# The Impact of the Automatic Balancing Mechanism for the Public Pension in Japan on the Extreme Elderly

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**Abstract**

Most developed countries are seeking ways to maintain a sustainable social security system. Japan is no exception. The "old-age dependency ratio" in Japan is currently 35 percent, and is expected to be 74 percent in 2050. Recently, the Japanese government has adopted an automatic balancing mechanism, which gradually reduces the real price of the public pension through a reduction of inflation adjustments. The reduction is a random process, so the elderly, in particular the extreme elderly, inherit significant uncertainties regarding the public pension. The objectives of this paper are threefold. First, we review the Japanese mortality and life expectancy emphasizing the growth in the extreme aged, and explain the underlying longevity issues that led to the automatic balancing mechanism. Second, by means of stochastic mortality and fertility modeling, we analyze how a mortality decline, particularly at extreme ages, will affect the future of public pensions in Japan. Third, we demonstrate, on the basis of the stochastic projections we made, how the automatic balancing mechanism will affect the financial security of people over age 100.

**Keywords:** Life expectancy; Longevity risk; Old-age dependency ratio; The Cairns-Blake-Dowd model; Value-at-Risk

## 1 Introduction

Most developed countries are seeking ways to maintain a sustainable social security system. Japan is no exception. Its old-age dependency ratio, that is, the ratio of the population aged 65 and more to the population aged 15 to 64, is currently 35%. In 2050, it is expected to be 74%, which is significantly higher than the corresponding projected values for the US (35%), the UK (38%) and Canada (43%).[1] The public

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[1] Source: The United Nations (2008).

pension system in Japan is pay-as-you-go, so its sustainability is highly subject to demographic changes. As a result of an aging population, there is a need to either raise contributions, reduce benefits, or both. However, none of these is an easy political decision.

In order to reduce people's apprehension for increasing contributions, in 2004 the government designed a new contribution schedule, which gradually increases contribution rates up to a maximum of 16,900 yen[2] per month for flat pensions and 18.3% of an annual income for earnings-related pensions. Under the new schedule, however, the benefit level may decrease according to the "automatic balancing mechanism." The mechanism gradually reduces the real price of the public pension through a reduction of benefit inflation adjustments. Since the increase in contribution rates is capped, the source of anxiety is now mainly the potential reduction of benefits.

The reduction of benefits is uncertain, because it depends heavily on the demographics of the future Japanese population. Nevertheless, in the government's financial report on the public pension system, deterministic demographic assumptions are used. More specifically, the report presents three different scenarios of the financial outcome (a best-estimate, high-cost estimate, and low-cost estimate) on the basis of different assumptions of mortality and fertility rates. These scenarios do not tell us the likelihood of deviating from the best-estimate, as there is no probability attached to them. Further, from these scenarios, we cannot understand how large the reduction of benefits will be in extreme circumstances.

We may have a better understanding of Japan's public pension by means of a stochastic analysis. Such an analysis has been performed in North America. For example, the Office of the Chief Actuary of the Social Security Administration (2004) in the United States created a discrete-time model to analyze the financial conditions of the social security system (OASDI). In particular, they use an AR(1) model for the annual rate of decrease in the central death rates and an ARMA(4,1) model for the total fertility rate, which is the sum of age-specific birth rates for women aged 14 to 49. Their results include not only the key demographic indicators but also the long-range actuarial balance of the OASDI.

An early attempt to analyze Japan's public pension has been provided by Kitamura and Nakajima (2004). Their model is largely continuous-time and is focused on the financial risk associated with the pension system. They present many useful results regarding the financial balance between contributions and benefits, but demographic risk, which will affect future benefits through the automatic balancing

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[2] \$1 = 83.45 yen as of September 30th, 2010

mechanism, is not extensively analyzed.

This paper presents a stochastic analysis of the longevity risk associated with the automatic balancing mechanism. Such analysis is particularly important for Japan, because for Japanese people the main source of post-retirement income is public pensions. According to the Ministry of Health, Labour and Welfare, as of 2008, the public pension is the only source of income for 61.2% of aged households. Furthermore, the automatic balancing mechanism has a compounding impact on the financial security of the elderly. This is because the longer the elderly live, the more they are affected by the reduction of pension benefits through the automatic balancing mechanism. Therefore, to fully understand the automatic balancing mechanism, an analysis of the longevity risk involved in the mechanism is crucial.

Another focus of this paper is the minimum benefit level provided by Japan's public pension. Under the automatic balancing mechanism, it is possible that the benefit level can go below some standard cost of living. This important issue, however, has not been extensively studied. Sakamoto (2008) discusses the minimum benefit level under the automatic balancing mechanism in Japan, Sweden and Germany. The discussion is mainly qualitative and the mechanism's potential impact on the minimum benefit level is not quantified. By means of stochastic modeling, in this paper we quantify the possibility that the amount of pension benefit falls below the minimum cost of living.

The remainder of this paper is organized as follows: Section 2 provides an overview of the public pension system in Japan, with a focus on the automatic balancing mechanism. Section 3 introduces time-series models of mortality and total fertility rates. These models allow us to make stochastic population projections, from which we can analyze the potential reduction in public pension benefits due to the automatic balancing mechanism. Section 4 quantifies the longevity risk associated with the automatic balancing mechanism, using measures including the longevity Value-at-Risk. Finally, Section 5 concludes the paper.

# 2 Japan's Pension System

## 2.1 An Overview

Before we perform a stochastic analysis, let us provide an overview of Japan's pension system. Japan's pension system consists of the following three tiers:

1. Tier 1, which is called the National Pension (NP) Scheme, is a universal public pension system, administered by the government. It provides a minimum level of old-age benefits (flat pensions). As of the end of March 2007, the NP Scheme has 70 million members, categorised into category 1 (self-employed people aged 20 to 59, unemployed people, students, etc), category 2 (employees aged 65 and below), and category 3 (spouses of category 2 employees). People with over 25 years enrollment are entitled to receive old-age benefits at the beginning of age 65. Its maximum benefit in 2009 is 792,100 yen per annum, which can be reduced depending on an enrollment period. As with the benefits, the contribution rate of the NP Scheme is also flat.

2. Tier 2 is public pension systems, also administered by the government. They provide further pensions (earnings-related pensions) for the workforce. Specifically, Tier 2 includes the Employees' Pension Insurance (EPI) Scheme in the private sector and the Mutual Aid Associations (MAA) in the public sector. In contrast to Tier 1, Tier 2 is only for members in category 2, employees in private or public sectors (38 million as of the end of March 2007). Its qualifying period for old-age EPI benefits and its pensionable age are the same as the NP's. Its annual benefit is calculated by multiplying 5.481/1000 of average pensionable remuneration[3] by the number of months enrolled, where average pensionable remuneration is defined as the average of revalued monthly pay and bonuses over enrolled months. As with the benefits, the contribution rates of the EPI Scheme and MAA are earnings-related.

3. Tier 3 is private pension plans, administered by employers or insurance companies.

This paper focuses on Tiers 1 and 2 schemes[4] because they involve the automatic balancing mechanism.

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[3] The maximum of pensionable remuneration is 620,000 yen and the minimum is 98,000 yen.

[4] According to the Ministry of Health, Labour and Welfare, the replacement ratio for an average household is 62.3% in 2009. They receive 223,000 yen per month, comprising 131,000 yen from Tier 1 and 92,000 yen from Tier 2.

To maintain the real price of pension benefits, the indexation of benefits was introduced in 1973. Tier 1 benefits are linked to the Consumer Price Index (CPI), while Tier 2 benefits are linked to both the wage index and the CPI since they provide earnings-related pensions rather than flat benefits. Both the CPI and the wage index were increasing during the period when Japan's economic growth was high, and contributed to maintaining the real price of the public pension.

However, in recent years, sustaining the public pension system has become difficult for two reasons: improvement in longevity and reduction in birthrates. These two issues can be seen from Table 1 in which we summarize the life expectancy at birth and the total fertility rate from 1955 to 2005. Over the past 50 years, the life expectancy for males has increased by 15.0 years, while that for females has increased by 17.7 years. On the other hand, the total fertility rate was nearly halved. Such a dramatic change in the demographic landscape means that, to ensure sustainability, Japan's public pension system needed to be amended, one way or another.

| Year | 1955 | 1965 | 1975 | 1985 | 1995 | 2005 |
|---|---|---|---|---|---|---|
| Male Life Expectancy | 63.60 | 67.74 | 71.73 | 74.78 | 76.38 | 78.56 |
| Female Life Expectancy | 67.75 | 72.92 | 76.89 | 80.48 | 82.85 | 85.52 |
| Total Fertility Rate | 2.37 | 2.14 | 1.91 | 1.76 | 1.42 | 1.26 |

Table 1: Sex-specific life expectancy at birth and total fertility rates for Japanese population, 1955–2005.

Although Japan's life expectancy is already very high and might reach close to the ceiling of life, it is expected to increase in the future. According to the official projections for Japan (medium projections), the life expectancy for males in 2055 is 83.67 and that for females is 90.34, that is, it may still increase by 5-year old. Thus, aging problem for the public pension plan is not only the problem in the past, but the problem in the future.

The cause of the improvement in life expectancy changed from an infectious disease to a chronic disease. In the first half of the 20th century, the life expectancy of an advanced nation had increased due to the mortality decline regarding an infectious disease. It has an impact on mortality rates for the elderly as well as those for the young and middle-aged people. In the second half of the 20th century, however, a chronic disease really matters in the increase in life expectancy for developed countries, such as Japan. This implies that the decline in mortality rates for the elderly mainly have affected, recently, and will affect the improvement in life expectancy.

Ishii (2008) investigates the contribution to the increase in Japan's life expectancy by age and sex. On the basis of the method proposed by Preston et al. (2001), he reveals that the contribution from the elderly aged over 75 has been significantly increasing since 1965. From 1990 to 1995, for example, the contribution from the elderly aged over 75 accounts for about 40% and 70% of the increase in the life expectancy for males and females, respectively. His results suggest that it is the reduction of mortality rates at old ages that really matters in our analysis.

As a result of improvement in longevity and reduction in birthrates, Japan's demographic structure had dramatically changed in the second half of the 20th century. One of the indicators to see the change in demographic structure is an old-age dependency ratio. Table 2 summarizes the old-age dependency ratio and the "extreme" old-age dependency ratio, the ratio of the population aged 75 and more to the population aged 15 to 64, from 1955 to 2005. The increase in both old-age dependency ratios has been accelerating over the past 50 years. The acceleration of extreme old-age dependency ratio is greater than that of old-age dependency ratio.

| Year | 1955 | 1965 | 1975 | 1985 | 1995 | 2005 |
|---|---|---|---|---|---|---|
| Old-age Dependency Ratio | 8.6% | 9.2% | 11.7% | 15.1% | 20.9% | 30.5% |
| Extreme Old-age Dependency Ratio | 2.5% | 2.8% | 3.7% | 5.7% | 8.2% | 13.8% |

Table 2: Old-age dependency ratio and extreme old-age dependency ratio for Japanese population, 1955-2005.

Let us take a look at old-age dependency ratios in the future. Table 3 summarizes two old-age dependency ratios, based on the official population projections for Japan (medium projections), from 2005 to 2055. Both ratios are expected to continue to increase. It is important to note that the increase in an extreme old-age dependency ratio is significant. In other words, the elderly over age 75 are expected to dominate the elderly. Without significant improvements in birthrate, Japan's public pension plan will embark on a critical stage.

| Year | 2005 | 2015 | 2025 | 2035 | 2045 | 2055 |
|---|---|---|---|---|---|---|
| Old-age Dependency Ratio | 30.5% | 44.0% | 51.2% | 59.2% | 72.5% | 79.4% |
| Extreme Old-age Dependency Ratio | 13.8% | 21.4% | 30.5% | 35.5% | 42.4% | 51.9% |

Table 3: Projected old-age dependency ratio and extreme old-age dependency ratio for Japanese population, 2005-2055.

## 2.2 The Automatic Balancing Mechanism

There are two options to keep the pay-as-you-go public pension system affordable in a rapidly aging population. One is to increase the contribution rates and the other is to reduce the pension benefits. No matter which way is used, the pension law needs to be amended. However, politicians are unlikely to support either option, since both may have a negative impact on their being elected. Sakamoto (2008), an actuary engaged in this reform, said

> "However, because of repeated measures taken to attain financial balance, the government faced an extremely difficult political situation. Specifically, because pension reforms that required increased burdens of people had been repeated, the reforms were ridiculed by the mass media as "mirage-like pensions." In addition, there was an ample possibility that the public might develope an allergic reaction of "What? Not again!" to similar measures"

As a result, it was decided to use a mechanism that will adjust the pension benefits gradually and automatically over time. In 2004, the Japanese government adopted the automatic balancing mechanism, which is fairly similar to that implemented in Sweden. Before the adoption of the automatic balancing mechanism, the benefit level is fully adjusted by the CPI and/or the wage index. However, with the mechanism in place, pensioners may not be able to receive fully adjusted pensions. The difference between the real price and the the partially inflation-adjusted price can be considered as the reduction of the benefit level. In other words, the automatic balancing mechanism ensures the sustainability of the public pension by gradually adjusting pension benefits, with the contribution rates remain fixed.

