Application of Mortality Models to Japan

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Abstract

When projecting future mortality trends, researchers may first develop models that fit observed historical data, and then use these models to project future mortality by estimating future model parameters or modifying the model assumptions. In this paper, four mortality models are examined: the Heligman-Pollard Model, the Mixed Weibull Model, the Lee-Carter Model and a simulation model. Using the Japan Life Table as input, this paper compares characteristics of each model, determines the model parameters and attempts to project future mortality. The strengths and limitations of each model are discussed. The usefulness of the Mixed Weibull Model and the simulation model is emphasized in comparison with other models.

1. Introduction

(1-1) Introduction

Mortality modeling is an old subject. One of the first and most widely known models, the Gompertz Law, was proposed in 1825. This law asserts that the force of mortality increases as an exponential function of age. A mortality model that describes the entire life table was first proposed by Thiele in 1872 (Higgins (2003)). Several other models have also been developed in the last several decades. In this paper, the author picked four mortality models that describe the entire mortality curve, examined their properties and tried to use them to project future mortality. Major focus is given to the Mixed Weibull Model and the simulation model.

(1-2) Japan Life Table

Before proceeding, some basic information on the Japan Life Tables should be provided. The Japan Life Table was first developed around 1900, using data from 1891 to 1898. Currently, the Ministry of Health, Labour and Welfare of Japan develops this table every five years. Data are obtained from the national census. The latest table (the Japan Life Table 19, or JLT19) is based on the census data in 2000. It should be noted that there are only 18 tables, since JLT07 was not developed due to World War II. Also, note that there are two JLT18 tables; one includes deaths caused by the Hanshin-Kobe earthquake on January 17, 1995, and the other excludes them. In this paper, the latter table is used. In addition, only male mortality tables are used and applied to each model.

2. Mortality Models

(2-1) Heligman-Pollard Model
(2-1-1) Model Description

Heligman and Pollard (1980) proposed the following mortality model, which describes the entire lifetime. They applied the model to fit the Australian Life Table.

$$\frac{q_x}{p_x} = A^{(x+B)^C} + De^{-E(\ln x - \ln F)^2} + GH^x. \qquad \ldots(1)$$

The three components of the formula represent early childhood mortality, accidental mortality and senescent mortality, respectively. The third component, GH^x, is interpreted as a discrete version of the Gompertz Law.

(2-1-2) Applying Heligman-Pollard to Japan

When applying the model to the Japan Life Table, parameters are determined by minimizing the square sum,

$$S2 = \sum_{x=0}^{X} \left(\frac{q_x}{\tilde{q}_x} - 1\right)^2 , \qquad \ldots(2)$$

where q_x represents the fitted mortality, while $\tilde{q}_x$ represents the observed mortality. For JLT19, X=112. The results are shown in Table 1. Graph 1a is the graphical presentation of the results. To explain the level of fit, Graph 1b shows the ratio between observed and fitted data. The relative difference of q(x) is as much as 10 percent, except in the early ages, where the relative difference may be up to 20 percent.

For ages 50 and over, more than 99.9 percent of the mortality is explained by the third component in equation (1) above. The first and second components are negligible. Therefore, the author applied a "simplified" Heligman-Pollard Model to the mortality of ages 50 and over,

$$\begin{aligned} \frac{q_x}{p_x} &\cong GH^x \\ &= H^{x-x_0}. \end{aligned} \qquad \ldots (3)$$

x_0= - lnG/lnH represents the age where q_x=0.5. It is a measure of the longest possible life span. Using this simplified model, the author estimated parameters for all JLT tables, which are shown in Table 2, Graph 2a and Graph 2b. In these calculations, ages 50 to 100 are used to calculate the least square sum. Mortality data after age 100 was not included because the last survivor age (omega) differs by table. Two things are worth mentioning. First, as shown in Graph 2a, the graph of H consists of three distinct segments, with the curve leveling off after 1965. Second, x_0 has been increasing since 1965.

In order to project the future mortality trends for elderly people, the author used the simplified Heligman-Pollard Model. It assumes that H is constant from 1965 and thereafter, and that x_0 will follow the current trend. To find the current trend of x_0, the following square sum is minimized.

$$S2 = \sum_{JLT12}^{JLT19} \sum_{x=50}^{100} \left(\frac{q_x}{\tilde{q}_x} - 1\right)^2 \qquad \ldots (4)$$

Table 3 shows the result: H equals 1.10704 and x_0 increases from 98.337 to 105.997. Graph 3 shows the trend of x_0. Linear regression between observation year (dependent variable, X) and x_0 (independent variable, Y) is performed to project x_0 for year 2025 (see Table 4).

In projecting the mortality table for the year 2025, we need to estimate the parameters A to F. Since our concern is elderly people and, also, in order to avoid the correlation problem among parameters, the parameters for year 2000 are used for the year 2025. In other words, it is assumed that no mortality improvement will occur for early childhood or for accidental death. Parameters A to F in the year 2000 are reestimated using G and H in Table 3. Final parameters for the year 2025 are summarized in Table 5. The result will be compared in a later section.

(2-2) Mixed Weibull Model
(2-2-1) Model Description

The Weibull Model is widely used in the area of reliability engineering for analyzing the lifetime of manufacturing products and parts. Let *S(t)* be the probability that a manufacturing product (such as bulbs, auto parts, etc.) is in operating condition (not failed) at time t. *S(t)* follows a (regular) Weibull Model if it is described as

$$S(t) = \exp\left[-\left(\frac{t-\gamma}{t_0}\right)^m\right] \qquad (t \geq \gamma)$$
$$= 1 \qquad (t \leq \gamma)$$

where … (5)

γ : *position parameter* $(\gamma \geq 0)$
t_0 : *scale parameter* $(t_0 > 0)$
m : *shape parameter* $(m > 0)$.

F(t) = 1-*S(t)* represents the accumulated probability of failure until t. In actuarial terms, *S(t)* is the survival function. The parameters are interpreted as follows. The position parameter represents the period in which there is no failure. Since S($t_0 + gamma$) = exp (-1) = 0.368, the scale parameter represents the time period in which the surviving products are about one-third of all products. $t_0 + gamma$ can be used as a measure of expected lifetime for the products. The shape parameter defines patterns of failure. Failure can be categorized either as

(a) m<1, early failure type
(b) m=1, accidental failure type, or
(c) m>1, worn-out failure type.

Since the force of mortality for the Weibull Model is written as

$$\mu(t) = -\frac{1}{S(t)}\frac{dS(t)}{dt} = \frac{m}{t_0}\left(\frac{t-\gamma}{t_0}\right)^{m-1}, \qquad \ldots(6)$$

it is a decreasing, constant or increasing function when m<1, m=1, or m>1, respectively.

A Mixed Weibull Model consists of two or more Weibull components combined in some fixed proportion. Kao proposed this type of distribution in his study of the lifetime of vacuum bulbs and discovered that the failure of bulbs is described as a mix of early failure and worn-out failure. Furukawa (1996) applied the Mixed Weibull Model with four components to the Japan Life Table and estimated its parameters. Furukawa also applied the Mixed Weibull Model to an ancient human (Jomon man, Japanese who lived several thousand years ago), an eighteenth-century Viennese person and some mammals and birds.

