_{x+1} = 0.8$

Calculate the expense-loaded premium using the equivalence principle.

(A) 670

(B) 940

(C) 1000

(D) 1300

(E) 1370

12. **(solution)**

Let G denote the expense-loaded premium
APV (actuarial present value) of benefits

$$=(0.1)(3000)v+(0.9)(0.2)(2000)v^2+(0.9)(0.8)1000v^2$$

$$=\frac{300}{1.04}+\frac{360}{1.04^2}+\frac{720}{1.04^2}=1286.98$$

APV of premium = G
APV of expenses $=0.02G+0.03G+15+(0.9)(2)v$

$$=0.05G+15+\frac{1.8}{1.04}$$

$$=0.05G+16.73$$

Equivalence principle: $G=1286.98+0.05G+16.73$

$$G=\frac{1303.71}{1-0.05}=1372.33$$

13. For a fully discrete 2-payment, 3-year term insurance of 10,000 on (x), you are given:

(i) $i = 0.05$

(ii) $q_x = 0.10$
$q_{x+1} = 0.15$
$q_{x+2} = 0.20$

(iii) Death is the only decrement.

(iv) Expenses, paid at the beginning of the year, are:

Policy Year	Per policy	Per 1000 of insurance	Fraction of premium
1	25	4.50	0.20
2	10	1.50	0.10
3	10	1.50	–

(v) Settlement expenses, paid at the end of the year of death, are 20 per policy plus 1 per 1000 of insurance.

(vi) G is the expense-loaded level annual premium for this insurance.

(vii) The single benefit premium for this insurance is 3499.

Calculate G, using the equivalence principle.

(A) 1597

(B) 2296

(C) 2303

(D) 2343

(E) 2575

13. **(solution)**

APV (actuarial present value) of benefits = 3499 (given)

$$\text{APV of premiums } = G + (0.9)(G)v$$
$$= G + \frac{0.9G}{1.05} = 1.8571G$$

APV of expenses, except settlement expenses,

$$= \left[25 + (4.5)(10) + 0.2G\right] + (0.9)\left[10 + (1.5)(10) + 0.1G\right]v + (0.9)(0.85)\left[10 + (1.5)(10)\right]v^2$$
$$= 70 + 0.2G + \frac{0.9(25 + 0.1G)}{1.05} + \frac{0.765(25)}{1.05^2}$$
=108.78+0.2857G

Settlement expenses are $20 + (1)(10) = 30$, payable at the same time the death benefit is paid.

$$\text{So APV of settlement expenses } = \left(\frac{30}{10{,}000}\right) \text{APV of benefits}$$
$$= (0.003)(3499)$$
$$= 10.50$$

Equivalence principle:

$$1.8571G = 3499 + 108.78 + 0.2857G + 10.50$$
$$G = \frac{3618.28}{1.8571 - 0.2857} = 2302.59$$

14. For a fully discrete 20-year endowment insurance of 10,000 on (50), you are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) The annual contract premium is 495.

(iv) Expenses are payable at the beginning of the year.

(v) The expenses are:

	Percent of Premium	Per Policy	Per 1000 of Insurance
First Year	35%	20	15.00
Renewal	5%	5	1.50

Calculate the actuarial present value of amounts available for profit and contingencies.

(A) 930

(B) 1080

(C) 1130

(D) 1180

(E) 1230

14. (solution)

$$\ddot{a}_{50:\overline{20}|} = \ddot{a}_{50} - {}_{20}E_{50}\,\ddot{a}_{70}$$
$$= 13.2668 - (0.23047)(8.5693)$$
$$= 11.2918$$

$$A_{50:\overline{20}|} = 1 - d\,\ddot{a}_{50:\overline{20}|} = 1 - \left(\frac{0.06}{1.06}\right)(11.2918)$$
$$= 0.36084$$

Actuarial present value (APV) of benefits $= 10{,}000 A_{50:\overline{20}|}$
$= 3608.40$

APV of premiums $= 495\,\ddot{a}_{50:\overline{20}|}$
$= 5589.44$

APV of expenses $= (0.35)(495) + 20 + (15)(10) + \left[(0.05)(495) + 5 + (1.50)(10)\right] a_{50:\overline{19}|}$
$= 343.25 + (44.75)(11.2918 - 1)$
$= 803.81$

APV of amounts available for profit and contingencies
= APV premium – APV benefits – APV expenses
= 5589.44 – 3608.40 – 803.81
= 1177.23

15. For a fully continuous whole life insurance of 1 on (x), you are given:

(i) $\delta = 0.04$

(ii) $\bar{a}_x = 12$

(iii) $Var\left(v^T\right) = 0.10$

(iv) ${}_oL_e = {}_oL + E$, is the expense-augmented loss variable,

where ${}_oL = v^T - \bar{P}\left(\bar{A}_x\right)\bar{a}_{\overline{T|}}$

$E = c_o + \left(g - e\right)\bar{a}_{\overline{T|}}$

$c_o =$ initial expenses

$g = 0.0030$, is the annual rate of continuous maintenance expense;

$e = 0.0066$, is the annual expense loading in the premium.

Calculate $Var\left({}_oL_e\right)$.

(A) 0.208

(B) 0.217

(C) 0.308

(D) 0.434

(E) 0.472

15. **(solution)**

$$\bar{P}\left(\bar{A}_x\right) = \frac{1}{\bar{a}_x} - \delta = \frac{1}{12} - 0.04 = 0.04333$$

$$\begin{aligned}
{}_oL_e &= {}_oL + E \\
&= v^T - \bar{P}\left(\bar{A}_x\right)\bar{a}_{\overline{T|}} + c_o + (g-e)\bar{a}_{\overline{T|}} \\
&= v^T - \bar{P}\left(\bar{A}_x\right)\left(\frac{1-v^T}{\delta}\right) + c_o + (g-e)\left(\frac{1-v^T}{\delta}\right) \\
&= v^T\left(1 + \frac{\bar{P}\left(\bar{A}_x\right)}{\delta} - \frac{(g-e)}{\delta}\right) - \frac{\bar{P}\left(\bar{A}_x\right)}{\delta} + c_o + \frac{(g-e)}{\delta}
\end{aligned}$$

$$Var\left({}_oL_e\right) = Var\left(v^T\right)\left(1 + \frac{\bar{P}\left(\bar{A}_x\right)}{\delta} - \frac{(g-e)}{\delta}\right)^2$$

Above step is because for any random variable *X* and constants *a* and *b*, $Var(aX+b) = a^2\,Var(X)$.
Apply that formula with $X = v^T$.

Plugging in,

$$\begin{aligned}
Var\left({}_oL_e\right) &= (0.10)\left(1 + \frac{0.04333}{0.04} - \frac{(0.0030 - 0.0066)}{0.04}\right)^2 \\
&= (0.10)(2.17325)^2 \\
&= 0.472
\end{aligned}$$