In mathematical terms, the automatic balancing mechanism can be described as follows. Given a CPI growth rate $c$, the pension benefits will be adjusted according to the formula below:

$$\text{adjusted indexation} = \begin{cases} \max(c+d, 0) & \text{if } c \text{ is positive} \\ c & \text{if } c \text{ is not positive,} \end{cases}$$

where $d$ is a negative number that determines how the indexation is adjusted. The deduction $d$ is the sum of two parameters, $d_1$ and $d_2$. Parameter $d_1$ reflects the effect of expected longevity improvements and is fixed to $-0.3\%$ by law. Parameter $d_2$ is linked to the growth rate of the number of contributing members in the pension schemes, and is determined by the geometrical average of growth rates over past three years. Note that the adjustment to the pension benefits can be positive or negative, depending on the sign of $c$. A parallel mathematical formula is defined for the wage index.

As of the time this paper is being written, the automatic balancing mechanism has not yet started. This is because the pension law stipulates that the benefit level must be maintained until the CPI is 1.7% higher than the CPI in 2005. The pension law also controls the stopping time of the mechanism. Specifically, it stipulates that the automatic balancing mechanism must stop if a replacement ratio for an average household becomes lower than 50%. According to the (medium variant) projections made in the government's financial report on the public pension system, the automatic balancing mechanism is expected to start in 2012 and finish in 2038. We will focus on this particular time period in the stochastic analysis.

# 3 The Models

## 3.1 Modeling Mortality Rates

We consider 58 years of mortality data (from 1950 to 2007), obtained from the Human Mortality Database (2010). The data are presented graphically in Figure 1. We observe that, for both genders, mortality has clearly improved.

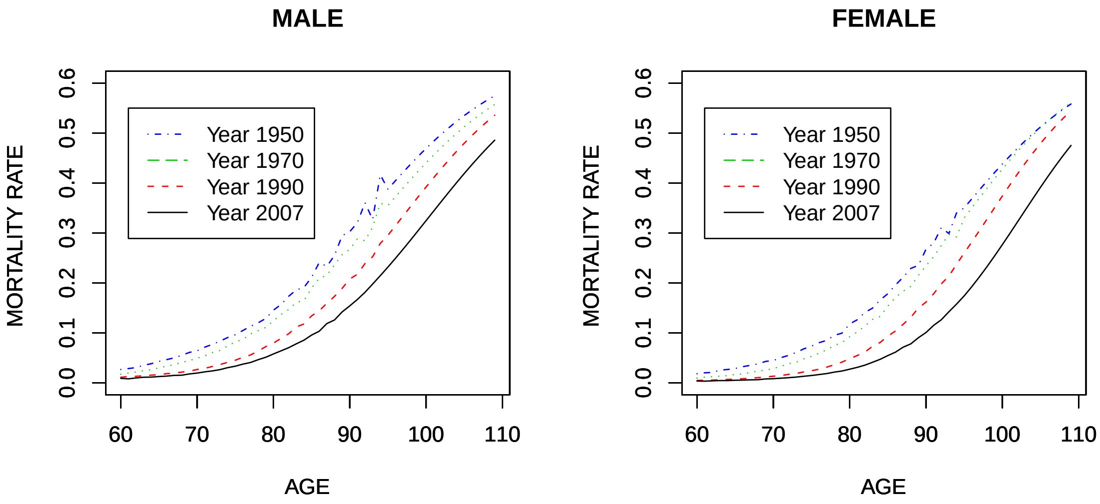

Figure 1: Sex-specific mortality rates from age 60 to 109 in years 1950, 1970, 1990 and 2007.

We use the two-factor model proposed by Cairns et al. (2006) to project future mortality rates. This model, which is also known as the Cairns-Blake-Dowd model, is a discrete-time model with two stochastic factors, one of which is a common factor that affects all the ages in an equal manner, the other of which has an effect that is proportional to age.

Mathematically, the model can be expressed as follows:

$$q_{x,t} = \frac{e^{A_1(t)+A_2(t)x}}{1+e^{A_1(t)+A_2(t)x}}, \tag{1}$$

where $q_{x,t}$ is the realized single-year death probability at age $x$ and time $t$, and $\{A_1(t)\}$ and $\{A_2(t)\}$ are discrete-time stochastic processes that are assumed to be measurable at time $t$. Alternatively, the model can be written as

$$\ln \frac{q_{x,t}}{1-q_{x,t}} = A_1(t) + A_2(t)x. \tag{2}$$

From this representation, we can see that $A_1(t)$ may be interpreted as an indication of the overall mortality level at time $t$, and that $A_2(t)$ may be interpreted as the steepness of the mortality curve (in logit scale) at time $t$.

The fitting procedure consists of two stages. First, for each year in the sample period of 1950 to 2007, we fit a mortality curve on the basis of equation (1). This is accomplished by minimizing the following sum of squared differences:

$$\sum_x \left( \ln \frac{q_{x,t}}{1-q_{x,t}} - \ln \frac{\hat{q}_{x,t}}{1-\hat{q}_{x,t}} \right)^2, \tag{3}$$

where $q_{x,t}$ and $\hat{q}_{x,t}$ are the actual and estimated probability of death (at age $x$ and time $t$), respectively. The summation is taken over the entire sample age range. By performing the minimization above for all years, we obtain a sequence of historic $A_1(t)$ and $A_2(t)$.

Figure 2 illustrates the fit of equation (2) to the actual mortality rates in year 2007. We observe that, over the age range of 60 to 109, the equation fits the crude mortality rates reasonably well.

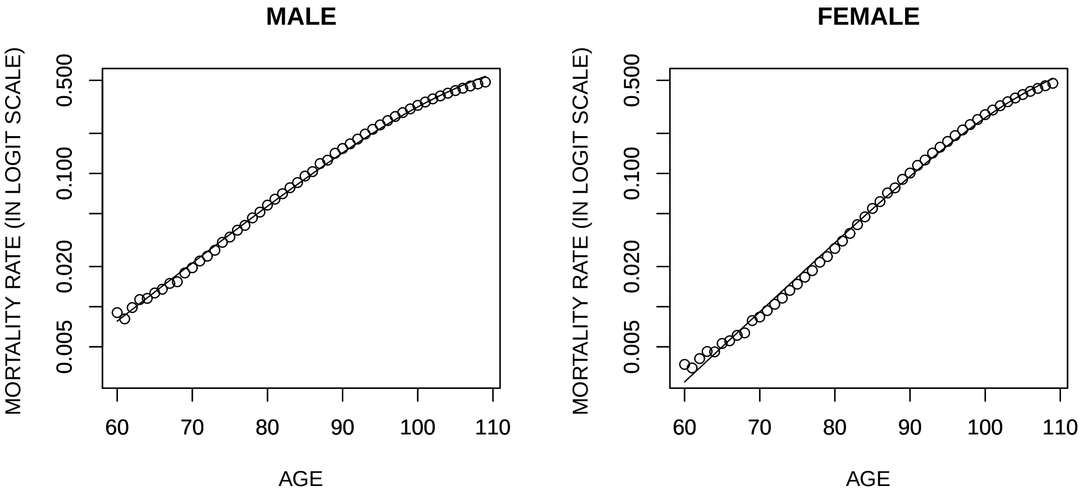

Figure 2: Actual and fitted mortality rates in 2007, males and females.

One drawback of the model is that it does not give a good fit to mortality rates at younger ages. In particular, the linearity shown in Figure 2 cannot be preserved if mortality data for younger ages (say 20 to 59) are involved in the estimation. However, we believe that this drawback is not so important to this study. This is partly because mortality improvement for younger ages has been very modest when compared with that for older ages, and partly because sensitivity analyses (not shown in this paper) indicate that the results are rather insensitive to changes in mortality rates at younger ages.

We fit equation (2) to historic mortality rates for the age range of 60 to 109.[5] In Figure 3 we show the estimated values of $A_1(t)$ and $A_2(t)$ for $t = 1950, \ldots, 2007$. We observe a clear trend in both series of $A_1(t)$ and $A_2(t)$. The downward trend in $A_1(t)$ reflects general improvements in mortality over time at all ages. The upward trend in $A_2(t)$ means that the mortality curve is becoming steeper; that is, mortality rates (in logit scale) are reducing faster at higher than lower ages.

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[5] In the preceding calculations, mortality rates below age 60 are assumed to be invariant over time beyond year 2007.

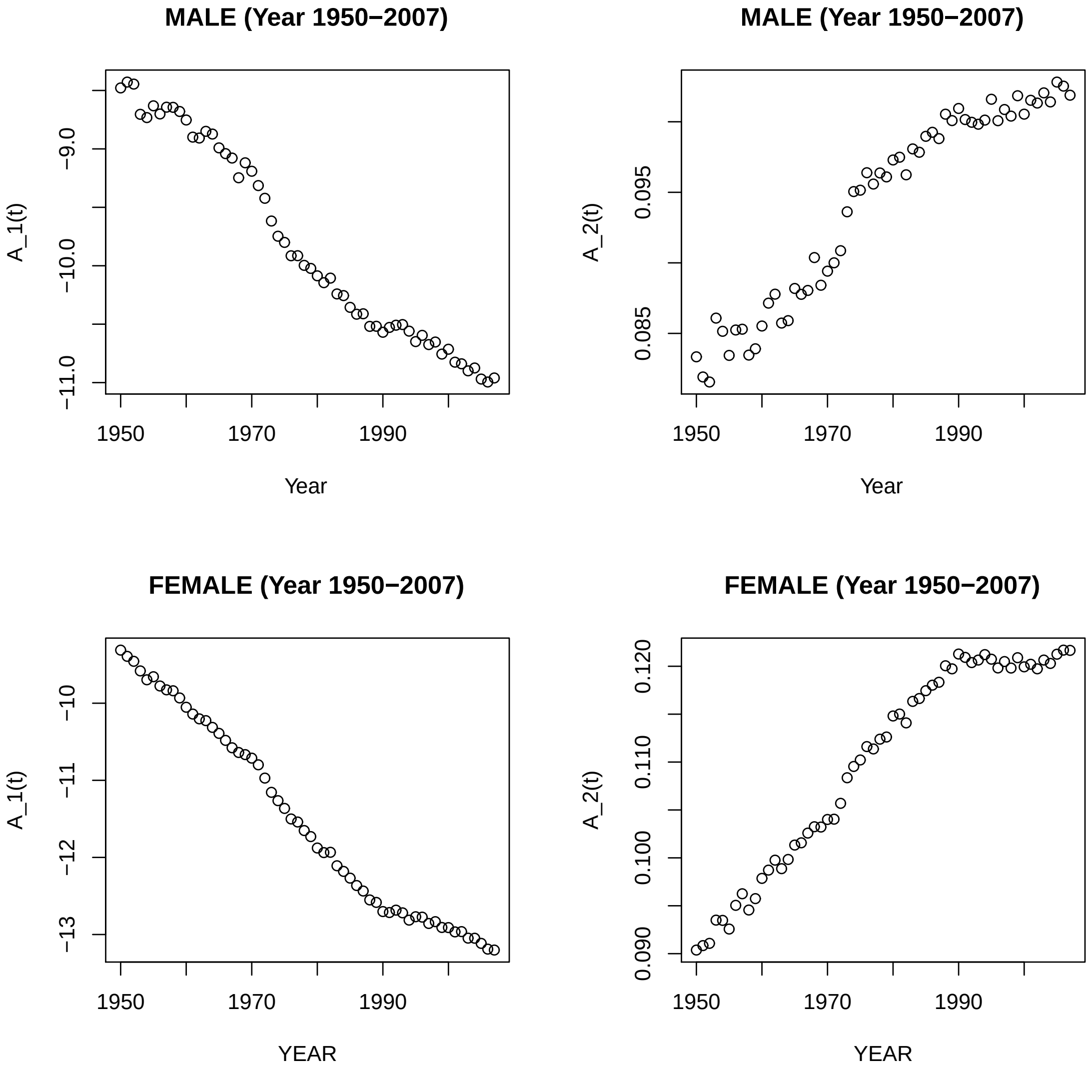

Figure 3: Estimated Values of $A_1(t)$ and $A_2(t)$, $t = 1950, 1951, \ldots, 2007$, males and females.

In the second stage, we model $\{A_1(t)\}$ and $\{A_2(t)\}$ with a bivariate time-series process. Given that the trends in $A_1(t)$ and $A_2(t)$ are highly linear, we consider the class of linear bivariate time-series models. On the basis of standard model selection criteria, we found that a 2-dimensional random walk with drift, that is,

$$A(t+1) = A(t) + \mu + CZ(t+1), \tag{4}$$

where $A(t) = (A_1(t), A_2(t))'$, $\mu = (\mu_1, \mu_2)'$ is a constant $2 \times 1$ vector, $C$ is a constant $2 \times 2$ upper triangular matrix, and $Z(t)$ is a 2-dimensional standard normal random variable, gives an adequate fit. We can estimate $\mu$ and $C$ by using the sample mean vector and the sample variance covariance matrix for the series of $D(t) = A(t+1) - A(t)$.

