

# Decision Making Under Uncertain and Risky Situations

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## **Abstract**

Decision making is certainly the most important task of a manager and it is often a very difficult one. The domain of decision analysis models falls between two extreme cases. This depends upon the degree of knowledge we have about the outcome of our actions. One “pole” on this scale is deterministic. The opposite “pole” is pure uncertainty. Between these two extremes are problems under risk. The main idea here is that for any given problem, the degree of certainty varies among managers depending upon how much knowledge each one has about the same problem. This reflects the recommendation of a different solution by each person. Probability is an instrument used to measure the likelihood of occurrence for an event. When probability is used to express uncertainty, the deterministic side has a probability of one (or zero), while the other end has a flat (all equally probable) probability. This paper offers a decision making procedure for solving complex problems step by step. It presents the decision analysis process for both public and private decision making, using different decision criteria, different types of information and information of varying quality. It describes the elements in the analysis of decision alternatives and choices, as well as the goals and objectives that guide decision making. The key issues related to a decision-maker's preferences regarding alternatives, criteria for choice and choice modes, together with the risk assessment tools, are also presented.

### **Keywords:**

Decision Making under Risk, Risk Management, Decision Making Technique, Bayesian Approach, Risk Measuring Tool.

## 1. Introduction

Modeling for decision making involves two distinct parties—one is the decision maker and the other is the model builder known as the analyst. The analyst is to assist the decision maker in his/her decision making process. Therefore, the analyst must be equipped with more than a set of analytical methods. Specialists in model building are often tempted to study a problem, and then go off in isolation to develop an elaborate mathematical model for use by the manager (i.e., the decision maker). Unfortunately the manager may not understand this model and may either use it blindly or reject it entirely. [1] The specialist may feel that the manager is too ignorant and unsophisticated to appreciate the model, while the manager may feel that the specialist lives in a dream world of unrealistic assumptions and irrelevant mathematical language. Such miscommunication can be avoided if the manager works with the specialist to develop first a simple model that provides a crude but understandable analysis. After the manager has built up confidence in this model, additional detail and sophistication can be added, perhaps progressively only a bit at a time. This process requires an investment of time on the part of the manager and sincere interest on the part of the specialist in solving the manager's real problem, rather than in creating and trying to explain sophisticated models. This progressive model building is often referred to as the bootstrapping approach and is the most important factor in determining successful implementation of a decision model. Moreover the bootstrapping approach simplifies the otherwise difficult task of model validating and verification processes. [2]

In deterministic models, a good decision is judged by the outcome alone. However, in probabilistic models, the decision maker is concerned not only with the outcome value but also with the amount of risk each decision carries. As an example of deterministic versus probabilistic models, consider the past and the future. Nothing we can do can change the past, but everything we do influences and changes the future, although the future has an element of uncertainty. Managers are captivated much more by shaping the future than the history of the past. [3]

Uncertainty is the fact of life and business. Probability is the guide for a “good” life and successful business. The concept of probability occupies an important place in the decision making process, whether the problem is one faced in business, in government, in the social sciences, or just in one's own everyday personal life. In very few decision making situations is perfect information—all the needed facts—available. Most decisions are made in the face of uncertainty. Probability enters into the process by playing the role of a substitute for certainty—a substitute for complete knowledge [4].

Probabilistic modeling is largely based on application of statistics for probability assessment of uncontrollable events (or factors), as well as risk assessment of your decision. The original idea of statistics was the collection of information about and for the state. The word statistics is not derived from any classical Greek or Latin roots, but from the Italian word for state. Probability has a much longer history. Probability is derived from the verb to probe meaning to “find out” what is not too easily accessible or understandable. The word “proof” has the same origin that provides necessary details to understand what is claimed to be true. Probabilistic models are viewed as similar to that of a game; actions are based on expected outcomes. The center of interest moves from the deterministic to probabilistic models using subjective statistical techniques for estimation, testing and predictions. In probabilistic modeling, risk means uncertainty for which the probability distribution is

known. Therefore risk assessment means a study to determine the outcomes of decisions along with their probabilities [4].

Decision makers often face a severe lack of information. Probability assessment quantifies the information gap between what is known, and what needs to be known for an optimal decision. The probabilistic models are used for protection against adverse uncertainty, and exploitation of propitious uncertainty. Difficulty in probability assessment arises from information that is scarce, vague, inconsistent or incomplete. A statement such as “the probability of a power outage is between 0.3 and 0.4” is more natural and realistic than its “exact” counterpart, such as “the probability of a power outage is 0.36342” [5].

It is a challenging task to compare several courses of action and then select one action to be implemented. At times, the task may prove too challenging. Difficulties in decision making arise through complexities in decision alternatives. The limited information-processing capacity of a decision-maker can be strained when considering the consequences of only one course of action. Yet, choice requires that the implications of various courses of action be visualized and compared. In addition, unknown factors always intrude upon the problem situation and seldom are outcomes known with certainty. Almost always, an outcome depends upon the reactions of other people who may be undecided themselves. It is no wonder that decision makers sometimes postpone choices for as long as possible. Then, when they finally decide, they neglect to consider all the implications of their decision [6, 7].