The Mixed Weibull Model with four components is described as follows:

$$S(t) = \sum_{i=1}^{4} p_i \exp\left[-\left(\frac{Max(t-\gamma_i, 0)}{t_{0i}}\right)^{m_i}\right]$$

where $\ldots$ (7)

$$\sum_{i-1}^{4} p_i = 1 \qquad (p_i > 0) \quad mix\,ratio$$

This is a model with sixteen parameters and fifteen degrees of freedom.

(2-2-2) Parameter Estimation

The Weibull plotting paper is a useful tool that helps determine Weibull parameters. The Weibull plotting paper uses X=ln(t-gamma) for its horizontal axis and Y=1/ln(ln(S(t))) for its vertical axis. If S(t) follows equation (5) above,

$$1/\ln(\ln(S(t)) = m\ln(t-\gamma) - mt_0 \qquad (t > \gamma)$$
$$Y = mX - mt_0. \qquad \ldots (8)$$

For the Mixed Weibull Models, a similar approach can be taken. After plotting the observed data on plotting paper, one can estimate the parameters for the first component by focusing on the first linear part on the paper. After determining the first component, the residual value is again plotted on the plotting paper, and then one determines the second component, and so on. These values would be used for the candidate values for the Weibull parameters.

Once the candidate values are obtained, one determines the Weibull parameters using the least square sum approach for S(x). This is not a simple step, and parameters are determined after a long trial-and-error process. Due to the redundant number of parameters in the model, there is no unique solution. Parameters are correlated, especially t's and gamma's, as well as p_3 and p_4 being correlated.

(2-2-3) Applying the Mixed Weibull Model to Japan

In this section, the Mixed Weibull Model is applied to JLT19. To estimate the Weibull parameters, assumptions are made *a priori* that m_1<1 and $gamma_1$=0. This assumption is made so that the first component represents premature death. Using plotting paper (Graph 5a), m_1 is determined to be somewhere around 0.3. In addition, by examining the line in Graph 5a, it is estimated that gamma2 would be around 15.

Using the above estimate, Weibull parameters are estimated using the least square sum approach. First, parameters for the first component were estimated by minimizing the square sum of the S(x) ratios for ages 1 to 15. Then, parameters for the second component were estimated by minimizing the square sum for ages 16 to 30. Finally, all parameters (including those for the first and the second components) were estimated using the least square sum for ages 1 to 100. In this process, S(x), d(x), q(x), ln(q(x)), as well as Weibull plottings, were monitored carefully, so that the final result would not deviate from the data to be fitted.

Although there are many possible solutions that fit equally well with the original data, Table 6 summarizes one result. To verify the fit, the results were presented in Graphs 4a to 4e. Also, Weibull plottings are given in Graphs 5a to 5d. For S(x) after age 90, an adjustment needed to be made. The Mixed Weibull Model in its original form does not fit well for q(x) in very old ages, because the first component of S(x) changes very gradually compared to the other components in very old ages. In order to adjust this problem, the first component of S(x) is smoothed at age 90 and thereafter, so that the S(x+1) /S(x) of the first component is the same as that of the fourth component. The same adjustment is made for the second component of S(x) at age 100 and thereafter. With these adjustments, fit improved significantly. Since the first and second components are small enough, their impact on S(x) is negligible. For reference, Graph 4f shows the q(x) before adjustment.

There are some important observations concerning the results. First, the fit of q(x), shown in Graph 4e, is very good. The relative difference between the observed curve and the fitted curve is within 5 percent for age 40 and thereafter. Before age 40, the relative difference can be as much as 20 percent. Fit in the early ages is more difficult than in the older ages, probably due to the smaller number of actual observations and the existence of the hump portion.

Decomposition of death into components figures as one interesting property of the Mixed Weibull Model. As shown in Table 6 and Graph 4d, the first component represents early, premature death as m_1=0.285. The second component, whose m_2 is 2.01, represents accidental death in adolescence. Since m_2 is higher than 1, this component has some aspects of the worn-out type of failure. The third and fourth components represent mortality in the elderly period. The third component accounts for 37 percent of the deaths and corresponds to deaths at age 40 and later (see Graph 4d). The fourth component accounts for 61 percent of the deaths and corresponds to the deaths at age 70 and later.

(2-2-4) Projection of Future Mortality

To observe the recent mortality trends of the Japan Life Table, the author determined the Weibull parameter for JLT15 (1980), JLT17 (1990) and JLT19 (2000) simultaneously, using the following bold assumptions. The three tables have the same t's and m's, which implies that they share components of the same shape. Only the mix of components (p's) and trigger age (gamma's) differ. Using the least square sum, the parameters are determined in Table 7. The parameter set in Table 7 for JLT19 is different from that of Table 6; however, the set is also valid for describing JLT19, although the fit is somewhat less than that in Table 6.

For the year 2025, only gamma3 and gamma4 are projected by extrapolating linearly from 1980 to 2000. Other parameters are left equal to those of the year 2000. The estimated parameters are also found in Table 7.

(2-3) Lee-Carter Model
(2-3-1) Model Description

The Lee-Carter Model is a typical relational model introduced by Carter and Lee in 1992. This model assumes that if we eliminate random factor e_t, the logarithm of the central death rate, or m_t, can formulate a family of curves with parameter k_t. By evaluating and forecasting the trend of k_t, the model can be applied to project future mortality. The model equation is as follows:

$$\log m_{x,t} = a_x + k_t \cdot b_x + e_{x,t} \ (t = 0,1,\ldots,T-1,\ x = 0,1,\ldots,N-1) \quad \ldots (9)$$

where

$m_{x,t}$: *central death rate of age x at time t*

a_x : *average shape of the age profile*

b_x : *pattern of deviations from age profile*

k_t : *level of mortality at time t*

$e_{x,t}$: *error term*

Parameters a, b, k are determined by minimizing the square sum of $e_{x,t}$. In practice, with the following conditions, one can determine these parameters uniquely, using the singular value decomposition technique in linear algebra. More precisely, if one assumes that the norms (square root of square sum of each element) of b and k are 1, and the sum of all elements of k is 0, the above parameters can be determined in the following way:

$$a = (a_x) = \left(\frac{1}{T}\sum_{t=0}^{T-1}\log(m_{x,t})\right) : \textit{average of } \log(m_{x,t}) \textit{ by } \mathrm{t}$$

$b : N-$*dimensional singular vector that corresponds to the principal component of* M'

$k : T-$*dimensional singular vector that corresponds to the principal component of* M'

where,

$$\mathrm{M}' = m'_{x,t} = (\log(m_{x,t}) - a_x) \ , \quad (N \times T \textit{ matrix})$$

Lee and Carter applied this model to U.S. mortality data and discovered that k_t has been changing linearly.

As an extension of the model, one may want to include the second or third components of the singular value decomposition. In this case, equation (9) above is reexpressed as follows:

$$\log m_{x,t} = a_x + {}^1k_t \cdot {}^1b_x + {}^2k_t \cdot {}^2b_x + {}^3k_t \cdot {}^3b_x + e_{x,t} \ (t = 0,1,\ldots,T-1,\ x = 0,1,\ldots,N-1) \quad \ldots (10)$$

where, for example, ^{2}k denotes T-dimensional singular vector that corresponds to the second component of M′.

(2-3-2) Applying Lee-Carter Model to Japan

Singular value decomposition was applied to the Japan Life Table, and parameters were estimated. Since the final ages differ by table, the author used the data from ages 0 to 100 and analyzed the mortality matrix M' (instead of the central death rate matrix) with 101 by 18 elements.