The estimates of $\mu$ and $C$ are presented in Table 4. The negativeness of $\hat{\mu}_1$ indicates a general reduction in mortality rates, while the positiveness of $\hat{\mu}_2$ indicates the increasing steepness of the mortality curves.

| | males | females |
|---|---|---|
| $\hat{\mu}$ | $\begin{pmatrix} -0.043548 \\ 0.000325 \end{pmatrix}$ | $\begin{pmatrix} -0.068241 \\ 0.000549 \end{pmatrix}$ |
| $V(=\hat{C}\hat{C}^t)$ | $\begin{pmatrix} 0.005419 & -0.000049 \\ -0.000049 & 0.000002 \end{pmatrix}$ | $\begin{pmatrix} 0.002783 & -0.000007 \\ -0.000007 & 0.000001 \end{pmatrix}$ |

Table 4: Estimates of $\mu$ and $V$ for both genders.

There is uncertainty involved in the estimation of parameters $\mu$ and $C$. We can quantify this piece of uncertainty by standard Bayesian methods, which treat the model parameters as random variables rather than fixed constants. Since there is no clear prior beliefs about the parameter values, we assume a non-informative prior distribution for parameters $\mu$ and $V$:

$$p(\mu, V) \propto |V|^{-3/2},$$

where $|V|$ is the determinant of the matrix $V$. Given the prior distribution and the data $D = \{D(1), D(2), ..., D(n)\}$, the posterior distributions for $\mu$ and $V$ are as follows:

$$\begin{aligned} V^{-1}|D &\sim \text{Wishart}(n-1, n^{-1}\hat{V}^{-1}), \\ \mu|V, D &\sim \text{MVN}(\hat{\mu}, n^{-1}V), \end{aligned}$$

where

$$\hat{\mu} = \frac{1}{n}\sum_{t=1}^{n} D(t),$$

and

$$\hat{V} = \frac{1}{n}\sum_{t=1}^{n}(D(t) - \hat{\mu})(D(t) - \hat{\mu})'.$$

We can incorporate parameter uncertainty into the projection of demographic quantities by simulations. In particular, when we simulate a sample path of future mortality rates, we first simulate a pair of $\mu$ and $V$ from the posterior distributions, and then use these values to generate the whole of that sample path. Given a sample path, we can calculate one possible value of the demographic quality of interest. So by generating a large number of sample paths, we can create for the demographic quantity an empirical distribution, from which we can obtain measures of uncertainty, such as confidence intervals. We refer readers to Cairns et al. (2006) for further details regarding the Bayesian methods we used.

Mortality fan charts, proposed by Blake, Dowd and Cairns (2008), effectively present simulated mortality rates. Figure 4-1 and 4-2 draw the fan charts of simulated mortality rates for Japanese males and females, respectively, at age 70, 80, 90 and 100. Each fan chart shows the central 90% prediction interval over 50 years with the highest and lowest bounds, the central 80% prediction intervals with the next highest and next lowest ones, and so on. The shading represents the likelihood of the outcome - the darker the shading, the more likely the outcome.

We obtain four observations from these fan charts. First, the fans for both males and females are wider as older, meaning that the volatility of mortality rates increases with age. Second, the upside volatility is greater than the downside volatility. This observation can be seen in the wideness of each fan chart. This is a reasonable characteristic because it is possible that mortality rates might increase due to a contagious disease, such as influenza, whereas that mortality rates might be hard to decrease because Japanese mortality rates are already extremely low. Third, all mortality rates have negative trends as years go by. Mortality rates for age 70 and 80 are highly likely to decrease over the forecast horizon; on the other hands, those for age 90 and 100 are not. Especially, mortality projections for age 100 involve significant uncertainties, including the possibility mortality rates even increase over 50 years. Fourth, comparing mortality rates for males and females, those for males involve larger volatility than those for females. The mortality rates for males are greater than those for females, so the former involves more significant volatility than the latter.

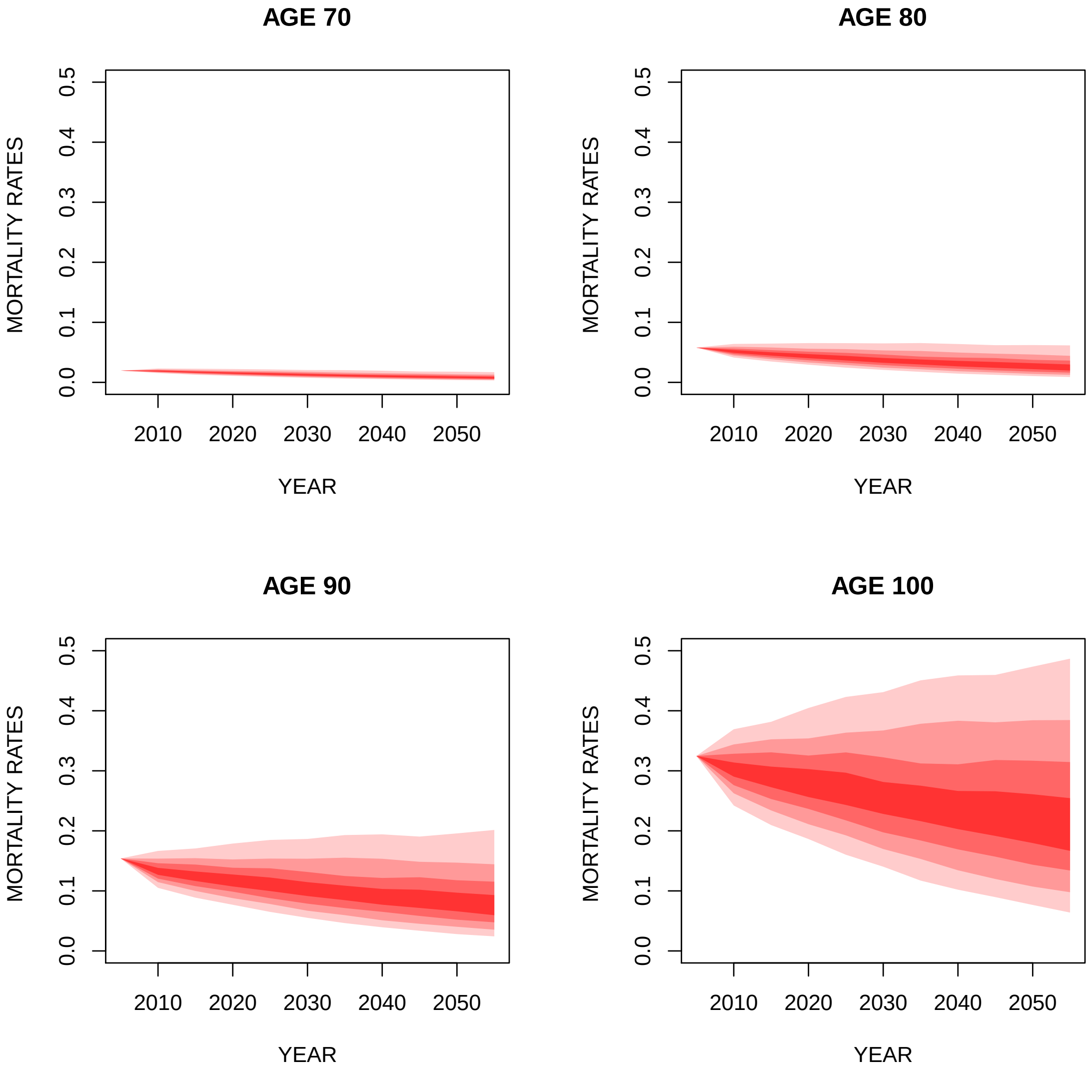

Figure 4-1: Mortality fan charts for Japanese males at age 70, 80, 90 and 100.

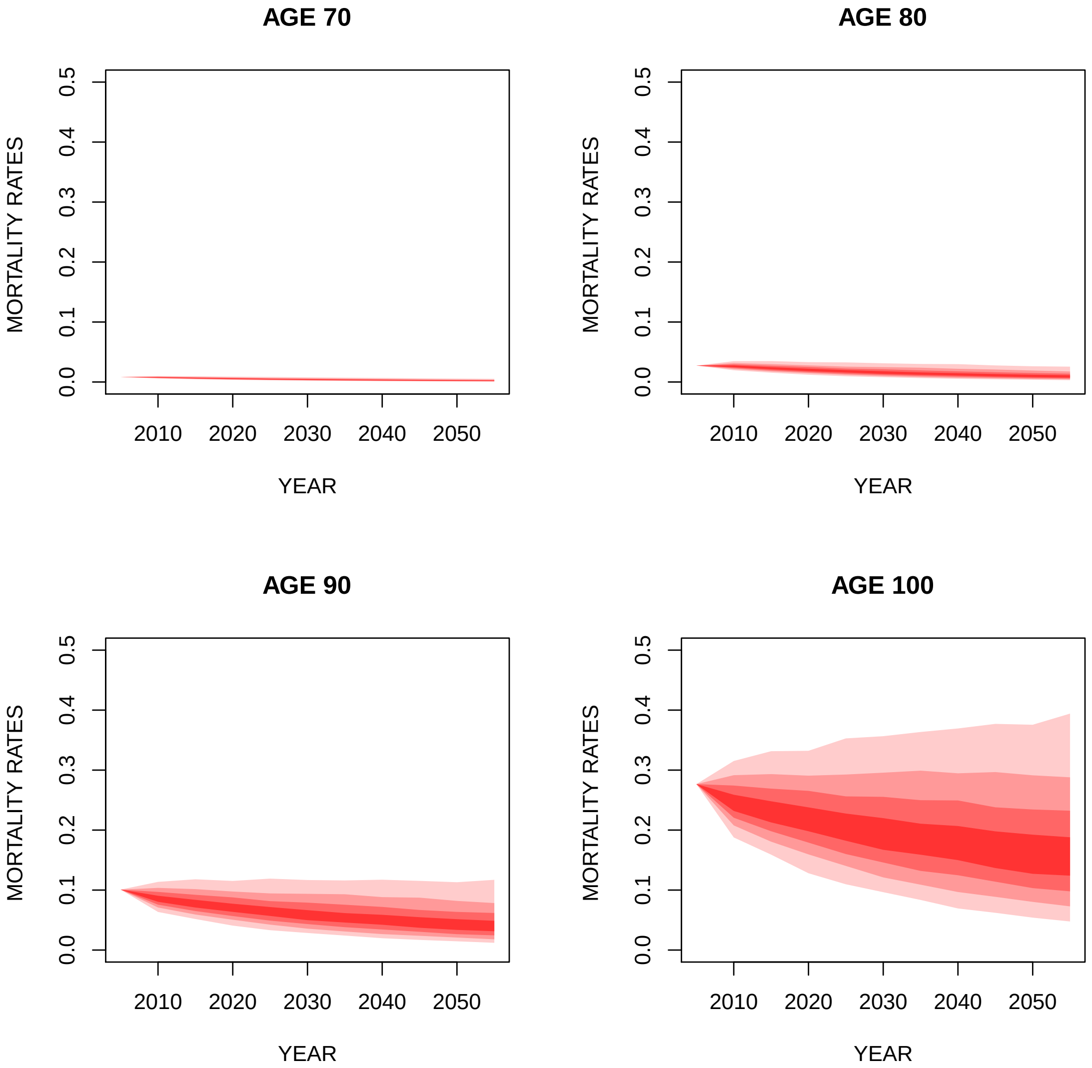

Figure 4-1: Mortality fan charts for Japanese females at age 70, 80, 90 and 100.

Using the Cairns-Blake-Dowd model and Bayesian methods, we obtain empirical distributions of life expectancy at age 60 in various future years (see Figures 5-1 and 5-2). Note that the dispersion is due partly to the stochastic uncertainty implied by equation (4) and partly to the uncertainty involved in estimating the parameters. As expected, both the projected life expectancy and the uncertainty associated with it increase with time. Also shown in Figures 5-1 and 5-2 are the corresponding official projections (solid lines for the medium variant and dotted lines for high and low variants). It appears that the method we use implies more longevity improvement than that used in the official projections, proposed by Kaneko et al. (2008)

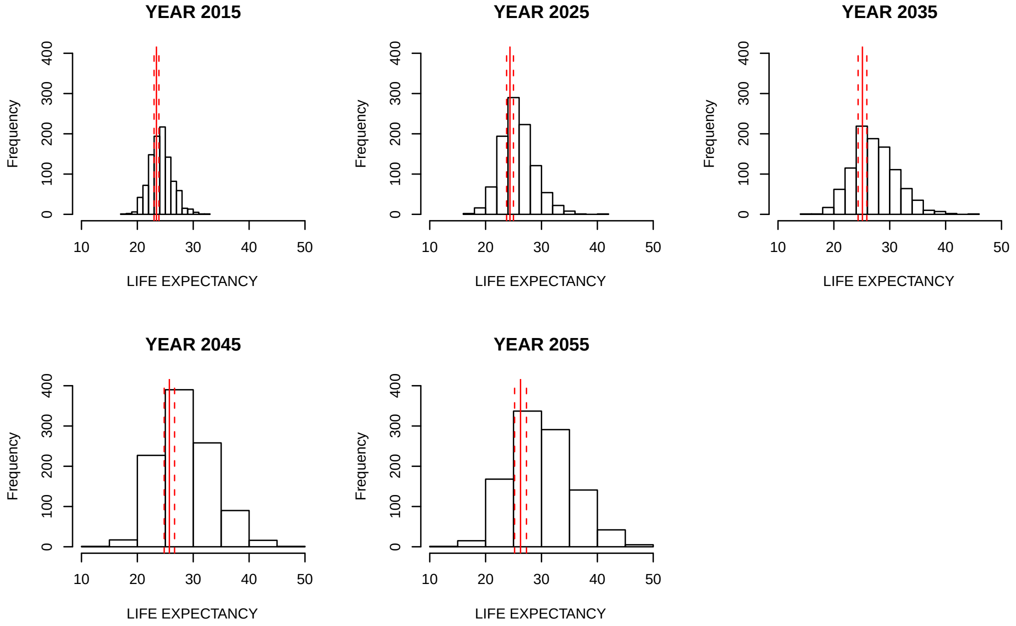

Figure 5-1: Life expectancy for males at age 60 in years 2015, 2025, 2035, 2045, and 2055.