Business decision making is almost always accompanied by conditions of uncertainty. Clearly, the more information the decision maker has, the better the decision will be. Treating decisions as if they were gambles is the basis of decision theory. This means that we have to trade off the value of a certain outcome against its probability. To operate according to the canons of decision theory, we must compute the value of a certain outcome and its probabilities; hence, determining the consequences of our choices. The origin of decision theory is derived from economics by using the utility function of payoffs. It suggests that decisions be made by computing the utility and probability, the ranges of options, and also lays down strategies for good decisions [3].

Objectives are important both in identifying problems and in evaluating alternative solutions. Evaluating alternatives requires that a decision maker’s objectives be expressed as criteria that reflect the attributes of the alternatives relevant to the choice. The systematic study of decision making provides a framework for choosing courses of action in a complex, uncertain or conflicting situation. The choices of possible actions, and the prediction of expected outcomes, derive from a logical analysis of the decision situation. A possible drawback in the decision analysis approach: You might have already noticed that the above criteria always result in selection of only one course of action. However, in many decision problems, the decision maker might wish to consider a combination of some actions. For example, in the investment problem, the investor might wish to distribute the assets among a mixture of the choices in such a way to optimize the portfolio's return [2-4].

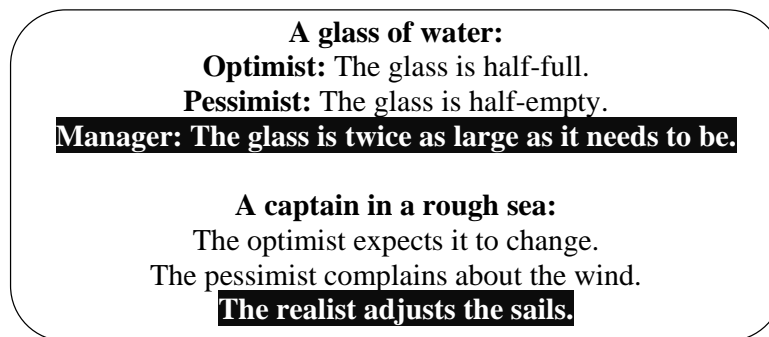
Relevant information and knowledge used to solve a decision problem sharpens our flat probability. Useful information moves the location of a problem from the pure uncertain “pole” towards the deterministic “pole.” Probability assessment is nothing more than the quantification of uncertainty. In other words, quantification of uncertainty allows for the communication of uncertainty between persons. There can be uncertainties regarding events, states of the world, beliefs and so on. Probability is the tool for both communicating

uncertainty and managing it. There are different types of decision models that help to analyze the different scenarios. Depending on the amount and degree of knowledge we have, the three most widely used types are:

- Decision making under pure uncertainty
- Decision making under risk
- Decision making by buying information (pushing the problem towards the deterministic “pole”)

In decision making under pure uncertainty, the decision maker has absolutely no knowledge, not even about the likelihood of occurrence for any state of nature. In such situations, the decision maker's behavior is purely based on his/her attitude toward the unknown [13]. Some of these behaviors are optimistic, pessimistic and least regret, among others. Consider three following known ideas about a glass of water and a captain in a rough sea:

**Figure 1**  
**Known Ideas about a Glass of Water and a Captain in a Rough Sea**



Optimists are right; so are the pessimists. It is up to you to choose which you will be [8, 9]. The optimist sees opportunity in every problem; the pessimist sees problem in every opportunity. Both optimists and pessimists contribute to our society. The optimist invents the airplane and the pessimist the parachute. Whenever the decision maker has some knowledge regarding the states of nature, he/she may be able to assign subjective probability for the occurrence of each state of nature. By doing so, the problem is then classified as decision making under risk. In many cases, the decision maker may need an expert's judgment to sharpen his/her uncertainties with respect to the likelihood of each state of nature. In such a case, the decision maker may buy the expert's relevant knowledge in order to make a better decision [10, 14]. The procedure used to incorporate the expert's advice with the decision maker's probabilities assessment is known as the Bayesian approach.

This paper presents the decision analysis process both for public and private decision making under different decision criteria, type and quality of available information. Basic elements in the analysis of decision alternatives and choice are described as well as the goals and objectives that guide decision making. In the subsequent sections, we will examine key issues related to a decision maker's preferences regarding alternatives, criteria for choice and choice modes.