The applied result of the Lee-Carter Model to JLT19 is shown in Graphs 6a to 6c. Singular value decomposition gives a fairly good approximation of JLT19 if one includes the principal, second and third components. The relative difference between fitted and observed q(x) is within 10 percent for ages 20 to 100. In the early ages, the fit is less good. The estimated parameters are shown in Graphs 7a and 7b. Graph 7a shows the shape of the deviation vectors ${}^1b$, ${}^2b$ and ${}^3b$. All elements in the first (principal) deviation vector, or ${}^1b$, have the same sign, which implies that mortality improves in all ages. Mortality improvement is more significant in the early ages and decreases as age increases. The second deviation vector, or ${}^2b$, is positive for ages 50 and above. This fact, combined with the fact that ${}^2k$ has decreased in the last 40 years, implies that mortality improvement in elderly ages has accelerated in recent years. Graph 7b shows the k parameters of the first three components. It is worth noting that ${}^1k$ consists of two lines connected at year 1950. This implies that the mortality improvement trend has changed significantly before and after World War II. The improvement trend is steady and prominent after the end of the World War II. Even though the effect is less significant, ${}^2k$ has also been decreasing for the last 40 years.

(2-3-3) Projection of Future Mortality

Table 8 is a tabular expression of Graph 7b. In order to project the mortality in year 2025, ${}^1k_t$ is determined using linear regression. Parameters ${}^2k_t$ and ${}^3k_t$ are assumed to remain at the level of year 2000. The projected result is examined in a later section.

(2-4) A Simulation Model
(2-4-1) Model Description

Furukawa (1996) proposed a Monte Carlo type simulation model that is based on the "Vitality" concept. He applied the model to the Japan Life Table and determined its parameters. The author has developed his own simulation model based upon Furukawa's model and ideas.

Regarding the aging process, there are two types of widely known theories (Held (2002)). One type is called an evolutionary theory, while the other is called a wear and tear theory. The latter theory states that aging is the result of mechanical or biochemical wear and tear on the human body and its organs, tissues and cells. The following model was developed in accordance with the wear and tear theory.

To develop a simulation model, the following set of assumptions was established. First, each life has its own vitality. Vitality is a measure of capacity of the human body in which various kinds of hazard factors occur. Vitality increases in the early ages, reaches its peak and levels off at the beginning of adolescence, then starts to decrease at the beginning of middle age. This assumption is based on the general observation that the functional capacity of the human organs, including respiratory system, heart, renal function and basal metabolism, decreases as time progresses once men reach their maturation stage. For example, the lung function of a healthy 70-year-old is about 50 percent that of a 30-year-old (Goldman and Ausiello (2004)). For simplicity, the increase and decrease of vitality are assumed to occur linearly. On the other hand, vitality is subject to hazard factors, which arise randomly. Hazard factors accumulate in the body as time goes by, with death occurring when all vitality is impacted by hazard factors.

The vitality functions and the hazard factor functions are described as follows. Let V(t) denote the vitality function at time t. V(t) is equal to u at t=0 and is equal to v at t=1, then increases linearly to 100 at t=15, levels off between t=15 and t=30, and then decreases linearly to 0 at t=130. Linear increase and decrease of vitality is assumed for simplicity. In addition, vitality function is assumed to be the same for all individuals; the model assumes there is no genetic or environmental difference among newborn babies. Heterogeneity might be introduced in a more sophisticated model; however, it is the subject of future research. Graphical representation of vitality is shown in Graph 8. U and v are parameters that have been introduced to create better fit for the early age mortality.

The hazard factor function, on the other hand, has a more complicated expression. Let H(t) denote the hazard factor function. Also, let $X_1(t)$, $X_2(t)$, and $X_3(t)$ denote the three hazard factors that occur at time t. These functions follow a stochastic process defined in the following recurring equation,

$$H(t) = H(t-1) + I * X_1(t) + X_2(t) + X_3(t) - (I-1) * X_1(t-1) \quad \text{... (11)}$$

where *I* denotes the impact parameter for the first hazard factor, and

$$
\begin{aligned}
X_1(t) &= B_1(t) * S_1(t), && where\ B_1(t) \approx Ber(p_1), S_1(t) \approx Exp(k_1) \\
X_2(t) &= 0 \quad (t \le 14) \\
&= B_2(t) * S_2(t)\ (t \ge 15), && where\ B_2(t) \approx Ber(p_2), S_2(t) \approx Exp(k_2) \\
X_3(t) &= 100 * B_3(t), && where\ B_3(t) \approx Ber(p_3).
\end{aligned}
\qquad \ldots(12)
$$

In the above expression, *Ber(p)* denotes a Bernoulli distribution with parameter p, and *Exp(k)* denotes exponential distribution with parameter k (note that the expected value of exponential distribution is 1/k). Exponential functions are used because they are easy to handle and because they have tails. Death occurs when H(t) > V(t) for the first time. The last term in equation (11) above describes recovery from the first hazard factor, if an individual endures the impact of the first hazard factor caused at t-1.

In equation (11), the first hazard factor expresses the effect of diseases and accidents that would arise in the course of an ordinary lifetime. It occurs fairly frequently (once every several years), and it threatens human life expectancy. The parameter p_1 refers to probability of occurrence. The hazard (or stress) caused by this incidence is I* S_1, where I is the impact parameter and S_1 is the size of the hazard. In the next year, the hazard is decreased by (I-1)* S_1 due to the recovery, and S_1 remains in the body. The second hazard function represents more severe damage to the body. It occurs much less frequently (probability p_2) than the first factor, and once it happens, all the damage remains in the body and no recovery is expected. The second hazard is assumed to occur at ages 15 and thereafter. The third hazard function expresses accidental death.

Graphs 9a to 9d give graphical explanation of how H(t) develops and when death occurs in this model. There are four patterns of death. Graph 9a shows a death from caducity. Even though no hazard factor occurred in the last several years, death occurred because vitality decreased gradually and finally went below H(t). Graph 9b shows a death from the first hazard factor in the elderly period. This is the typical death pattern in this model. Graph 9c shows a death from the second hazard factor (serious disease or accident). Graph 9d shows a death caused by the third factor, which represents accidental death.

In summary, the obtained model has eight parameters, u, v, I, p_1, p_2, p_3, k_1 and k_2. By simulating V(t) and H(t) and counting the number of deaths at each age, one can create a distribution of death d(x), and other mortality functions.

(2-4-2) Applying the Model to Japan

The above model is applied to JLT19, and parameters are determined so as to minimize the square sum of d(t). Also, the author considered other indicators, such as e(0) (expected lifetime), the shape of s(t), d(t), q(t) and log(q(t)), to produce a good fit. The result was obtained after long trial-and-error processes and, therefore, it may not be the best solution. However, the author believes that this solution can help to understand and assess the validity of the model.

To determine the parameters, at least 50,000 lives are simulated for each run. The author used the following approach to determine the parameters. The value of u is set so that expected d(0) is equal to the observed d(0). More specifically,

$$u = -\ln((\tilde{q}(0) - p_3) / p_1) * \frac{I}{k_1}, \qquad \ldots(13\text{-}1)$$

where the tilde symbol implies the observation data. Similarly, the value of v can be set so that expected d(1) is equal to the observed d(1), or

$$v = -\ln((\tilde{q}(1) - p_3) / p_1) * \frac{I}{k_1} + \frac{p_1}{k_1} \qquad \ldots(13\text{-}2)$$

The value of k_3 is set equal to the minimum q(x) of the observed data. The impact parameter is set equal to I=5, after trial-and-error process for JLT19.