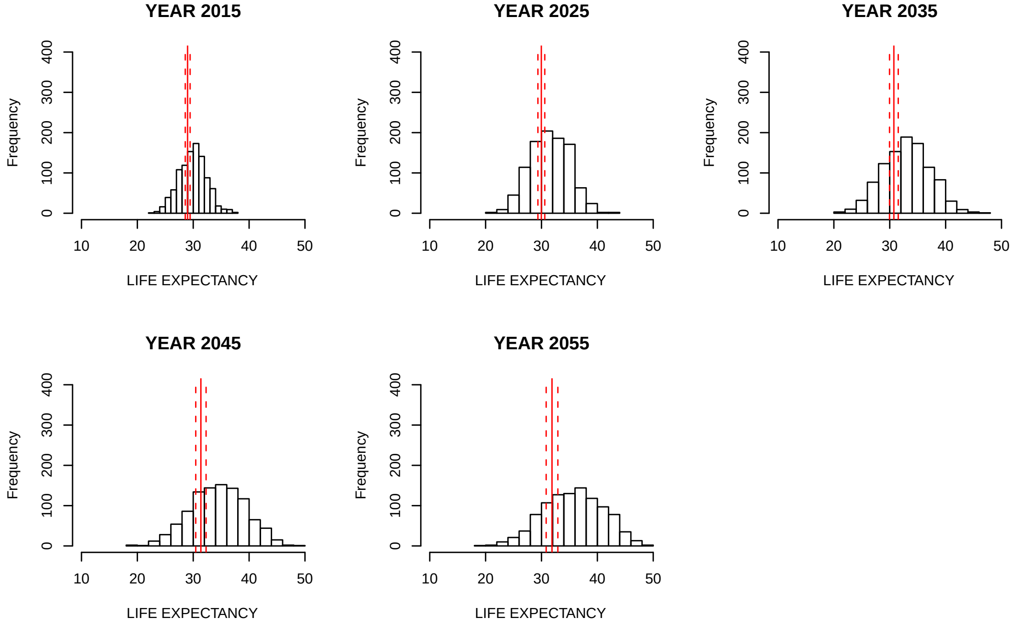

Figure 5-2: Life expectancy for females at age 60 in years 2015, 2025, 2035, 2045, and 2055.

## 3.2 Modeling the Total Fertility Rate

Let us first look at the historic total fertility rates, shown graphically in Figure 6. We observe that the trend is fairly linear. By using the Box-Jenkin's (1976) method, we find that a first order difference stationary process gives a good fit to the logarithm of the historic rates.

However, we do not believe that a purely statistical approach is appropriate for modeling the total fertility rate. This is because over the past 50 years, fertility rates in Japan have been significantly affected by a series of events which are unlikely to recur in the future. For instance, the peak from late 1960s to early 1970s is primarily due to the babyboom effect. Another example is the sudden drop in year 1966, the last year of the Fiery Horse[6], during which the birth of baby girls is believed to have adverse effects on fathers. The year of the Fiery Horse comes every sixty years, so the next one will be 2026. However, we believe that its effect on the total fertility rate would be a lot smaller as the younger generations are far less superstitious.

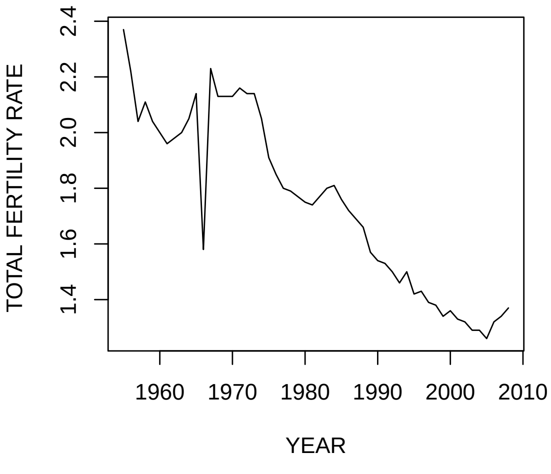

Figure 6: Total fertility rates from 1955 to 2008.

It is not straightforward to project future fertility rates. The official projection made by the Japanese government is largely qualitative, with a careful consideration

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[6] We refer readers to Kaneko (2009) for further details on relations between the Fiery Horse and the total fertility rate.

of cohort-specific characteristics. Factors such as a first marriage age, a divorce rate and a childbearing age are taken into account. In Figure 7 we show the high, medium and low official projections of the total fertility rate. The official projection predicts that the total fertility rate will approach 1.26 in the long run, with a prediction interval from 1.05 to 1.5.

Note that, in recent years, the total fertility rates in Japan have been improving. Kaneko (2009) examines this upward trend is a long- or short-term effect and states

> ”From a long-term perspective, if the rise in fertility currently being observed is purely caused by type-H period effects, the period fertility rate should decline again within the next several years and will ultimately not significantly change the long-term outlook for the fertility rate. In fact, according to observation of the monthly development, fertility rates have already been turning around to a downward trend again for at least half a year since December 2008.”

In his paper, type-H period effects are defined as the change in fertility rates caused by a cohort reacting to certain events, such as the Fiery Horse, which leads to adjusting the schedule of childbearing. More details are discussed in his paper.

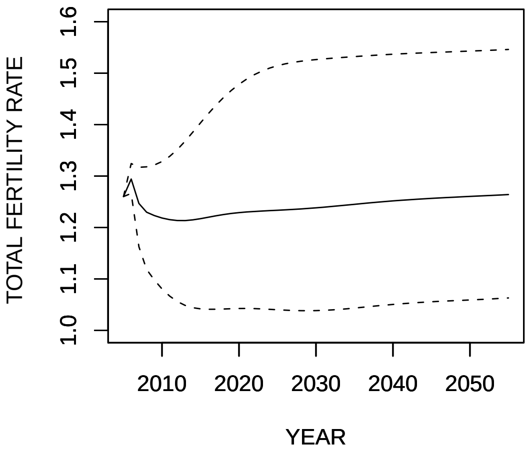

Figure 7: High, medium and low projections of total fertility rates from 2005 to 2055.

The official projection is carefully done, but it is not stochastic, by which we mean we cannot generate sample paths of the total fertility rate directly from it.

Here we expand the official projection to a stochastic one. Specifically, we consider the following process:

$$\ln(B(t+1)) - \ln(1.26) = \phi_1(\ln(B(t) - \ln(1.26))) + \sigma W(t+1), \tag{5}$$

where $B(t)$ is the total fertility rate in year $t$, $\phi_1$ and $\sigma$ are constants, and $\{W(t)\}$ is a sequence of iid standard normal random variables. It can be shown easily that the unconditional mean of $\ln(B(t))$ is $\ln(1.26)$, so the process ensures that in the long run the projected total fertility rates are in line with the official projection. The autoregressive parameter $\phi_1$ and the volatility parameter $\sigma$ are estimated from historic total fertility rates from year 1977 when the peak due to the babyboomers was passed. We obtain $\hat{\phi}_1 = 0.9569$ and $\hat{\sigma} = 0.0222$.

In Figure 8 we show 100 sample paths of the total fertility rate. The sample paths, on average, revert from 1.38 (the level in 2008) to 1.26 (the long-run official medium projection). We also observe that the uncertainty implied by equation (5) is roughly in line with the high and low official projections.

Some may consider these paths are unlikely to be realistic since they remain extremely low. However, these are consistent with the official projections, which are deeply examined. Additionally, according to the Delphi survey (2006), the experts in different major state fertility rates remain low in 50 years. For example, the estimates of fertility rates in 2050 are 1.28 for demography, 1.25 for medical science, 1.24 for sociology, and 1.25 for economics. The standard deviations are 0.21 for demography, 0.23 for medical science, 0.13 for sociology, and 0.17 for economics. The stochastic paths in this paper are not far from their views.

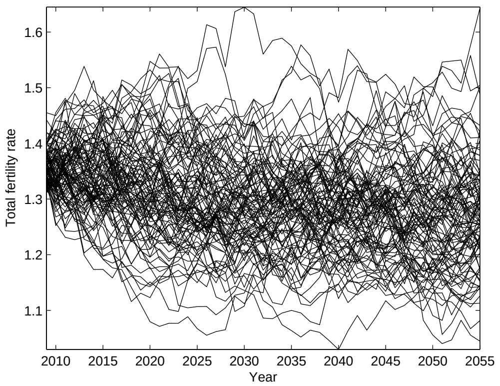

Figure 8: 100 sample paths of total fertility rates simulated from the assumed model.

## 3.3 Projecting the Population Structure

Given the mortality and fertility models, we are now ready to make a population projection. First, let us consider the aggregate population $P(t)$ in year $t$. In year $t > 2005$, the total population can be expressed by the following formula:

$$P(t) = P(2005) + F(t) - G(t), \tag{6}$$

where $F(t)$ and $G(t)$ represent the births and deaths from year 2005 to $t$, respectively. By generating sample paths of mortality and fertility rates from the stochastic models, we can obtain values of $F(t)$, $G(t)$ and hence $P(t)$. The population in the base year is obtained from the total population estimated from the national census conducted by Japan's Ministry of Internal Affairs and Communications (2005). Figure 9 displays the sex-specific population structure of Japan in 2005.

Note that, in the formula above, the effects of immigration and emigrations are not taken into account. This is because the net migration (as a percentage of the total population) in Japan is only 0.11%, which is significantly smaller than that in other developed countries, for example, 0.36% in the United States and 0.33% in the United Kingdom.[7] In addition, the net migration in Japan is extremely low compared to the births. According to the official projections, the births account for more than 95% in 2006 and 90% in 2025 in the growth of the total population. For ease of explanation, we do not consider the terms of migrations.

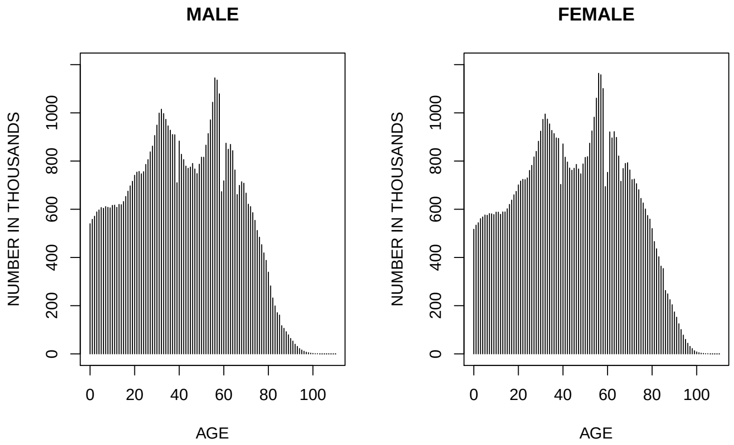

Figure 9: The sex-specific population structure of Japan in 2005.

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[7] A positive net migration means the immigrants outnumber the emigrants.

Next, we proceed to the projection of each age component. We let $C_x^{(m)}(t)$ and $C_x^{(f)}(t)$, $x = 0, 1, ..., 109$, be the number of males and females aged $x$ in year $t$, respectively. Then, ignoring migration, the corresponding values in year $t+1$ can be calculated easily by using the projected survival probabilities. For example, we have

$$C_{x+1}^{(m)}(t+1) = C_x^{(m)}(t) \times p_x^{(m)}(t),$$

where $p_x^{(m)}(t)$ is the probability that a male aged $x$ in year $t$ will survive to age $x+1$.

All that remains is to calculate the number of newborns. The calculation requires age-specific fertility rates. In Figure 10 we show the distribution of fertility rates among age groups for the past 9 years. We observe that the distribution has been fairly stationary, although there has been a minor shift to right, which is possibly because of the tendency of late marriage. In this study, we assume that the age-distribution of fertility rates in year 2005 will remain unchanged over time. In addition, the following two assumptions are made:

- the childbearing ages are from 15 to 49 inclusive;
- the ratio of male to female fertility rates is 1.054; this assumption is also used in the official population projection.