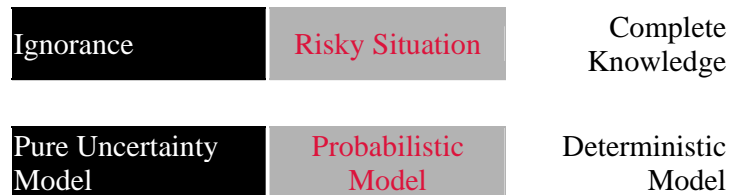
## 2. Decision Making Under Pure Uncertainty

In decision making under pure uncertainty, the decision maker has no knowledge regarding any of the states of nature outcomes, and/or it is costly to obtain the needed information. In such cases, the decision making depends merely on the decision maker's personality type [11].

### 2.1 Continuum of Pure Uncertainty and Certainty

The domain of decision analysis models falls between two extreme cases. This depends upon the degree of knowledge we have about the outcome of our actions as shown below [12]:

**Figure 2**  
**Uncertainty and Certainty Domain**



One “pole” on this scale is deterministic. The opposite “pole” is pure uncertainty. Between these two extremes are problems under risk. The main idea here is that for any given problem, the degree of certainty varies among managers depending upon how much knowledge each one has about the same problem. This reflects the recommendation of a different solution by each person. Probability is an instrument used to measure the likelihood of occurrence for an event. When you use probability to express your uncertainty, the deterministic side has a probability of one (or zero), while the other end has a flat (all equally probable) probability. For example, if you are certain of the occurrence (or non-occurrence) of an event, you use the probability of one (or zero). If you are uncertain, and would use the expression “I really don't know,” the event may or may not occur with a probability of 50 percent. This is the Bayesian notion that probability assessment is always subjective. That is, the probability always depends upon how much the decision maker knows. If someone knows all there is to know, then the probability will diverge either to one or zero. The decision situations with flat uncertainty have the largest risk. For simplicity, consider a case where there are only two outcomes, with one having a probability of  $p$ . Thus, the variation in the states of nature is  $p \times (1-p)$ . The largest variation occurs if we set  $p = 50\%$ , given each outcome an equal chance. In such a case, the quality of information is at its lowest level. Due to statistics science the quality of information and variation are inversely related. That is, larger variation in data implies lower quality data (i.e., information). In this tutorial several techniques for decision making under risky, deterministic and uncertain situation are presented. These techniques will enable managers to challenge with nondeterministic outcomes of nature [15-18].

## 2.2 Source of Errors in Decision Making

The main sources of errors in risky decision making problems are: false assumptions, not having an accurate estimation of the probabilities, relying on expectations, difficulties in measuring the utility function, and forecast errors. Consider the following stereotype investment decision making example. In order to shorten the description we present the example in the form of Table 1.

**TABLE 1**  
**The Investment Decision Making Example**

|         |         | States of Nature |          |           |     |
|---------|---------|------------------|----------|-----------|-----|
|         |         | Growth           | Medium G | No Change | Low |
|         |         | G                | MG       | NC        | L   |
| Actions | Bonds   | 12%              | 8        | 7         | 3   |
|         | Stocks  | 15               | 9        | 5         | -2  |
|         | Deposit | 7                | 7        | 7         | 7   |

The states of nature are the states of economy during one year [19]. The problem is to decide what action to take among three possible courses of action with the given rates of return as shown in the body of the table [20].

## 2.3 Personality Types and Decision Making:

**2.3.1 Pessimism, or Conservative (MaxMin).** Worse case scenario. Bad things always happen to me.

**TABLE 2**  
**MaxMin Course of Action**

|                                     |   |    |   |
|-------------------------------------|---|----|---|
|                                     | B | 3  |   |
| a) Write min # in each action row,  | S | -2 |   |
| b) Choose max # and do that action. | D | 7  | * |

**2.3.2 Optimism, or Aggressive (MaxMax).** Good things always happen to me.

**TABLE 3**  
**MaxMax Course of Action**

|                                     |   |    |   |
|-------------------------------------|---|----|---|
|                                     | B | 12 |   |
| a) Write max # in each action row,  | S | 15 | * |
| b) Choose max # and do that action. | D | 7  |   |

**2.3.3 Coefficient of Optimism (Hurwicz's Index).** Middle of the road: I am neither too optimistic nor too pessimistic.

- a) Choose  $\alpha$  between zero and one. One means optimistic and zero means pessimistic,
- b) Choose largest and smallest # for each action,
- c) Multiply largest payoff (row-wise) by  $\alpha$  and the smallest by  $(1-\alpha)$ ,
- d) Pick action with largest sum.

For example, for  $\alpha = 0.7$ , we have:

**TABLE 4**  
**Coefficient of Optimism Course of Action**

|          |                             |
|----------|-----------------------------|
| <b>B</b> | $(.7*12) + (.3*3) = 9.3$    |
| <b>S</b> | $(.7*15) + (.3*-2) = 9.9 *$ |
| <b>D</b> | $(.7*7) + (.3*7) = 7$       |

This method is a linear combination of all nature outcomes and can easily be extended for  $n$  outcomes and  $k$  nature states with their associate  $i=1,2,3,\dots,k$  values [13, 21, 22].