There is a weak relation among some parameters, which helps determine those parameters. On average, death occurs when V(t) = E[H(t)]. It is expected that the solution of this equation (denoted by T) is close to other indicators such as expected lifetime or mode of d(x). For JLT19, e(0) =77.73 and the mode of d(x) = 84. Since T = 77.96, T happens to be close to expected life expectancy at birth. Detailed calculation of E[H(t)] and T is found in Appendix A.

Table 9 summarizes the parameters for JLT19, and Graphs 10a to 10e give graphical expressions. A total of 2,500,000 lives were simulated to determine the final distribution. The ratio of q(x), shown in Graph 10e, implies that the relative difference is less than 10 percent for ages 50 and above. For younger ages, the fit is not as good. However, judging from the shape of s(x), d(x) and log(q(x)), the fit is acceptable.

(2-4-3) Projection of Future Mortality

To project future mortality using this simulation model, the parameters for JLT15 (1980) and JLT17 (1990) are also determined with some conditions. For JLT15 and JLT17, k_2 and p_2 are assumed to be the same as those of JLT19. This assumption is made for two reasons. The first is to focus our attention to either k_1 or p_1, since our concern is the mortality improvement in elder ages and such improvement is observed in the improvement of either k_1 or p_1. The second reason is to reduce the number of parameters to an estimate to simplify the calculation. I is set equal to that of JLT19. The value of p_3 is set equal to the minimum q(x). U and v are determined using equations (13-1) and (13-2) above.

Table 10 is the result of this parameter estimation. For projecting the parameters for year 2025, p_1 and k_1 are projected by extrapolating the trend between JLT15 and JLT19. Other parameters, u, v, I, p_2, k_2 and p_3, are set equal to those of JLT19.

3. Comparison of the Models

(3-1) Projection Result
(3-1-1) Simple Projection

Projected mortality tables for year 2025 are shown in Table 11a. The projection is based upon the mortality improvement of each model q(x) between year 2000 and 2025. Model q(x) is shown in Table 11b. Also in Table 11a, life expectancies at some selected ages (x=0, 50, 70 and 80) are given for comparison.

Please note that the projection result is obtained by assuming that the current trend of key model parameters will continue into the near future. No additional consideration was given for any future change of the current trend.

In Table 11a, two additional projections are given for comparison. These projections are based on a simple method. For each age, it is assumed that mortality improvement in the last T years will continue for the next 25 years. In mathematical terms, it is described as

$$
\begin{aligned}
{}^{2025}q(x) &= (mortality\ \ improvement) \times {}^{2000}\tilde{q}(x) \\
&= \left(\frac{{}^{2000}\tilde{q}(x)}{{}^{2000-T}\tilde{q}(x)}\right)^{\frac{25}{T}} \times {}^{2000}\tilde{q}(x). \qquad \ldots (14)
\end{aligned}
$$

In this paper, the mortality tables developed in this manner refer to simple projections. Simple projection "a" (SP-a) uses T=25, while simple projection "b" (SP-b) uses T=10. These two tables are used as benchmarks.

Graphs 11a and 11b compare the JLT19 (2000) with the two simple projections. Notably, for ages 80 and above, SP-b produces lower mortality than SP-a. Further, the difference seems to widen as the age increases. This implies that, in this age range, the mortality improvement of the last 10 years is more significant than that of the last 25 years. Further research will be necessary if such rapid improvement in mortality at the very old ages continues in the future.

However, from age 20 to 50, SP-b doesn't show much mortality improvement. At age 32, for example, mortality worsened from year 1990 (q(32)= 0.00083) to year 2000 (q(32)= 0.00088). Further research is indicated to find whether this is a temporary phenomenon or not.

(3-1-2) Heligman-Pollard Model

Graphs 11c and 11d compare the Heligman-Pollard (HP) Model with SP-a and SP-b. Mortality improvement is significant at age 30 and above. Life expectancies of HP found in Table 11a are almost equal to or are higher than any other projections for all selected ages. For example, e(50, HP)=35.18, while e(50, SP-a) = 34.28. It might be that the projected mortality improvement by HP is excessive. The probable reason for this over-estimation of mortality improvement is that parameter x_0 is extrapolated using linear regression. Even though Graph 2b shows a linear trend, one should be cautious in assuming that x_0 will continue to grow linearly without any restriction.

(3-1-3) Mixed-Weibull Model

Graphs 11e and 11f show the projection result of the Mixed Weibull Model. Compared to SP-a, mortality improvement is less significant for ages 40 to 70, and more significant for age 80 and thereafter.

(3-1-4) Lee-Carter Model

Graphs 11g and 11h show the projection result of the Lee-Carter Model. At age 60 and below, mortality improvement is more significant than that of SP-a and less significant at age 60 and above. The Lee-Carter model produced the lowest life expectancy among all projections for all selected ages. This implies that large improvement in mortality at younger ages is less significant than small improvement at older ages. For projected mortality for ages 60 to 90, the Lee-Carter Model produces smaller mortality improvement when compared to other methods.

(3-1-5) Simulation Model

Graphs 11i and 11j show the projection results of the simulation model. Interestingly, the simulation model produces very close projection to that of SP-a.

(3-1-6) Comparison of Four Model Result

Graphs 11k and 11l compare the projection results of the four models. Since our interest is mainly in the middle to older age mortality projection, comparison is made at age 50 and later.

To compare the mortality level, the relative mortality to SP-a is plotted in Graph 11m. If this value is larger than one, it implies that the improvement is less significant than that of SP-a. This graph clearly demonstrates that the Lee-Carter Model shows relatively high mortality compared to other models, the simulation model produced a close simulation to SP-a, and the Heligman-Pollard Model shows low relative mortality compared to the other models.

(3-2) Model Comparison

The Heligman-Pollard Model is a typical parameter model that is easily understandable for actuaries. It has a Gompertz Law component that actuaries are familiar with. Also, the model contains two other components that correspond to early age and adolescent age mortality, respectively. The fit seems to be fair. At age 50 and above, the relative difference between fitted and observed mortality is below the 10 percent range.

The Mixed Weibull Model is also a parameter model; however, it can be applied to a wider variety of settings than the Heligman-Pollard Model. This flexibility comes from the unlimited number of components that can be included in the model, and that each component can represent a different type of distribution by changing the shape parameter. They can be either early failure type ($m<1$), accidental failure type ($m=1$) or worn-out failure type ($m>1$).

When the parameters are determined in an appropriate manner, the result has a very good fit, as shown in Section (2-2-3). For age 40 and above, the relative difference was within the 5 percent range. Parameter determination is somewhat difficult, however, due to the redundant number of parameters and correlation among them. There is no unique solution. Different sets of parameters might be valid. For mortality projection, some kind of model simplification might be needed.

The Lee-Carter Model is a popular relational model. One advantage is that it is easy to apply for determining parameters. Even though one may need a computer program to solve a singular value decomposition problem, it is basically an application of linear algebra. However, when applied to the Japan Life Table, the Lee-Carter Model produced excessive mortality improvement in the early ages, which may require additional consideration.

A Monte-Carlo simulation model generally refers to a broad range of models that incorporate stochastic simulation using random variables. This type of model is very flexible and can incorporate various kinds of assumptions. The simulation model has the potential power of explaining the dynamics of aging process, while other models have limited explanation power. This is the most appealing point of developing simulation models.