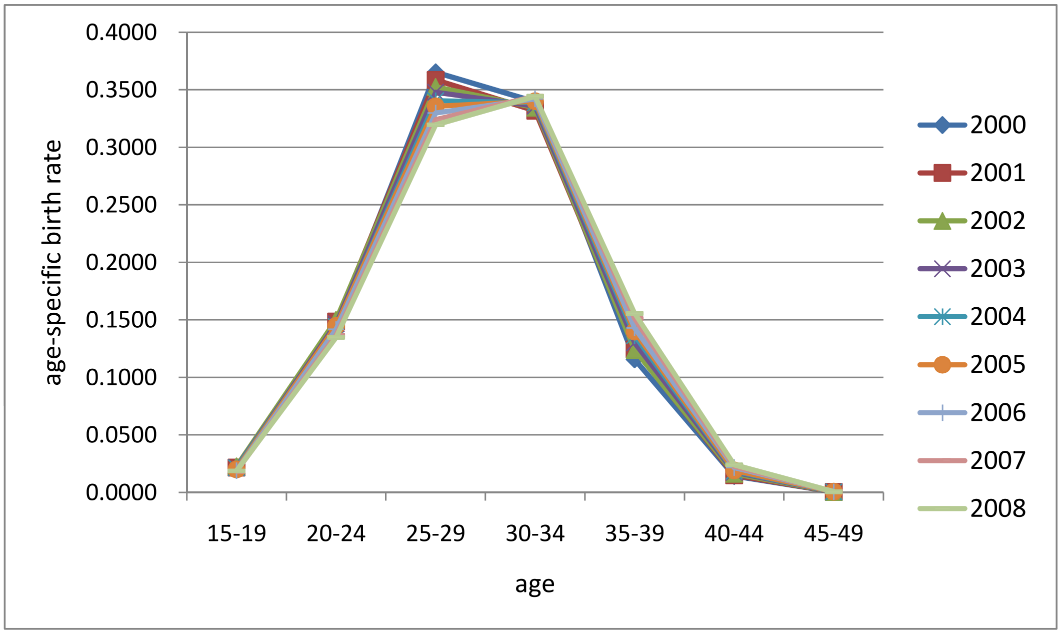

Figure 10: Distributions of fertility rates among age groups, 2000 to 2008.

Denote $f_x^{(m)}(t)$ and $f_x^{(f)}(t)$ as male and female fertility rates at age $x$ and in year $t$, respectively. Assuming that deaths are uniformly distributed within a calendar year, the number of newborns from, for example, women aged 30, can be calculated as follows:

$$C_{30}^{(f)}(t)\Big[\frac{1}{2}f_{30}^{(m)}(t)+\frac{1}{2}p_{30}^{(f)}(t)f_{30}^{(m)}(t)\Big]+C_{30}^{(f)}(t)\Big[\frac{1}{2}f_{30}^{(f)}(t)+\frac{1}{2}p_{30}^{(f)}(t)f_{30}^{(f)}(t)\Big], \qquad (7)$$

where the first term is the number of male newborns and the second term is the number of female newborns.

Finally, we project the population structure using a Leslie matrix, which is widely used in fields as diverse as botany, zoology and demography. Leslie matrices for males and females are defined as follows:

$$M^{(m)}(t)=\begin{pmatrix} 0 & 0 & \frac{p_0^{(m)}(t)}{2}\Big[0+\frac{p_{15}^{(f)}(t)f_{15}^{(m)}(t)}{2}\Big] & \frac{p_0^{(m)}(t)}{2}\Big[\frac{f_{15}^{(m)}(t)}{2}+\frac{p_{16}^{(f)}(t)f_{16}^{(m)}(t)}{2}\Big] & \cdots \\ p_0^{(m)}(t) & 0 & 0 & \cdots & \\ 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & p_{15}^{(m)}(t) & 0 & \cdots \\ 0 & 0 & 0 & p_{16}^{(m)}(t) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

$$M^{(f)}(t)=\begin{pmatrix} 0 & 0 & \frac{p_0^{(f)}(t)}{2}\Big[0+\frac{p_{15}^{(f)}(t)f_{15}^{(f)}(t)}{2}\Big] & \frac{p_0^{(f)}(t)}{2}\Big[\frac{f_{15}^{(f)}(t)}{2}+\frac{p_{16}^{(f)}(t)f_{16}^{(f)}(t)}{2}\Big] & \cdots \\ p_0^{(f)}(t) & 0 & 0 & \cdots & \\ 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & p_{15}^{(f)}(t) & 0 & \cdots \\ 0 & 0 & 0 & p_{16}^{(f)}(t) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Let $K^{(m)}(t)=(C_0^{(m)}(t),...,C_{109}^{(m)}(t))'$ and $K^{(f)}(t)=(C_0^{(f)}(t),...,C_{109}^{(f)}(t))'$. These two vectors contain information about the entire population structure in year $t$. On the basis of $K^{(m)}(2005)$ and $K^{(f)}(2005)$, we can obtain the vectors $K^{(m)}(t)$ and $K^{(f)}(t)$, for $t=2006,\ldots$, by the following equations:

$$\begin{aligned} K^{(m)}(t) &= M^{(m)}(t)\cdots M^{(m)}(2006)M^{(m)}(2005)K^{(m)}(2005), \\ K^{(f)}(t) &= M^{(f)}(t)\cdots M^{(f)}(2006)M^{(f)}(2005)K^{(f)}(2005). \end{aligned}$$

Given a projection of the population structure, we can estimate the old-age dependency ratio. By simulating sample paths of future mortality and fertility rates, we can further obtain a large number of simulated population structures, from which an empirical distribution of old-age dependency ratios can be derived.

Figure 11 displays the simulated distributions of old-age dependency ratios in 2015, 2025, 2035, 2045, and 2055. Also shown in the diagrams are the corresponding distributions based on the official estimates of mortality and fertility rates. Specifically, the solid lines indicate the ratios derived from the official medium variant projections, while the dotted lines indicate the ratios derived from the official high and low variant projections.

Our simulation results involve larger uncertainties than the official population projections'. This difference comes from mortality projections for the extreme elderly. Since the total fertility rates are simulated consistent with the official population projections', the effect of the total fertility rates is small. In addition, as the fan charts in Figure 4-1 and 4-2 shown, the uncertainties for age 70 and 80 are not as significant as those for age 90 and 100. In other words, the mortality projections for the extreme elderly really matter in the projections of old-age dependency ratios, suggesting that sustaining the public pension scheme may become more difficult if the mortality rates for the extreme elderly improve than we expected.

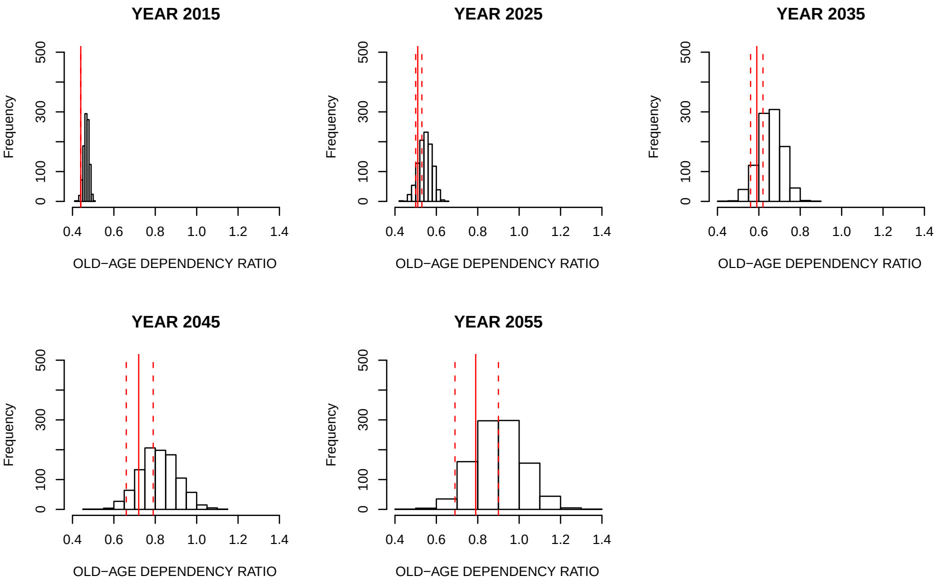

Figure 11: Simulated distributions of old-age dependency ratios in years 2015, 2025, 2035, 2045, and 2055.

# 4 Longevity Risk

## 4.1 Adjusted Indexations

Recall that the adjusted indexations under the automatic balancing mechanism consist of two components: one is the decline in mortality rates; the other is the decline in the number of the insured. The former is fixed at $-0.3\%$, but the latter is a random process depending on the demographics of the future population. Here we calculate the adjusted indexations on the basis of the simulated population structures.

Our primary objective is to examine the effects of demographic assumptions on the mechanism. As such, we need to keep all non-demographic assumptions deterministic. In particular, we assume that both the CPI growth rate and the wage growth rate are constant over time. To give readers an idea about how our results may change under different economic environments, we consider four deterministic scenarios: 1.0%, 1.5%, 2.0%, and 2.5% for both growth rates. Further, we assume that the number of contributing members in the public pension schemes is the same as the total population from age 20 to 60.

Some may argue that these scenarios are higher than recent circumstances in Japan. However, if we assume lower inflation rates, the results would be less interesting because the adjustments would not be triggered.[8] Moreover, the economic assumptions are consistent with those in the official reports of the public pension schemes. The government assumes the CPI and wage growth rate as 1.0% and 2.5% for the medium scenario, respectively. Thus, we use these four deterministic scenarios.

The mean and standard deviation of the deductible rates $d$ are plotted in Figure 12. The mean values over the period are all smaller (more negative) than $-0.3\%$, reflecting the future decrease in the number of contributing members. The deductible rates peak in around year 2035, when the children of the babyboomers reach age 60. The standard deviation of the deductible rates is zero before 2029 because it is assumed the mortality rates below age 60 are constant over time. Before the projected newborns reach age 20, there is no uncertainty associated with the population from age 20 to 60. Thereafter, the standard deviation continues to increase.

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[8] This is the point criticized by some authorities in Japan. Under the deflation era, the automatic balancing mechanism may not work well.

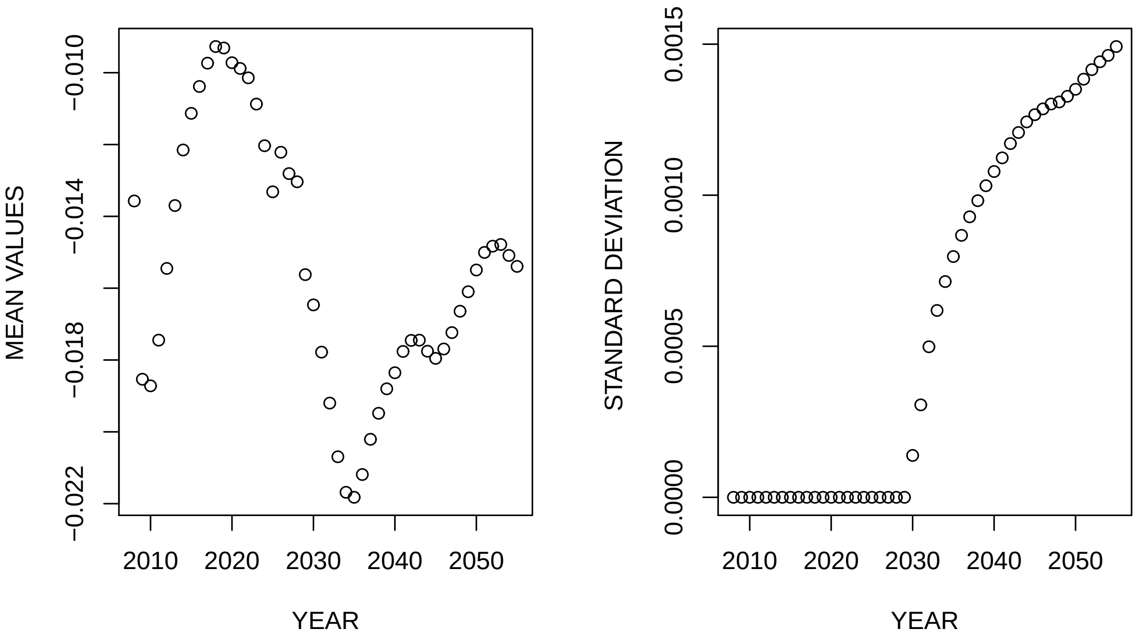

Figure 12: Mean and standard deviation of the simulated deductible rates.

## 4.2 Longevity Risk for the Extremely Aged

Japan has one of the most rapidly aging populations in the world. Most of the baby-boomers have already retired. At this moment, the living conditions for the majority of the elderly are not very hard. Figure 13 displays the distributions of the elderly by their self-assessed living conditions.[9] We observe that the weight of those who answer very hard or a little hard decreases as older. For example, for the elderly aged over 80, 96% of the elderly regard their living conditions as not very hard. 82% of the elderly regard their lives as common or very good. There are two main factors that attribute to the comfortable living conditions for the extreme elderly.

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[9] Source: Cabinet Office, Japan (2010).