**2.3.4 Minimize Regret: (Savag's Opportunity Loss)**

**Some managers think as follows:** I hate regrets and therefore I have to minimize my regrets. My decision should be made so that it is worth repeating. I should only do those things that I feel I could happily repeat. This reduces the chance that the outcome will make me feel regretful, or disappointed, or that it will be an unpleasant surprise. Regret is the payoff on what would have been the best decision in the circumstances minus the payoff for the actual decision in the circumstances. Therefore, the first step is to set up the regret table:

- a) Take the largest number in each state of nature column (say, L).
- b) Subtract all the numbers in that state of nature column from it (i.e.,  $L - X_{i,j}$ ).
- c) Choose maximum number of each action.
- d) Choose minimum number from step (d) and take that action.

**TABLE 5**  
**The Regret Matrix**

| The Regret Matrix |         |       |       |       |     |
|-------------------|---------|-------|-------|-------|-----|
|                   | G       | MG    | NC    | L     |     |
| Bonds             | (15-12) | (9-8) | (7-7) | (7-3) | 4 * |
| Stocks            | (15-15) | (9-9) | (7-5) | (7+2) | 9   |
| Deposit           | (15-7)  | (9-7) | (7-7) | (7-7) | 8   |



## **2.4 Limitations of Decision Making Under Pure Uncertainty**

Decision analysis in general assumes that the decision maker faces a decision problem where he or she must choose at least and at most one option from a set of options. In some cases this limitation can be overcome by formulating the decision making under uncertainty as a zero-sum two-person game. In decision making under pure uncertainty, the decision-maker has no knowledge regarding which state of nature is “most likely” to happen. He or she is probabilistically ignorant concerning the state of nature; therefore he or she cannot be optimistic or pessimistic. In such a case, the decision maker invokes consideration of security. Notice that any technique used in decision making under pure uncertainties is appropriate only for the private life decisions. Moreover, the public person (i.e., you, the manager) has to have some knowledge of the state of nature in order to predict the probabilities of the various states of nature. Otherwise, the decision maker is not capable of making a reasonable and defensible decision in this case [\[23-26\]](#).

### 3. Decision Making Under Risk

Risk implies a degree of uncertainty and an inability to fully control the outcomes or consequences of such an action. Risk or the elimination of risk is an effort that managers employ. However, in some instances the elimination of one risk may increase some other risks. Effective handling of a risk requires its assessment and its subsequent impact on the decision process. The decision process allows the decision-maker to evaluate alternative strategies prior to making any decision. The process is as follows:

- 1) The problem is defined and all feasible alternatives are considered. The possible outcomes for each alternative are evaluated.
- 2) Outcomes are discussed based on their monetary payoffs or net gain in reference to assets or time.
- 3) Various uncertainties are quantified in terms of probabilities.
- 4) The quality of the optimal strategy depends upon the quality of the judgments. The decision maker should identify and examine the sensitivity of the optimal strategy with respect to the crucial factors.

Whenever the decision maker has some knowledge regarding the states of nature, he/she may be able to assign subjective probability estimates for the occurrence of each state. In such cases, the problem is classified as decision making under risk [27]. The decision maker is able to assign probabilities based on the occurrence of the states of nature. The decision making under risk process is as follows:

- a) Use the information you have to assign your beliefs (called subjective probabilities) regarding each state of the nature,  $p(s)$ ,
- b) Each action has a payoff associated with each of the states of nature  $X(a,s)$ ,
- c) Compute the expected payoff, also called the return ( $R$ ), for each action

$$R(a) = \sum_{i=1}^n X(a_i, s_i) \cdot p(s_i)$$

- d) We accept the principle that we should minimize (or maximize) the expected payoff,
- e) Execute the action which minimizes (or maximizes)  $R(a)$ .

#### 3.1 Expected Payoff

The actual outcome will not equal the expected value. What you get is not what you expect, i.e. the “Great Expectations!”

- a) For each action, multiply the probability and payoff and then,
- b) Add up the results by row,
- c) Choose largest number and take that action.

**TABLE 6**  
**The Expected Payoff Matrix**

|          | <u>G (0.4)</u> | <u>MG (0.3)</u> | <u>NC (0.2)</u> | <u>L (0.1)</u> | Exp. Value |
|----------|----------------|-----------------|-----------------|----------------|------------|
| <b>B</b> | 0.4(12) +      | 0.3(8) +        | 0.2(7) +        | 0.1(3) =       | 8.9        |
| <b>S</b> | 0.4(15) +      | 0.3(9) +        | 0.2(5) +        | 0.1(-2) =      | 9.5*       |
| <b>D</b> | 0.4(7) +       | 0.3(7) +        | 0.2(7) +        | 0.1(7) =       | 7          |

### 3.2 The Most Probable States of Nature

This method is a simple way for decision making under risk but it is good for non-repetitive decisions. The steps of this method are as follows:

- Take the state of nature with the highest probability (subjectively break any ties),
- In that column, choose action with greatest payoff.