In order to develop an appropriate model, however, one needs to have a deep understanding of the model subject. In creating a model, one needs to interpret the observed phenomenon in terms of the model parameters and setting. Through this process, one can find ways to understand and explain the dynamics behind the subject. Parameter determination also requires consideration. The author used spreadsheets for his analysis; however, it was a long trial-and-error process and not effective. One needs to utilize a non-linear least square sum program that incorporates the Monte-Carlo simulation process. Least square sum optimization is much more time-consuming for the simulation model than for the Heligman-Pollard and the Mixed Weibull Models.

The model developed by the author contains eight parameters. The author intended to make the model as simple as possible and limit the number of parameters to as few as those of the Heligman-Pollard model. The fitted result is fair and comparable to that of the Heligman-Pollard in elder ages. The relative difference between observed and fitted is in the 10 percent range in these ages.

The simulation model developed in this paper is based on the simple idea of decreasing vitality (after age 30) and accumulating hazard factor. The author was encouraged by the fact that even such a simple model has a fair fit with observed mortality. Vitality and hazard factor concepts seem to be useful for developing a simulation model. It goes without mentioning that one may develop different types of simulation model from different perspectives.

4. Conclusions

Of the four mortality models discussed, the author believes that the Heligman-Pollard Model and the Lee-Carter Models are widely known and applied, while the Mixed Weibull Model and the simulation model may not be. The Mixed Weibull Model is a useful model for its good fit and its applicability. It produces better fit than the Heligman-Pollard Model or the simulation model. The Mixed Weibull Model can be applied to various subjects by including or reducing the number of components and choosing appropriate shape parameters. It will serve as one valid model for mortality studies.

The author developed a simulation model inspired by Furukawa's early research and results. One goal of this paper was to develop a simple model with eight to ten parameters that is comparable to other three mortality models. Even though the obtained simulation model is based on a set of bold assumptions and still in development, it was shown that the simulation model has a fair fit. It will serve as a useful mortality model.

One appealing point of the simulation model is its potential power of explaining the human aging process, which strongly encourages us in further research. The simulation model can incorporate dynamic processes of how we age and die, while the other three models can not. The study of the aging process is a major subject in gerontology. By developing a simulation model consistent with medical knowledge, one can better understand and explain the dynamics of mortality.

Following is a list of further research topics regarding the development of the simulation model:

(1) Develop a useful nonlinear optimization program that incorporates a Monte Carlo simulation process.
(2) Apply the model to various mortality tables, including sex, nationality and time of observation, and verify the validity of the model structure. The current model contains three hazard factor functions. It may be appropriate to modify these functions so that the same model structure can be applied to various tables.
(3) Develop a deep understanding of the aging process and interpret the knowledge in terms of vitality and hazard factors. These two key concepts may be redefined so as to be consistent with the medical knowledge.
(4) Examine the distribution of the hazard factor functions. In this paper, exponential distributions are used because of their simplicity. Other

distributions, which might be more explainable from medical point of view, may substitute for the exponential distributions.

(5) Research on the shape of vitality function. Again, this should be undertaken with the help of medical knowledge. In addition, different vitality functions might be assumed for each individual. Heterogeneity or frailty might be introduced.

(6) Maximum survival age is set equal to 130. This is the age when vitality function becomes zero. In the future, this limit may be extended. This is another parameter one can incorporate in the model.

(7) The model should be validated by the cause of death analysis. In the current model, there are four patterns of death. The distribution of the patterns should be consistent with actual statistics. A more sophisticated simulation model can include the hazard factors that represent major causes of death (such as cancer, cardiovascular diseases, suicide, etc.).

(8) If a simulation model is developed appropriately, it can be used not only for determining the distribution of mortality, but also for determining the distribution of human healthiness. For any t, distribution of H(t) represents the distribution of health conditions of the observed group. This direction of future research can be incorporated with the study of biological age.

(9) Find a way to appropriately fit the hump portion of the observed mortality at age 20 and 30. In the current model, mortality hump is not modeled satisfactorily.

Finally, since there is no one valid mortality model, researchers should try as many approaches as possible, review their strong and weak points, and carefully choose the appropriate ones for their purpose. A simulation model, as well as the Mixed Weibull model, will be strong approaches that should be utilized for mortality studies. This paper is an attempt at the development of such model.

References

Adachi, Y. (1997). "A Study on Mortality Rates," Bulletins of the Institute of Actuaries of Japan, Vol. 50(2): 69-94 (in Japanese).

Carter, L., and Lee, R. D. (1992). "Modeling and Forecasting U.S. Mortality: Differences in Life Expectancy by Sex," *International Journal of Forecasting,* 8, No. 3 (November): 393-412.

Furukawa, T. (1996). *Mathematical Theories on Longevity.* Tokyo, Japan: Asakura Book Store (in Japanese).

Goldman, L., and Ausiello, D. (2004). *Cecil Textbook of Medicine* 22 ed. Philadelphia: Saunders.

Held, G. (2002). "Research on the Aging Process: Survey," *North American Actuarial Journal* 6(3): 30-37.

Heligman, L., and Pollard, J. H. (1980). "The Age Pattern of Mortality," *Journal of the Institute of Actuaries,* 107:49-80.

Higgins, T. (2003). "Mathematical Models on Mortality," Paper presented at the Workshop on Mortality Modeling and Forecasting, found on the Internet at http://www.acpr.edu.au/Publications/Mortality%20talk%20-%20mathematical%20equations.pdf

Johns, B. L. (1998). "A Model for Analyzing the Impact of Selective Lapsation on Mortality," *North American Actuarial Journal,* 2(1): 79-86.

Lee, R. (2000). "The Lee-Carter Method for Forecasting Mortality, with Various Extensions and Applications," *North American Actuarial Journal,* 4(1): 80-92.

Lipkovich, I., and Smith, E. P. (2002). "Biplot and Singular Value Decomposition Macros for Excel ©," *Journal of Statistical Software,* Vol. 7, Issue 5, found on the Internet at http://www.jstatsoft.org/index.php?vol=7

Tuljapurkar, S., and Boe, C. (1998). "Mortality Change and Forecasting: How Much and How Little Do We Know?" *North American Actuarial Journal,* 2(4): 13-47.

Watcher, K., and Finch, C., ed. (1997). *Between Zeus and the Salmon: The biodemography of Longevity*. Washington, D.C.: National Academy Press.

Wong-Fupuy, C., and Haberman, S. (2004). "Projecting Mortality Trends: Recent Developments in the United Kingdom and the United States," *North American Actuarial Journal,* 8(2): 56-83.