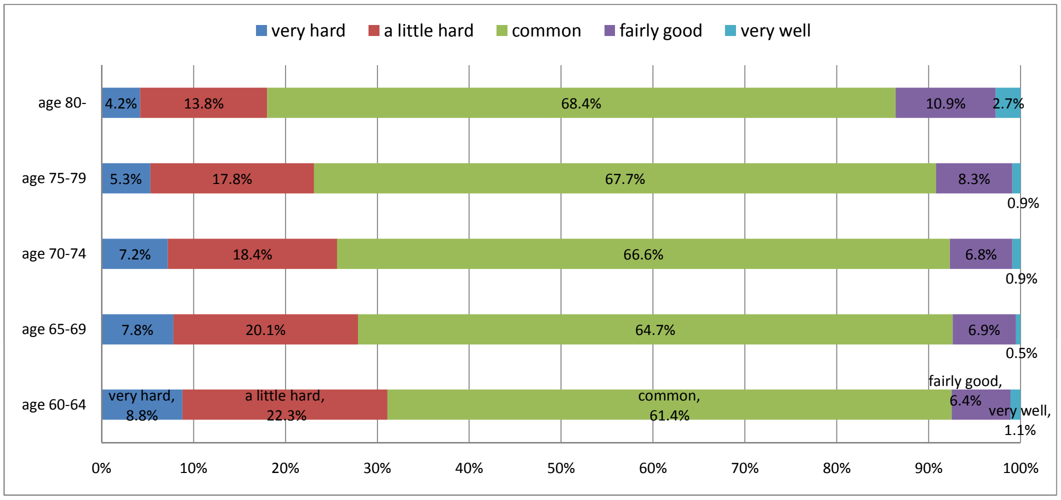

Figure 13: Distributions of the elderly by their self-assessed living conditions.

The first tier is the public pension. Figure 14 displays the average income per aged household (household consisting of one or more elderly over age 65) in 2008.[10] The diagram indicates that the major source of income is the public pension, which accounts for 71% of the average income. The second largest source of income is wage earnings. However, as one gets older, in particular for the extreme elderly, it may be more difficult to depend upon wage earnings due to health and related issues.

Note that the income from private pensions, grouped under "others" is extremely small. This is because most corporate sponsored pension plans in Japan allow retirees to select lump-sum benefits, and most retirees choose lump-sum benefits instead of monthly payments for life. Moreover, retirees in Japan tend to use their lump-sum benefits to clear their mortgage debt. Any fund that remains after mortgage repayment is usually put into a savings account because the elderly in Japan are risk-averse in general. Since the interest rate in Japan has been extremely low, funds in a savings account generate little return, and therefore cannot be depended on as a major source of post-retirement income.[11]

Moreover, companies are unwilling to take longevity risk. Defined benefit plans in Japan tend to offer annuities payable only for a fixed term, say 10, 15, or 20 years from age 60. Even if the elderly select annuities, they cannot receive defined benefits after age 80. Thus, for the extreme elderly, defined benefit pensions cannot be the main source of income: public pensions are definitely the main source of income.

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[10] Source: Cabinet Office, Japan (2010).

[11] Figure 14 does not include investment returns due to these specific conditions in Japan.

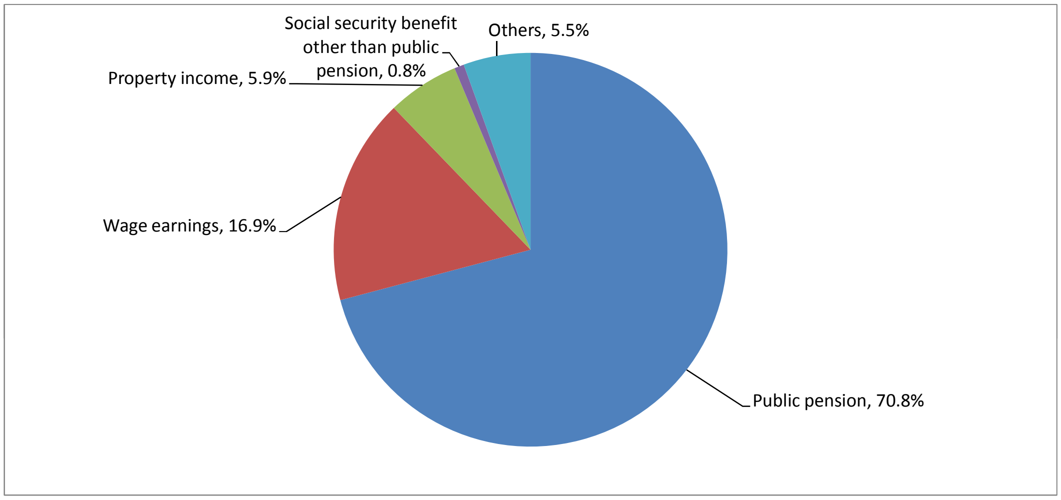

Figure 14: Distribution of sources of income for aged households in 2008.

In addition to the public pension, individual savings also play a vital role in the living conditions for the elderly. Figure 15 displays the savings for aged households in 2008.[12] We observe that nearly two-thirds of the elderly possess individual savings of more than 10,000,000 yen. Only 20.3% have less than 5,000,000 yen; these elderly are likely to live on the public pension or family supports only.

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[12] Source: Cabinet Office, Japan (2010).

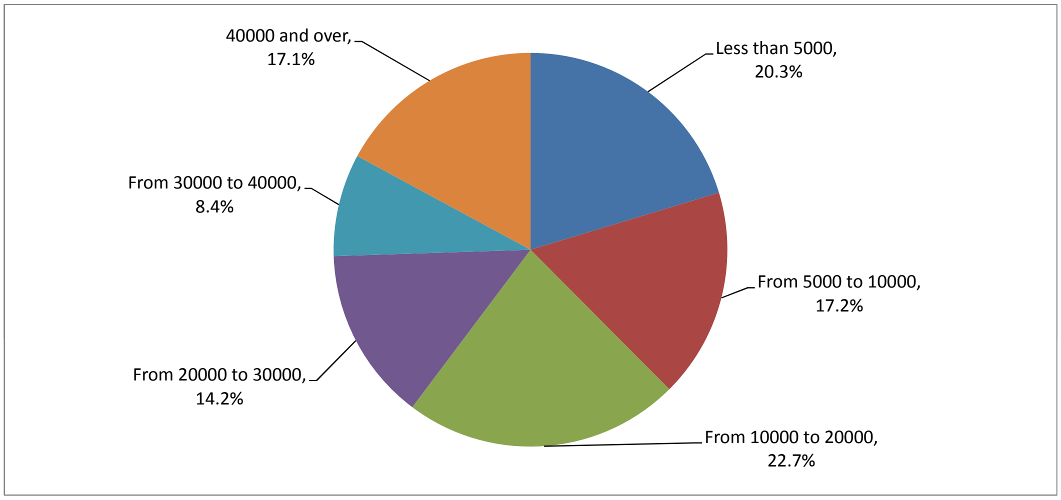

Figure 15: Savings for the aged households (in thousand yen).

Due to the automatic balancing mechanism, however, the future elderly may not be able to obtain the current level of public pension benefits. So, they may need to depend on other sources of income if they wish to achieve the same living standard as the current elderly's. It is therefore particularly important to understand the impact of longevity risk on future pension benefits through the automatic balancing mechanism.

We have seen that other sources of post-retirement income include corporate sponsored pensions, wage earnings, and family supports. However, as we mentioned, most corporate sponsored pensions are in the form of annuity-certain and are used to clear mortgage debts. It is also difficult, particularly for the elderly at advanced ages, to earn a salary. We expect the significance of family supports will gradually reduce, due to the increasing number of nuclear families and population aging. Therefore, it is most likely that the future elderly will use their savings to top up the reduction in public pension benefits. In this paper, we evaluate the potential reduction in public pensions relative to individual savings among the elderly.

Here we quantify the longevity risk associated with the automatic balancing mechanism. In particular, we estimate the central tendency and the extreme characteristics of the reduction in public pension benefits due to longevity risk, and then compare the reduction with the individual savings among the elderly. We focus on the longevity risk for two particular age groups, over age 80 and over age 100, because it is likely that these people will depend only on the public pension for their living income.

Let $R(x,t)$ and $Q(x,t)$ be the real price of pension benefits with and without the automatic balancing mechanism, at age $x$ and in year $t$, respectively. We set $R(x, 2005) = Q(x, 2005) = 2,400,000$ yen, close to the average amount of public pension benefits in 2005. With the automatic balancing mechanism, the real price of annual public pensions depends on future populations, so $R(x,t)$ may follow different paths depending on the realized values of mortality and fertility rates. In contrast, $Q(x,t)$ is fixed since we assume that both the CPI growth rate and the wage growth rate are constant over time.

We define the deficit function $L^{80}(m)$ in year $2005 + m$ as follows:

$$L^{80}(m) = \sum_{t=0}^{\infty}\{[Q(80+t, 2005+m) - R(80+t, 2005+m)]v^t \prod_{s=0}^{t} p_{80+s}(2005+m)\},$$

where $v = \frac{1}{1+i}$ is a discount factor and $p_x(t)$ is a one-year survival probability at age $x$ and in year $t$. We assume $i = 0.03$ in our calculations. The deficit function, $L^{80}(m)$, can be considered as the present value of the difference between the real price of benefits with and without the automatic balancing mechanism. The central characteristics of longevity risk are measured by the expectation of $L^{80}(m)$, while the extreme characteristics are measured by the longevity Value-at-Risk (VaR) at $100p\%$ level, defined as follows:

$$VaR_p[L^{80}(m)] = inf\{y \in R; Pr[L^{80}(m) > y] \leq 1-p\}.$$

The longevity VaR can be interpreted to mean that the probability of having a deficit greater than the longevity VaR is $p\%$. The deficit function $L^{100}(m)$ for the elderly over age 100 is similarly defined.

Figures 16-1, 16-2, 16-3, and 16-4 display the expected values of $L^{80}(m)$ in years 2015, 2020, 2025, and 2030, assuming that the CPI growth rate is 1.0%, 1.5%, 2.0%, and 2.5%, respectively.

From the diagrams we observe that the expected deficits for females are larger than those for males. This observation results from the fact that mortality rates for females are smaller than those for males. Another interesting observation is that the expected deficits for females are close to those for males who are 5 years older. For example, the expected longevity risk for females in 2025 is close to that for males in 2020.

These diagrams indicate that, for a given CPI growth rate, the expected deficits appear to increase linearly with time. The increase in the expected deficits is faster under a higher assumed CPI growth rate. So, the automatic balancing mechanism will have a greater impact on a high inflation economic environment, which future generations may encounter. This outcome can be explained as follows. As seen in Figure 12, the projected deductible rates are in between -0.8% and -2.2%. Thus, if the CPI growth rate is low, say 1.0%, then the impact by the automatic balancing mechanism is very limited because the adjusted indexation cannot go below zero. On the other hand, if the CPI growth rate is high, say 2.5 percent, the automatic balancing mechanism has a full impact on the real price of benefits.

Figures 17-1, 17-2, 17-3, and 17-4 display the longevity VaR for those who reach age 80 in years 2015, 2020, 2025, and 2030, assuming that the CPI growth rate is 1.0%, 1.5%, 2.0%, and 2.5%, respectively.

As in the diagrams for the expected deficits, we observe here a sex differential in the longevity VaR and an increase in the longevity VaR over time. We observe that the difference between Figures 16-1 and 17-1 is not very large, but Figure 16-4 and Figure 17-4 are highly different. We can also explain this outcome by the range (-0.8 - -2.0%) of the simulated deductible rates. In particular, if the CPI growth rate is only 1.0%, then most of the actual indexations will be zero, implying that there is little uncertainty. Thus, the tail risk is not very large in this case. On the other hand, if the CPI growth rate is 2.5%, the actual indexations involve huge uncertainty, which makes the longevity VaR a lot larger.

If we compare longevity risk with individual savings in Figure 15, we can quickly understand the percentage of the elderly who will be able to enjoy the current elderly's living standard in the future, at the expense of their savings. For example, for males who reach age 80 in 2030, the expected longevity risk is between 500,000 and 1,000,000 yen. Figure 15 implies that almost two-thirds of the elderly can achieve the living standard among the current elderly if they consume their individual savings. On the other hand, if we take into account the extreme change in the demographics using longevity VaR, the elderly may require more than 1,000,000 yen, according to Figure 17-4. In extreme demographic changes, the number of the elderly who can enjoy the current elderly's living standard may be limited.

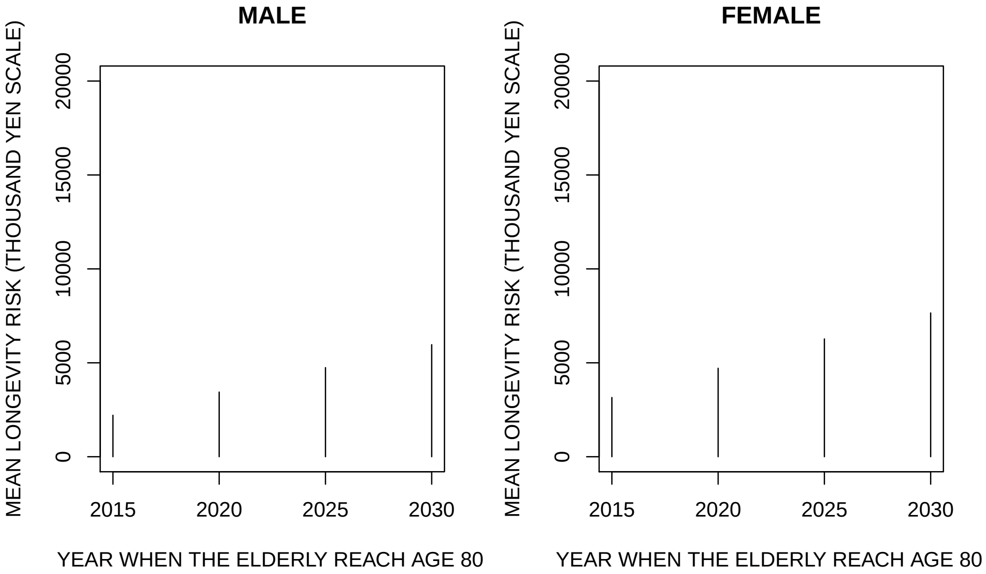

Figure 16-1: Expected deficits for those who reach age 80 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 1.0%.