In our numerical example, there is a 40 percent chance of growth so we must buy stocks with payoff 15 and expected payoff 0.6.

### 3.3 Expected Opportunity Loss (EOL)

The steps of this method are as follows:

- Set up a loss payoff matrix by taking largest number in each state of nature column (say L), and subtract all numbers in that column from it,  $L - X_{ij}$ ,
- For each action, multiply the probability and loss then add up for each action,
- Choose the action with smallest EOL.

**TABLE 7**  
**The Expected Opportunity Loss Matrix**

| Loss Payoff Matrix |                |                |                |                |            |
|--------------------|----------------|----------------|----------------|----------------|------------|
|                    | <u>G (0.4)</u> | <u>MG(0.3)</u> | <u>NC(0.2)</u> | <u>L (0.1)</u> | <u>EOL</u> |
| <b>B</b>           | 0.4(15-12) +   | 0.3(9-8) +     | 0.2(7-7) +     | 0.1(7-3)       | 1.9        |
| <b>S</b>           | 0.4(15-15) +   | 0.3(9-9) +     | 0.2(7-5) +     | 0.1(7+2)       | 1.3*       |
| <b>D</b>           | 0.4(15-7) +    | 0.3(9-7) +     | 0.2(7-7) +     | 0.1(7-7)       | 3.8        |

Note that the result is coincidentally the same as Expected Payoff and Most Probable States of Nature.

### 3.4 Computation of the Expected Value of Perfect Information (EVPI)

EVPI helps to determine the worth of an insider who possesses perfect information. Recall that EVPI is equal to EOL.

- Take the maximum payoff for each state of nature,
- Multiply each case by the probability for that state of nature and then add them up,
- Subtract the expected payoff from the number obtained as Expected Payoff.

**TABLE 8**  
**EVPI Computation Matrix**

|    |           |       |
|----|-----------|-------|
| G  | 15(0.4) = | 6.0   |
| MG | 9(0.3) =  | 2.7   |
| NC | 7(0.2) =  | 1.4   |
| L  | 7(0.1) =  | 0.7   |
|    | +         | ----- |
|    |           | 10.8  |

Therefore,  $EVPI = 10.8 - \text{Expected Payoff} = 10.8 - 9.5 = 1.3$ . Verify that  $EOL = EVPI$ . The efficiency of the perfect information is defined as  $100 [EVPI / (\text{Expected Payoff})]\%$ . Therefore, if the information costs more than 1.3 percent of investment, don't buy it. For example, if you are going to invest \$100,000, the maximum you should pay for the information is  $[100,000 * (1.3\%)] = \$1,300$ .

### 3.5 We Know Nothing (the Laplace Equal Likelihood Principle)

Every state of nature has an equal likelihood. Since we don't know anything about the nature, every state of nature is equally likely to occur:

- For each state of nature, use an equal probability (i.e., a Flat Probability),
- Multiply each number by the probability,
- Add action rows and put the sum in the Expected Payoff column,
- Choose largest number in step (c) and perform that action.

**TABLE 9**  
**Laplace Equal Likelihood Principle Matrix**

|         | G        | MG      | NC      | L        | Exp. Payoff |
|---------|----------|---------|---------|----------|-------------|
| Bonds   | 0.25(12) | 0.25(8) | 0.25(7) | 0.25(3)  | 7.5 *       |
| Stocks  | 0.25(15) | 0.25(9) | 0.25(5) | 0.25(-2) | 6.75        |
| Deposit | 0.25(7)  | 0.25(7) | 0.25(7) | 0.25(7)  | 7           |

### **3.6 A Discussion on Expected Opportunity Loss (Expected Regret)**

Comparing a decision outcome to its alternatives appears to be an important component of decision making. One important factor is the emotion of regret. This occurs when a decision outcome is compared to the outcome that would have taken place had a different decision been made. This is in contrast to disappointment, which results from comparing one outcome to another as a result of the same decision. Accordingly, large contrasts with counterfactual results have a disproportionate influence on decision making. Regret results compare a decision outcome with what might have been. Therefore, it depends upon the feedback available to decision makers as to which outcome the alternative option would have yielded. Altering the potential for regret by manipulating uncertainty resolution reveals that the decision making behavior that appears to be risk averse can actually be attributed to regret aversion. There is some indication that regret may be related to the distinction between acts and omissions. Some studies have found that regret is more intense following an action, than an omission. For example, in one study, participants concluded that a decision maker who switched stock funds from one company to another and lost money would feel more regret than another decision maker who decided against switching the stock funds but also lost money. People usually assigned a higher value to an inferior outcome when it resulted from an act rather than from an omission. Presumably, this is as a way of counteracting the regret that could have resulted from the act.