Appendix A

The Derivation of E[H(t)]

E[H(t)] is expressed in the following way.

$$E[H(t)] = p_1/k_1 * (t+I) + p_2/k_2 * (t-14) + 100 * p_3 * (t+1), \ (t \geq 14). \qquad \ldots(A1)$$

This equation is obtained by recursion.

$$E[H(0)] = I * p_1/k_1 + 100 * p_3$$

$$E[H(1)] = E[H(0)] + I * p_1/k_1 + 100 * p_3 - (I-1) * p_1/k_1$$

$$= (I+1) * p_1/k_1 + 100 * p_3 * 2$$

For $t < 15$,

$$E[H(t)] = E[H(t-1)] + I * p_1/k_1 + 100 * p_3 - (I-1) * p_1/k_1 \qquad \ldots(A2)$$

$$= p_1/k_1 * (t+I) + 100 * p_3 * (t+1)$$

For $t \geq 15$,

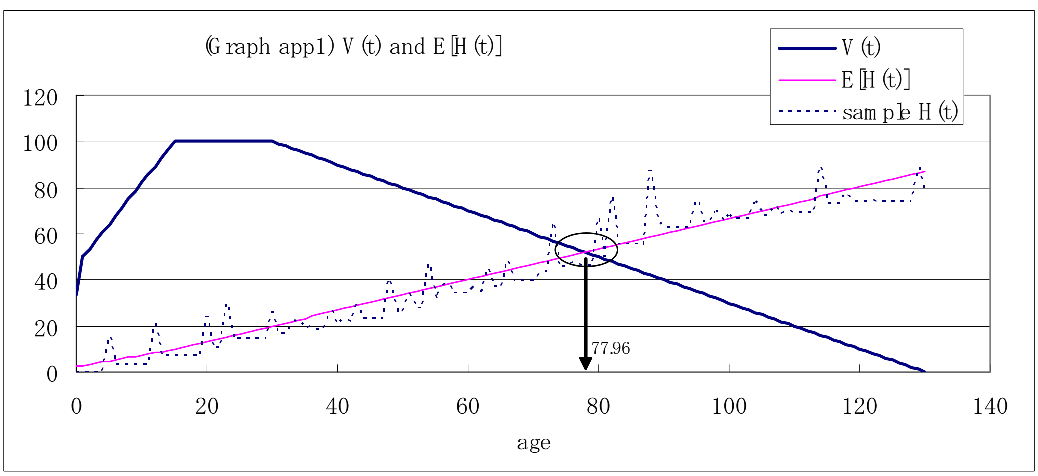
$$E[H(t)] = E[H(t-1)] + I * p_1/k_1 + p_2/k_2 + 100 * p_3 - (I-1) * p_1/k_1$$

$$= p_1/k_1 * (t+I) + p_2/k_2 * (t-14) + 100 * p_3 * (t+1)$$

On the other hand, $V(t) = 130 - t \ \ (t \geq 30)$ and therefore, E[H(t)]=V(t) is solved as

$$T = \frac{130 - I * p_1/k_1 + 14 * p_2/k_2 - 100 * p_3}{1 + p_1/k_1 + p_2/k_2 + 100 * p_3} \qquad \ldots(A3)$$

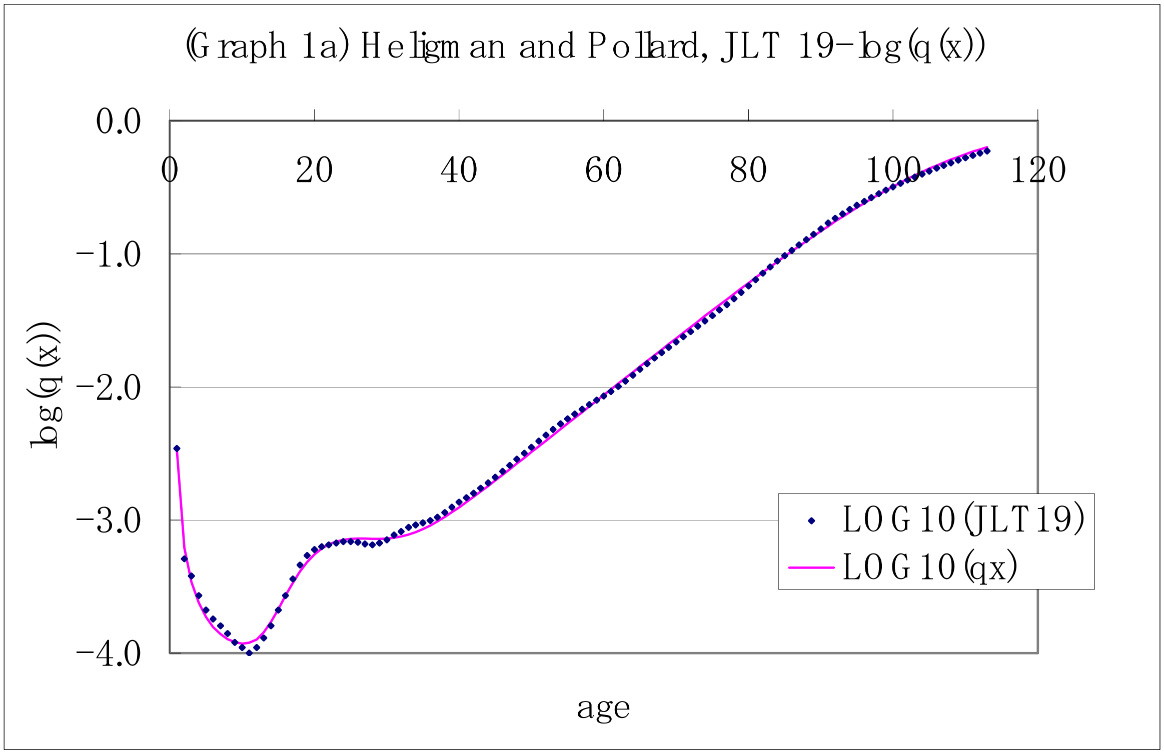
Using the parameters in Table 10, T is determined as T=77.96. See following graph for the idea.

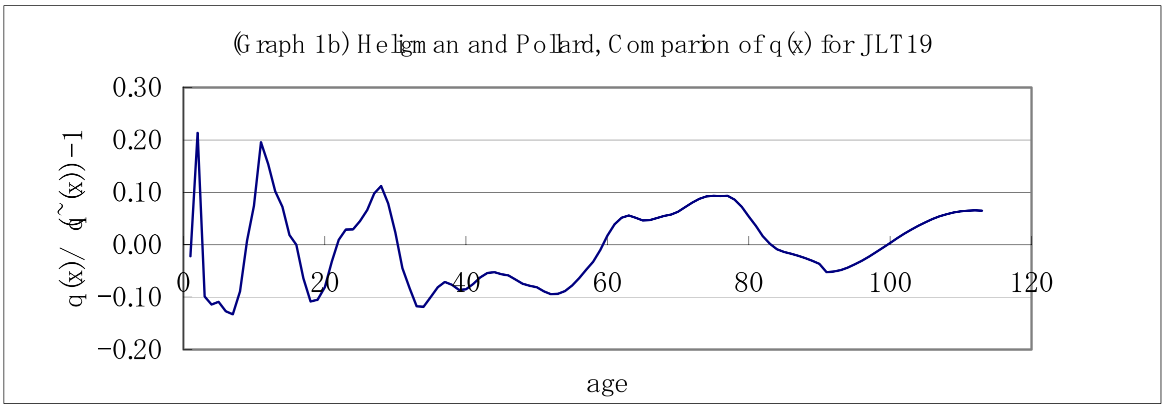


Tables and Graphs

(Table 1) Heligman and Pollard, Summary of Parameters for JLT19

A	0.00068
B	0.15370
C	0.13201
D	0.00045
E	7.65618
F	22.43161
G	0.0000245
H	1.10475
X0=-lnG/lnH	106.55239
S2	0.59580