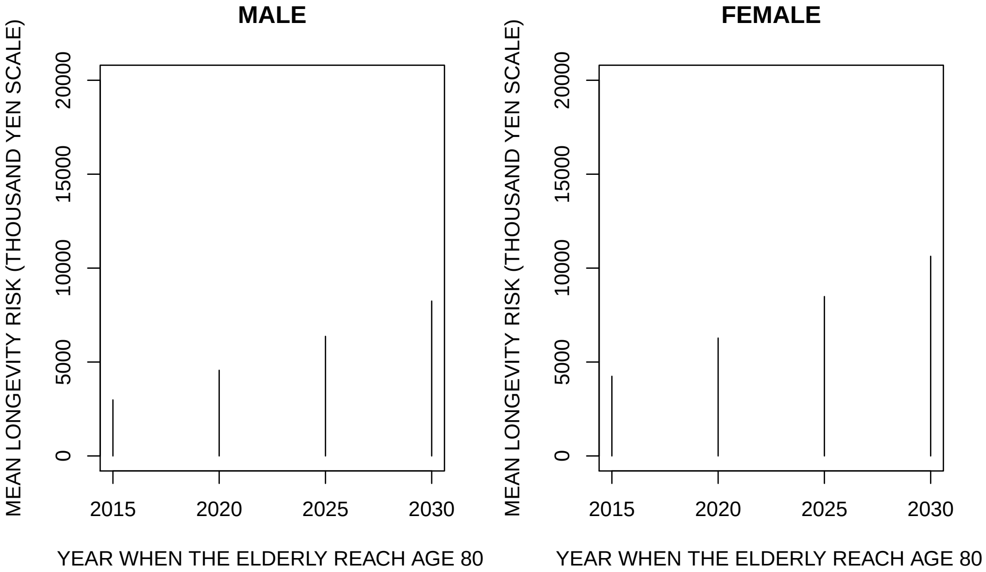

Figure 16-2: Expected deficits for those who reach age 80 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 1.5%.

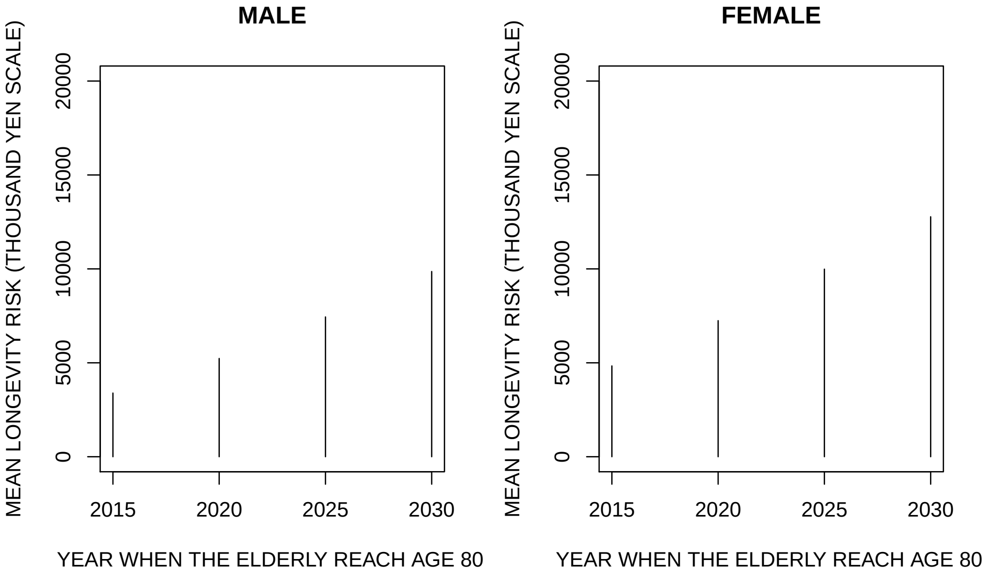

Figure 16-3: Expected deficits for those who reach age 80 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 2.0%.

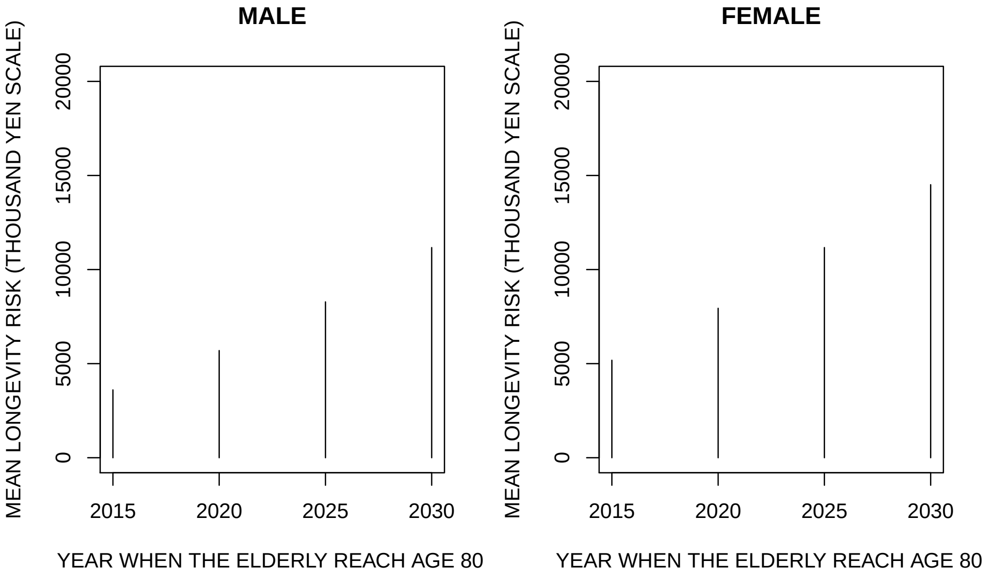

Figure 16-4: Expected deficits for those who reach age 80 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 2.5%.

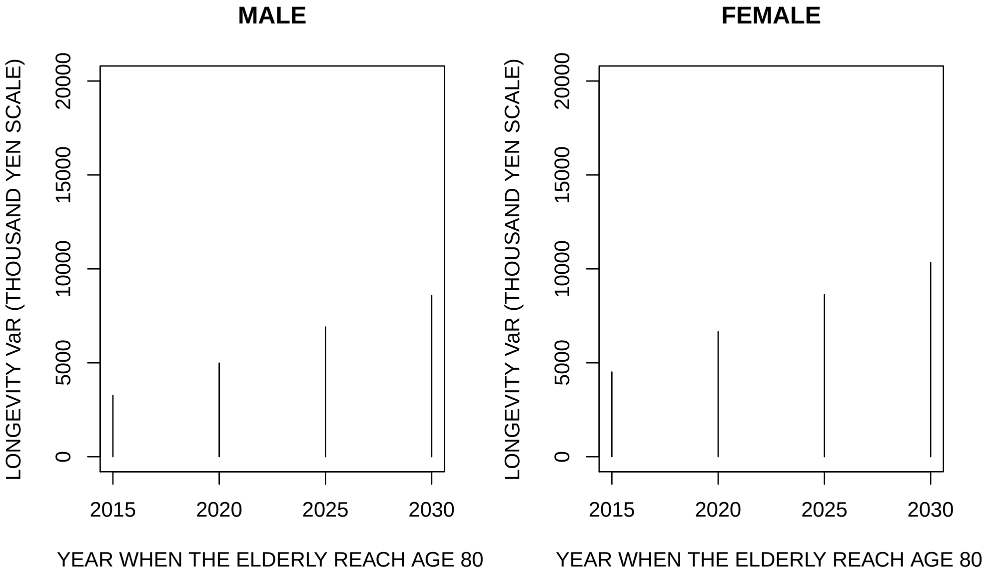

Figure 17-1: Longevity VaR for those who reach age 80 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 1.0%.

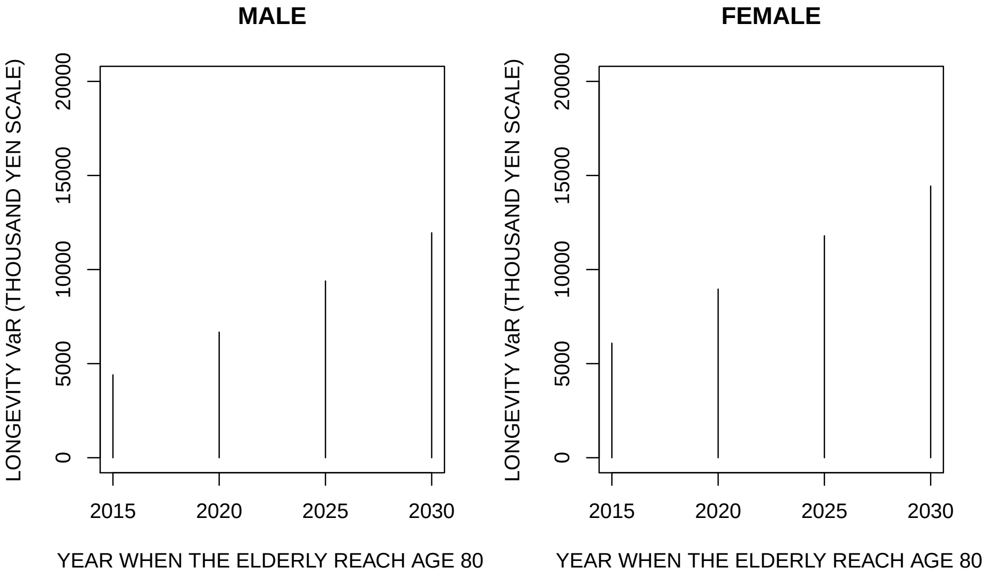

Figure 17-2: Longevity VaR for those who reach age 80 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 1.5%.

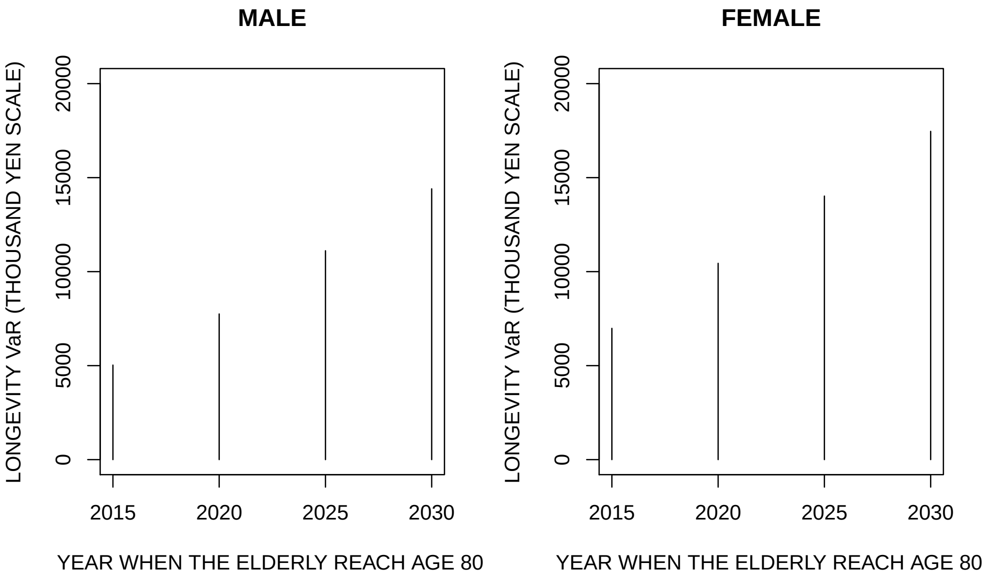

Figure 17-3: Longevity VaR for those who reach age 80 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 2.0%.