#### 4. Making a Better Decision by Buying Reliable Information (Bayesian Approach)

In many cases, the decision maker may need an expert's judgment to sharpen his/her uncertainties with respect to the probable likelihood of each state of nature. For example, consider the following decision problem a company is facing concerning the development of a new product:

**TABLE 10**  
**Buying Reliable Information**

|    |                 | States of Nature |            |           |
|----|-----------------|------------------|------------|-----------|
|    |                 | High Sales       | Med. Sales | Low Sales |
|    |                 | A(0.2)           | B(0.5)     | C(0.3)    |
| A1 | (develop)       | 3000             | 2000       | -6000     |
| A2 | (don't develop) | 0                | 0          | 0         |

The probabilities of the states of nature represent the decision maker's (e.g., manager's) degree of uncertainty and personal judgment on the occurrence of each state. We will refer to these subjective probability assessments as "prior" probabilities.

The expected payoff for each action is:

$$A1 = 0.2(3000) + 0.5(2000) + 0.3(-6000) = \$ -200$$

$$A2 = 0.$$

So the company chooses A2 because of the expected loss associated with A1, and decides not to develop. However, the manager is hesitant about this decision. Based on "nothing ventured, nothing gained," the company is thinking about seeking help from a marketing research firm. The marketing research firm will assess the size of the product's market by means of a survey. Now the manager is faced with a new decision to make—which market research company should he/she consult? The manager has to make a decision as to how "reliable" the consulting firm is. By sampling and then reviewing the past performance of the consultant, we can develop the following reliability matrix:

**TABLE 11**  
**Reliability Matrix**

|                               |    | What Actually Happened in the Past |     |     |
|-------------------------------|----|------------------------------------|-----|-----|
|                               |    | A                                  | B   | C   |
| What the Consultant Predicted | Ap | 0.8                                | 0.1 | 0.1 |
|                               | Bp | 0.1                                | 0.9 | 0.2 |
|                               | Cp | 0.1                                | 0.0 | 0.7 |

All marketing research firms keep records (i.e., historical data) of the performance of their past predictions. These records are available to their clients free of charge. To construct a reliability matrix, you must consider the marketing research firm's performance records for similar products with high sales. Then, find the percentage of which products the marketing research firm correctly predicted would have high sales (A), medium sales (B) and little (C) or almost no sales. Their percentages are presented by  $P(A_p|A) = 0.8$ ,  $P(B_p|A) = 0.1$ ,  $P(C_p|A) = 0.1$ , in the first column of the above table, respectively. Similar analysis should be conducted to construct the remaining columns of the reliability matrix. Note that for consistency, the entries in each column of the above reliability matrix should add up to one. While this matrix provides the conditional probabilities such as  $P(A_p|A) = 0.8$ , the important information the company needs is the reverse form of these conditional probabilities. In this example, what is the numerical value of  $P(A|A_p)$ ? That is, what is the chance that the marketing firm predicts A is going to happen, and A actually will happen? This important information can be obtained by applying the Bayes Law (from your probability and statistics course) as follows:

- a) Take probabilities and multiply them “down” in the above matrix,
- b) Add the rows across to get the sum,
- c) Normalize the values (i.e., making probabilities adding up to one) by dividing each column number by the sum of the row found in Step b,

To illustrate the procedure note the calculation of  $P(A|A_p)$ . The results are shown in Table 12.

$$P(A_p|A) = \frac{P(A_p \cap A)}{P(A)} = \frac{P(A_p) \cdot P(A|A_p)}{P(A)}$$

$$P(A_p) = P(A) \cdot P(A_p|A) + P(B) \cdot P(B_p|B) + P(C) \cdot P(C_p|C) = 0.2 \times 0.8 + 0.5 \times 0.1 + 0.3 \times 0.1 = 0.24$$

$$P(A|A_p) = \frac{P(A_p|A) \cdot P(A)}{P(A_p)} = \frac{0.2 \times 0.8}{0.24} = 0.667$$

**TABLE 12**  
**Bayes Law**

| 0.2               | 0.5               | 0.3               |      |
|-------------------|-------------------|-------------------|------|
| A                 | B                 | C                 | SUM  |
| $0.2(0.8) = 0.16$ | $0.5(0.1) = 0.05$ | $0.3(0.1) = 0.03$ | 0.24 |
| $0.2(0.1) = 0.02$ | $0.5(0.9) = 0.45$ | $0.3(0.2) = 0.06$ | 0.53 |
| $0.2(0.1) = 0.02$ | $0.5(0) = 0$      | $0.3(0.7) = 0.21$ | 0.23 |
| A                 | B                 | C                 |      |
| $(.16/.24)=.667$  | $(.05/.24)=.208$  | $(.03/.24)=.125$  |      |
| $(.02/.53)=.038$  | $(0.45/.53)=.849$ | $(.06/.53)=.113$  |      |
| $(.02/.23)=.087$  | $(0/.23)=0$       | $(0.21/.23)=.913$ |      |

- d) Draw the decision tree. Many managerial problems, such as this example, involve a *sequence of decisions*. When a decision situation requires a series of decisions, the payoff table cannot accommodate the multiple layers of decision making. Thus, a decision tree is needed [8, 28].