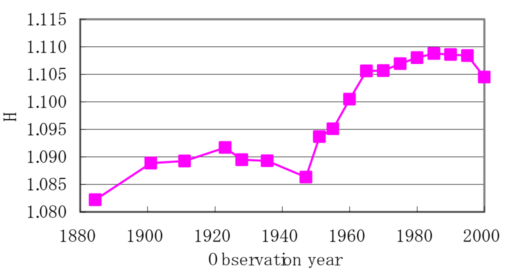


(Table 2) Estimated Parameters of Simplified Heligman and Pollard

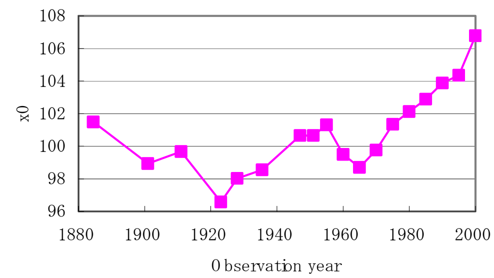
table observation year	JLT01M 1891−98	JLT02M 1899−03	JLT03M 1909−13	JLT04M 1921−25	JLT05M 1926−30	JLT06M 1935−36	JLT08M 1947	JLT09M 1950−52	JLT10M 1955
G	0.000329	0.000220	0.000200	0.000209	0.000224	0.000218	0.000240	0.000122	0.000100
H	1.0822	1.0889	1.0892	1.0917	1.0895	1.0893	1.0863	1.0937	1.0951
x0=−lnG / lnH	101.48	98.93	99.67	96.57	98.02	98.56	100.65	100.65	101.31
S2	0.1526	0.3577	0.3111	0.1745	0.0887	0.1739	0.0193	0.0161	0.0447

table observation year	JLT11M 1960	JLT12M 1965	JLT13M 1970	JLT14M 1975	JLT15M 1980	JLT16M 1985	JLT17M 1990	JLT18M (e 1995	JLT19M 2000
G	0.000073	0.000050	0.000044	0.000034	0.000028	0.000024	0.000022	0.000022	0.000025
H	1.1005	1.1056	1.1057	1.1070	1.1080	1.1088	1.1086	1.1084	1.1045
x0=−lnG / lnH	99.49	98.70	99.77	101.34	102.12	102.88	103.88	104.35	106.77
S2	0.0627	0.0525	0.0496	0.1355	0.1681	0.3374	0.2086	0.1317	0.1557

(Graph 2a) Trend of H



(Graph 2b) Trend of x0

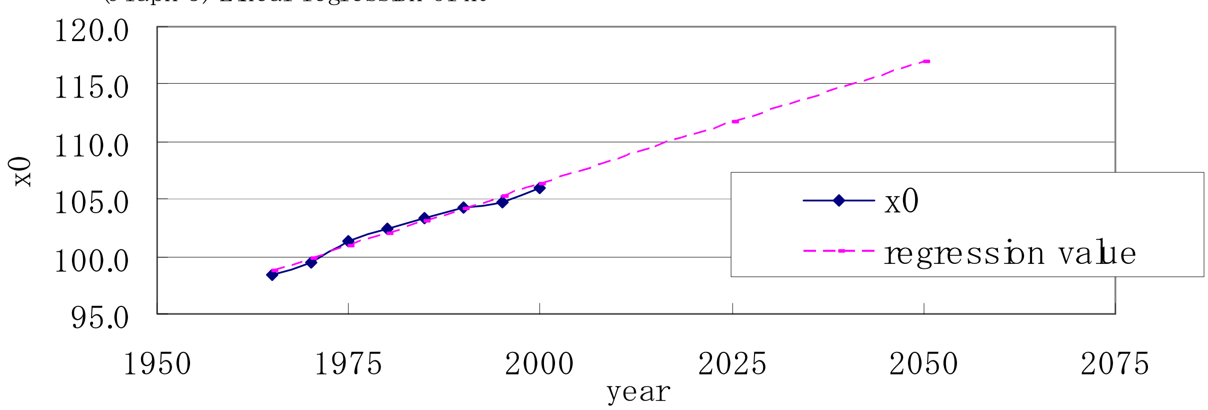


(Table 3) Estimation of H and x0's

H	1.10704
S2	1.3497559

table observation year	JLT12M 1965	JLT13M 1970	JLT14M 1975	JLT15M 1980	JLT16M 1985	JLT17M 1990	JLT18M (ee) 1995	JLT19M 2000
G	0.000045	0.000041	0.000034	0.000030	0.000027	0.000025	0.000024	0.000021
H	1.10704	1.10704	1.10704	1.10704	1.10704	1.10704	1.10704	1.10704
x0=−lnG / lnH	98.337	99.413	101.320	102.391	103.369	104.316	104.735	105.997

(Graph 3) Linear regression of x0



(Table 4) Regression Summary

Regression statistics	
R	0.9887
R2	0.9775
Adjusted R2	0.9738
Standard error	0.4310
Number of obs	8

Analysis of Variance

	Freedom	Difference	Variance	Variance ratio	F-value
Regression	1	48.4247	48.4247	260.7036	0.0000
Residue	6	1.1145	0.1857		
Total	7	49.5392			

	Coefficient	Standard error	t	P-value	lower 95%	upper 95%
Intercept	-323.2626	26.3685	-12.2594	0.0000	-387.7841	-258.7411
X value	0.2148	0.0133	16.1463	0.0000	0.1822	0.2473

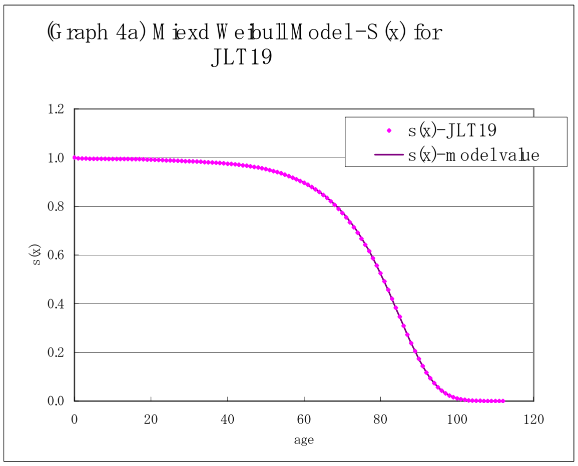
(Table 5) Parameter Summary for 2025

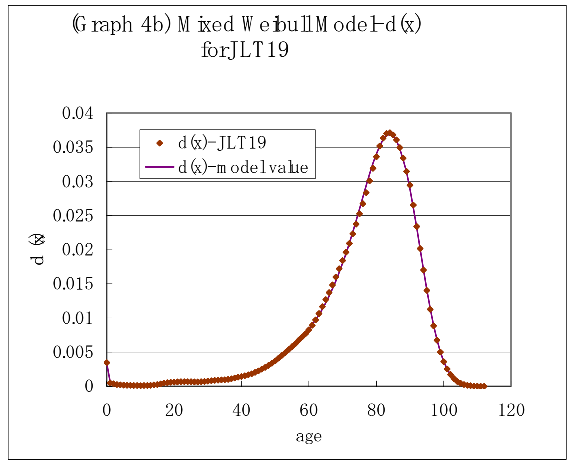
	JLT19(2000)*	2025(projected)
A	0.00067	0.00067
B	0.13884	0.13884
C	0.12678	0.12678
D	0.00047	0.00047
E	6.15186	6.15186
F	23.35454	23.35454
G	0.0000208	0.0000118
H	1.10704	1.10704
x0=-lnG/lnH	105.99694	111.61172
S2	0.80993	