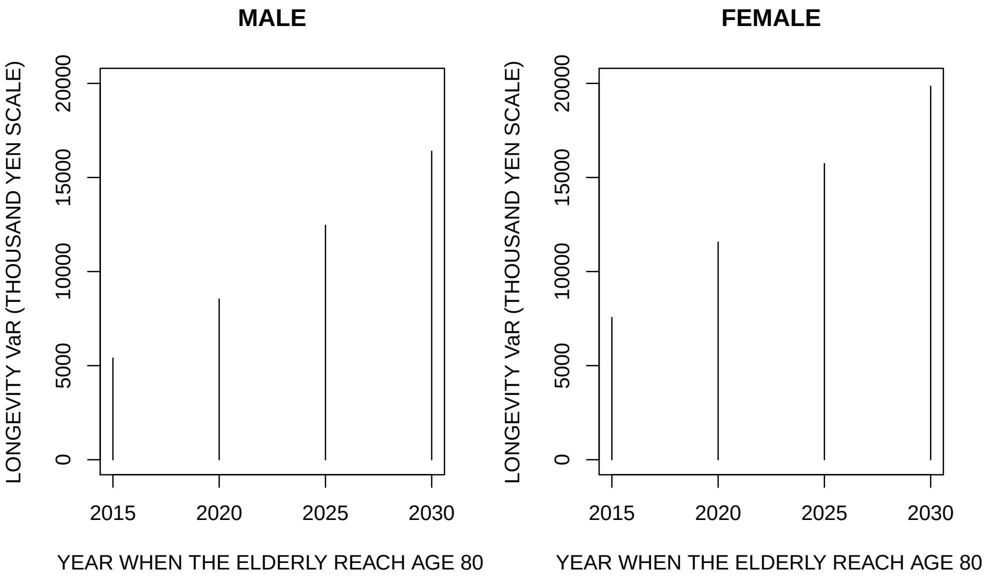

Figure 17-4: Longevity VaR for those who reach age 80 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 2.5%.

Similarly, we estimate the longevity risk for the elderly who reach age 100 in 2015, 2020, 2025, and 2030. Figures 18-1, 18-2, 18-3, and 18-4 display the expected longevity risk for males and females under the assumption that the CPI growth rates are 1.0%, 1.5%, 2.0%, and 2.5%, respectively. In addition, Figures 19-1, 19-2, 19-3, and 19-4 display the longevity VaR for males and females under the same four CPI assumptions.

The expected longevity risk and the longevity VaR for the elderly aged 100 are not as large as those for aged 80, mainly because of a shorter remaining lifetime during which public pensions are received. The values of longevity VaR are smaller than 5,000,000 yen. This amount is not very large, but at age 100, individual savings might have already depleted.

The government may not be able to afford to cover the longevity risk for the extreme elderly because of increasing financial deficits. Without the support from the government, the extreme elderly must prepare for the longevity risk under tier 3 – private pension plans. Specifically, tier 3 consists of defined benefit (DB) plans, defined contribution (DC) plans, and personal pensions available at insurance companies.

We believe that the second efficient tools to manage longevity risk, next to the public pension plan, is DB plans. Still, the same trend from DB plans to DC plans as the US and the UK exists in Japan. This is mainly due to increasing and unpredictable burdens of DB plans. If Japan leaves this trend as it is, the extreme elderly will have to prepare for the longevity risk by themselves. This is not an efficient way to manage longevity risk from an aspect of total economy in Japan.

One of the solutions is to delay the pensionable age of DB plans. Most of the DB plans provide pensions starting at age 60; however, age 60 may be too young to receive defined benefits. According to the ”Annual Report on the Aging Society:2010”, 73% of the male elderly aged 60 to 64 are employed, and 50.1% of the elderly aged 65 to 69 are employed. For females, 43.5% of the elderly aged 60 to 64 and 28.2% of the elderly aged 65 to 70 are employed. With wage earnings, they can survive with them and DB plans may concentrate on managing the longevity risk by delaying the pensionable age, say, to age 65 or 70.

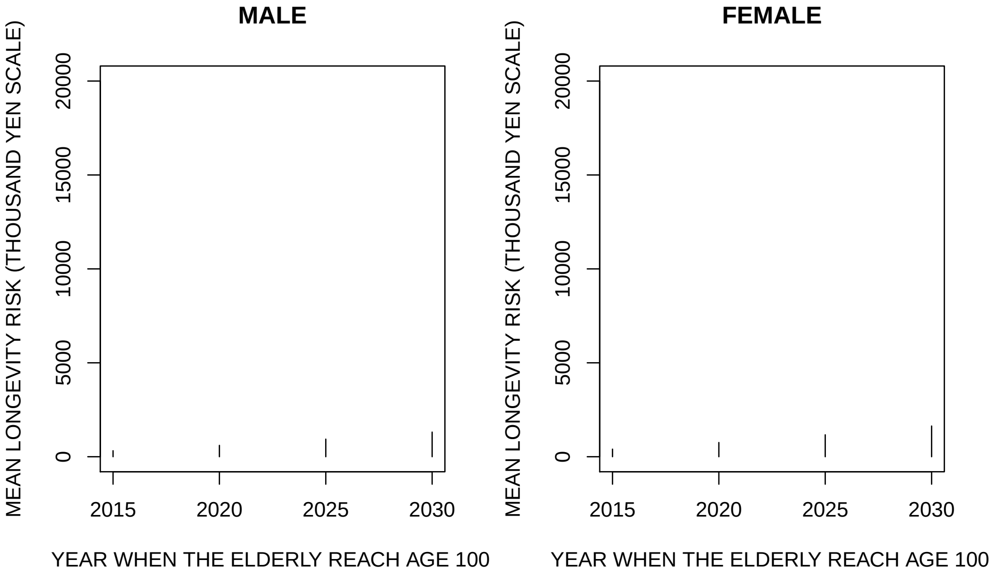

Figure 18-1: Expected deficits for those who reach age 100 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 1.0%.

Figure 18-2: Expected deficits for those who reach age 100 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 1.5%.

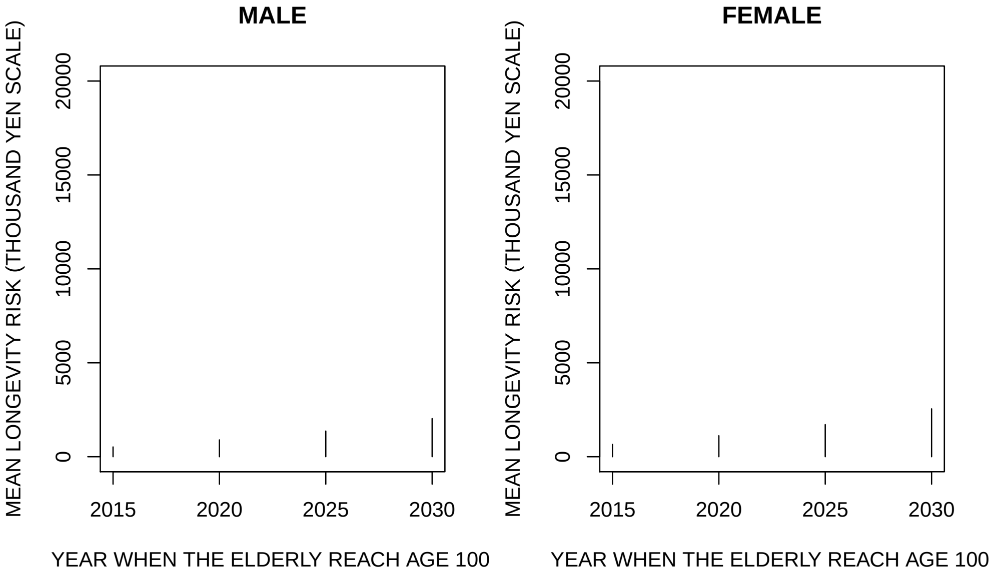

Figure 18-3: Expected deficits for those who reach age 100 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 2.0%.

Figure 18-4: Expected deficits for those who reach age 100 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 2.5%.

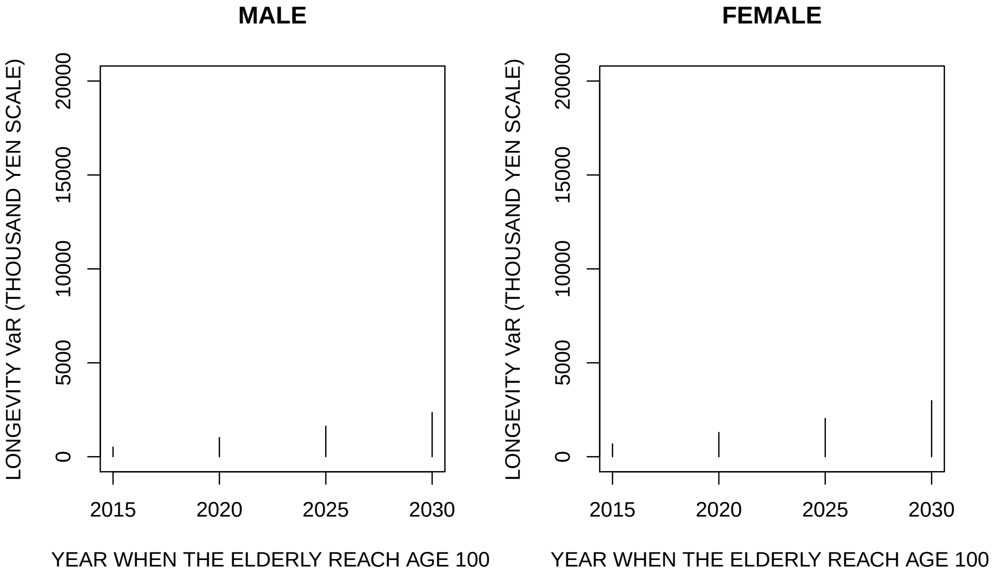

Figure 19-1: Longevity VaR for those who reach age 100 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 1.0%.

Figure 19-2: Longevity VaR for those who reach age 100 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 1.5%.

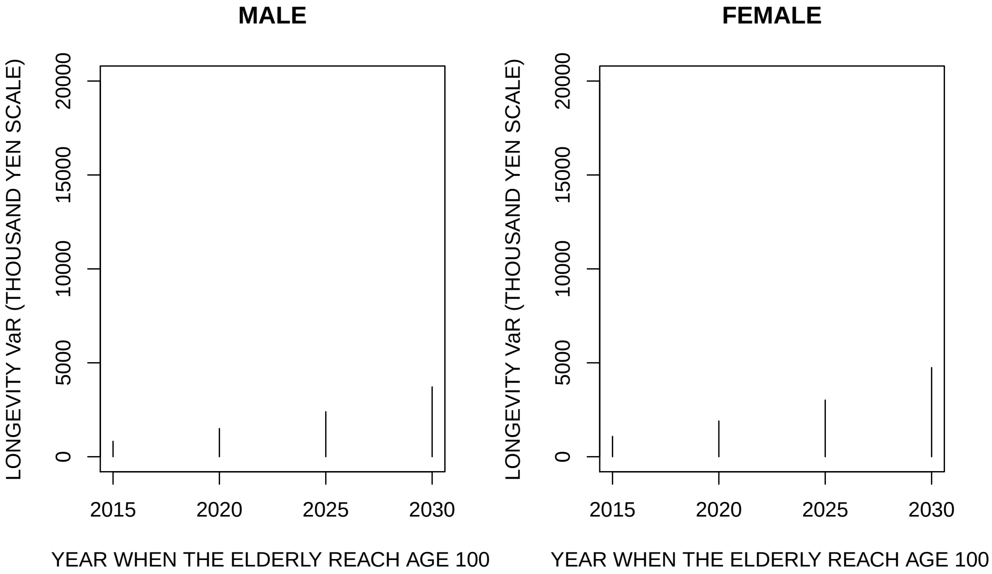

Figure 19-3: Longevity VaR for those who reach age 100 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 2.0%.

Figure 19-4: Longevity VaR for those who reach age 100 in 2015, 2020, 2025, and 2030, assuming the CPI growth rate is 2.5%.

# 5 Conclusion

In this paper, we propose two indicators, expected deficits and longevity VaR, to measure the longevity risk associated with the Japanese automatic balancing mechanism. These indicators can help Japanese people quantify their longevity risk exposure and indicate how much money they should save for a reasonable post-retirement living. To calculate expected deficits and longevity VaR, we introduce two stochastic models, one for mortality and the other for fertility, from which a distribution of the adjusted indices associated with the automatic balancing mechanism can be obtained.

In this paper, demographic factors are assumed to be stochastic while economic factors are assumed to be deterministic. However, stochastic analysis of both factors would provide useful results as well. In such analysis, we should consider the interactions between demographic and economic factors. For example, the consumption behaviour of the elderly will have a significant impact on CPI in the aging society like Japan. This is because the risk that reduces future benefits from the public pension system may affect the elderly consumption. Stochastic analysis with more sophisticated models is the future research of this paper.

This paper focuses on the longevity risk for each individual, but for the government, examining the aggregate longevity risk is also useful. It is important to note that this risk is non-diversifiable because it applies to all the elderly in the public pension schemes. In extreme demographic situations, that is, when both mortality and fertility rates turn out to be very low, the government may be forced by political pressure to provide additional benefits in order to compensate for the significant decline in public pensions arising from the automatic balancing mechanism. It may be difficult (and too late) to collect additional contributions from the contributing members at that time. The methods we propose in this paper can be applied to the estimation of the aggregate longevity risk associated with the public pension, helping the government to better plan for the future.

Many Japanese are worried about their future standard of living due to the rapidly aging population. Although the financial report on the public pension system states that the public pension can be sustained over 100 years, some are doubtful about the underlying assumptions and thus the financial health of the system. The stochastic analysis provided in this paper can partially answer the questions regarding post-retirement living standards in the future. However, we believe that significant further research is needed to solve other problems associated with the public pension schemes in Japan.

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