## 5. Decision Tree and Influence Diagram

### 5.1 Decision Tree Approach

A decision tree is a chronological representation of the decision process. It utilizes a network of two types of nodes: decision (choice) nodes (represented by square shapes), and states of nature (chance) nodes (represented by circles). Construct a decision tree utilizing the logic of the problem. For the chance nodes, ensure that the probabilities along any outgoing branch sum to one. Calculate the expected payoffs by rolling the tree backward (i.e., starting at the right and working toward the left). You may imagine driving your car; starting at the foot of the decision tree and moving to the right along the branches. At each *square* you have control, to make a decision and then turn the wheel of your car. At each *circle*, fortune takes over the wheel and you are powerless. Here is a step-by-step description of how to build a decision tree:

- 1) Draw the decision tree using squares to represent decisions and circles to represent uncertainty,
- 2) Evaluate the decision tree to make sure all possible outcomes are included,
- 3) Calculate the tree values working from the right side back to the left,
- 4) Calculate the values of uncertain outcome nodes by multiplying the value of the outcomes by their probability (i.e., expected values).

On the tree, the value of a node can be calculated when we have the values for all the nodes following it. The value for a choice node is the largest value of all nodes immediately following it. The value of a chance node is the expected value of the nodes following that node, using the probability of the arcs. By rolling the tree backward, from its branches toward its root, you can compute the value of all nodes including the root of the tree. Put these numerical results on the decision tree results in a graph like what is presented following. Determine the best decision for the tree by starting at its root and going forward. Based on the preceding decision tree, our decision is as follows:

- Hire the consultant, and then wait for the consultant's report. If the report predicts either high or medium sales, then go ahead and manufacture the product. Otherwise, do not manufacture the product.

Check the consultant's efficiency rate by computing the following ratio:

$$\text{Consultant's Efficiency Rate} = \frac{\text{(Expected payoff using consultant dollars amount)}}{\text{EVPI}}$$

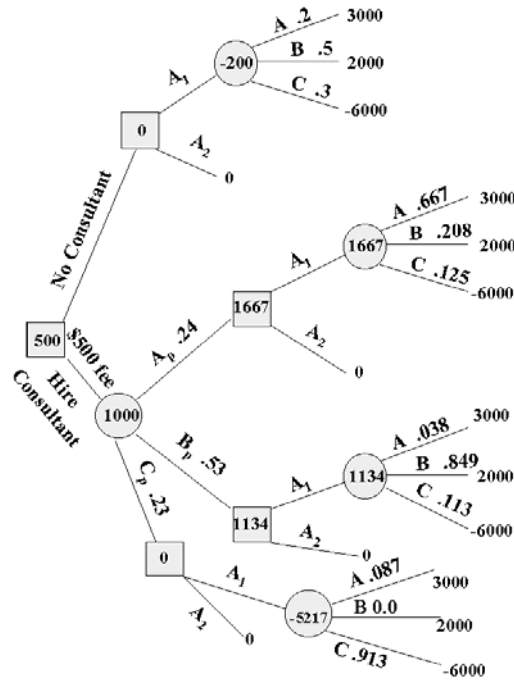
Using the decision tree, the expected payoff if we hire the consultant is:

$$\begin{aligned} \text{EP} &= 1000 - 500 = 500, \\ \text{EVPI} &= .2(3000) + .5(2000) + .3(0) = 1600. \end{aligned}$$

Therefore, the efficiency of this consultant is:  $500/1600 = 31\%$

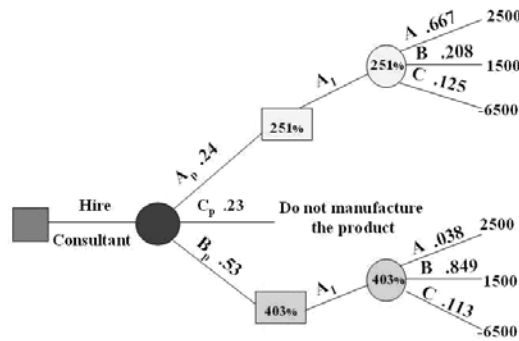


**Figure 3**  
A Typical Decision Tree



If the manager wishes to rely solely on the marketing research firm's recommendations, then we assign flat prior probability (as opposed to (0.2, 0.5, 0.3) used in our numerical example). Clearly the manufacturer is concerned with measuring the risk of the above decision, based on decision tree. Coefficient of Variation as Risk Measuring Tool and Decision Procedure: Based on the above decision, and its decision-tree, one might develop a coefficient of variation (C.V) risk-tree, as depicted below:

**Figure 4**  
Coefficient of Variation as a Risk Measuring Tool and Decision Procedure



Notice that the above risk-tree is extracted from the decision tree, with C.V. numerical value at the nodes relevant to the recommended decision. For example the consultant fee is already subtracted from the payoffs. From the above risk-tree, we notice that this consulting

firm is likely (with probability 0.53) to recommend Bp (medium sales), and if you decide to manufacture the product then the resulting coefficient of variation is very high (403 percent), compared with the other branch of the tree (i.e., 251 percent).

Clearly one must not consider only one consulting firm; rather one must consider several potential consulting firms during the decision making planning stage. The risk decision tree then is a necessary tool to construct for each consulting firm in order to measure and compare to arrive at the final decision for implementation.

## **5.2 The Impact of Prior Probability and Reliability Matrix on Your Decision**

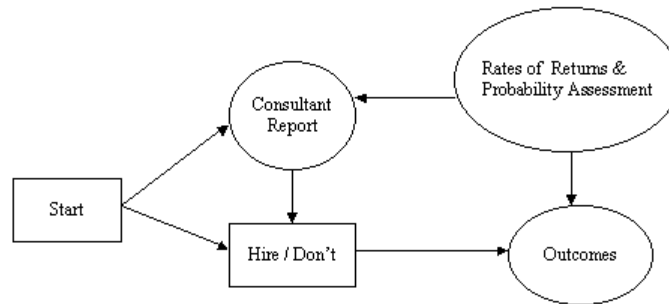
To study how important your prior knowledge and/or the accuracy of the expected information from the consultant in your decision of our numerical example is, the main considerations are as below.

- Consider a flat prior, without changing the reliability matrix.
- Consider a perfect reliability matrix (i.e., with an identity matrix), without changing the prior.
- Consider a perfect prior, without changing the reliability matrix.
- Consider a flat reliability matrix (i.e., with all equal elements), without changing the prior.
- Consider the consultant prediction probabilities as your own prior, without changing the reliability matrix [8].

## **5.3 Influence Diagrams**

As can be seen in the decision tree examples, the branch and node description of sequential decision problems often become very complicated. At times it is downright difficult to draw the tree in such a manner that preserves the relationships that actually drive the decision. The need to maintain validation, and the rapid increase in complexity that often arises from the liberal use of recursive structures, have rendered the decision process difficult to describe to others. The reason for this complexity is that the actual computational mechanism used to analyze the tree, is embodied directly within the trees and branches. The probabilities and values required to calculate the expected value of the following branch are explicitly defined at each node. Influence diagrams are also used for the development of decision models and as an alternate graphical representation of decision trees. The following figure depicts an influence diagram for our numerical example.

**Figure 5**  
**The Influence Diagram for the Numerical Example**



In the influence diagram above, the decision nodes and chance nodes are similarly illustrated with squares and circles. Arcs (arrows) imply relationships, including probabilistic ones. Finally, decision trees and influence diagrams provide effective methods of decision making because they:

- Clearly lay out the problem so that all options can be challenged
- Allow us to analyze fully the possible consequences of a decision
- Provide a framework to quantify the values of outcomes and the probabilities of achieving them
- Help us to make the best decisions on the basis of existing information and best guesses

## 6. Conclusions

Most people often make choices out of habit or tradition, without going through the decision making process steps systematically. Decisions may be made under social pressure or time constraints that interfere with a careful consideration of the options and consequences. Decisions may be influenced by one's emotional state at the time a decision is made. When people lack adequate information or skills, they may make less than optimal decisions. Even when or if people have time and information, they often do a poor job of understanding the probabilities of consequences. Even when they know the statistics; they are more likely to rely on personal experience than on information about probabilities. The fundamental concerns of decision making are combining information about probability with information about desires and interests.

This paper presented the decision analysis process both for public and private decision making under different decision criteria, type, and quality of available information. Basic elements in the analysis of decision alternatives and choice were described as well as the goals and objectives that guide decision making. The theoretic and practical importance of decision making under risky and uncertain situation made us allocate this paper to a quick tour about decision making under uncertain, risky and deterministic situations. Due to this fact, famous techniques have been reviewed and their weaknesses and strengths have been surveyed. We used a unique numerical example for suitable expression of these techniques, so the reader can clearly touch on the difference and similarity points of discussed methods. At the end of each section, a series of threats and opportunities of considered techniques, as well as their limitations and abilities, has been reviewed. The steps of the discussed methods are introduced briefly at the beginning of each section. This tutorial has been written through a simple literature so it can help managers to understand decision making concepts and to make better decisions in uncertain conditions that will open a new window in their minds.

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