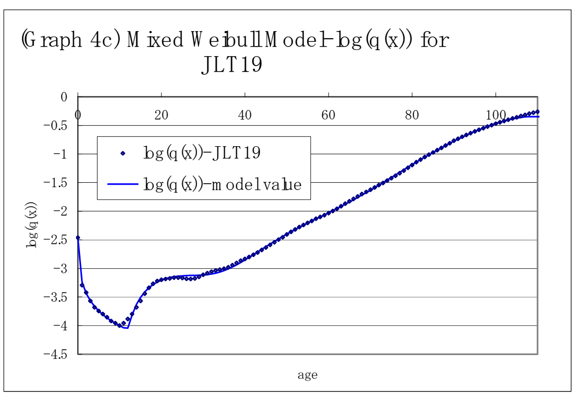
* Parameter A to F are re-estimated using G and H in table 3

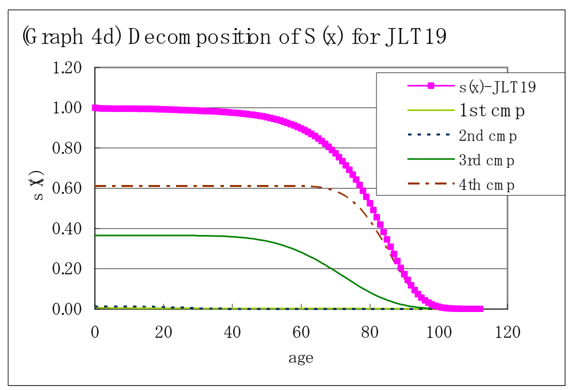
(Table 6) Mixed Weibull Model Parameters for JLT19

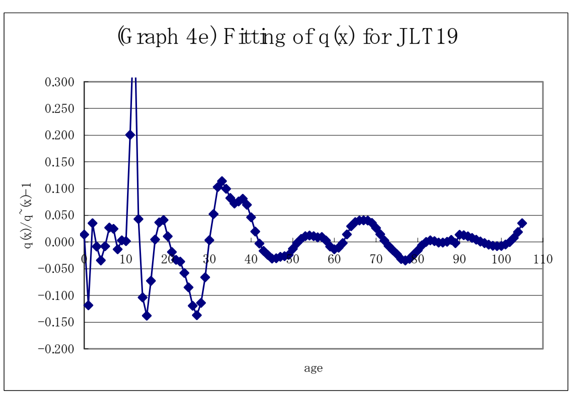
	p	m	t	gamma
1st cmp	0.01016	0.28534	23.20442	0.000010
2nd cmp	0.01247	2.01246	16.78188	12.63157
3rd cmp	0.36521	4.95782	62.23411	12.63157
4th cmp	0.61216	4.95782	37.32033	50.06431

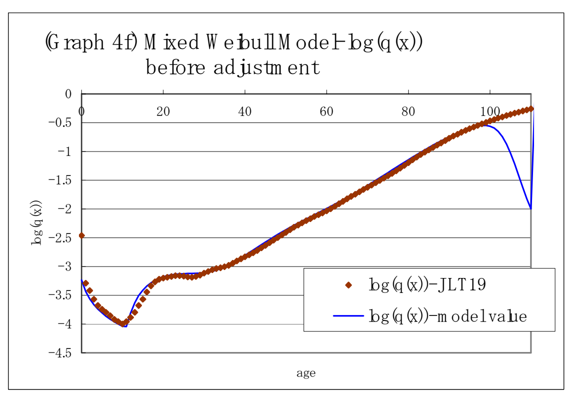


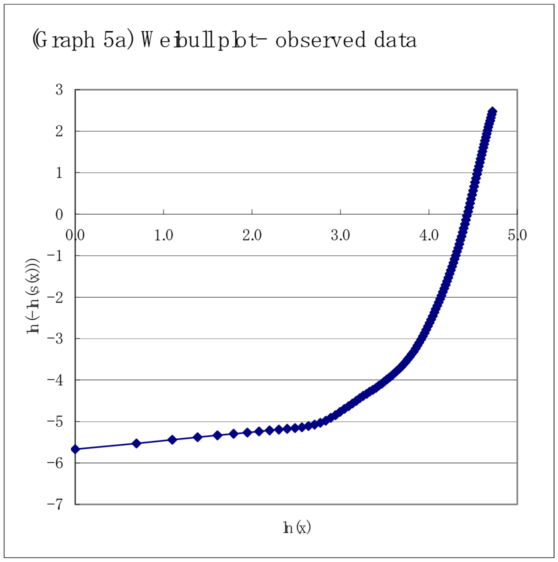


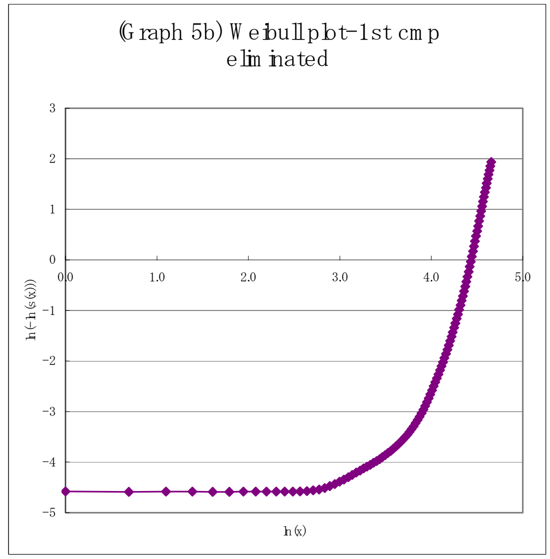


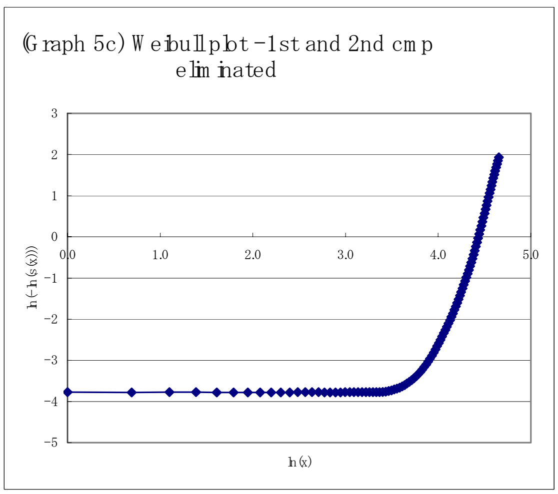


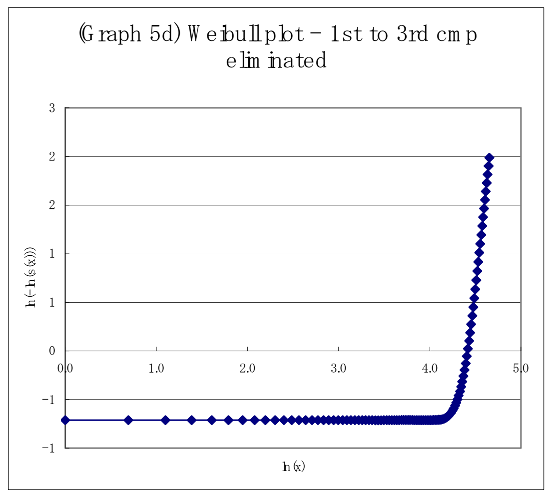





(Graph 5a) Weibull plot- observed data
ln(-ln(s(x)))
ln(x)


(Graph 5b) Weibull plot-1st comp eliminated
ln(-ln(s(x)))
ln(x)

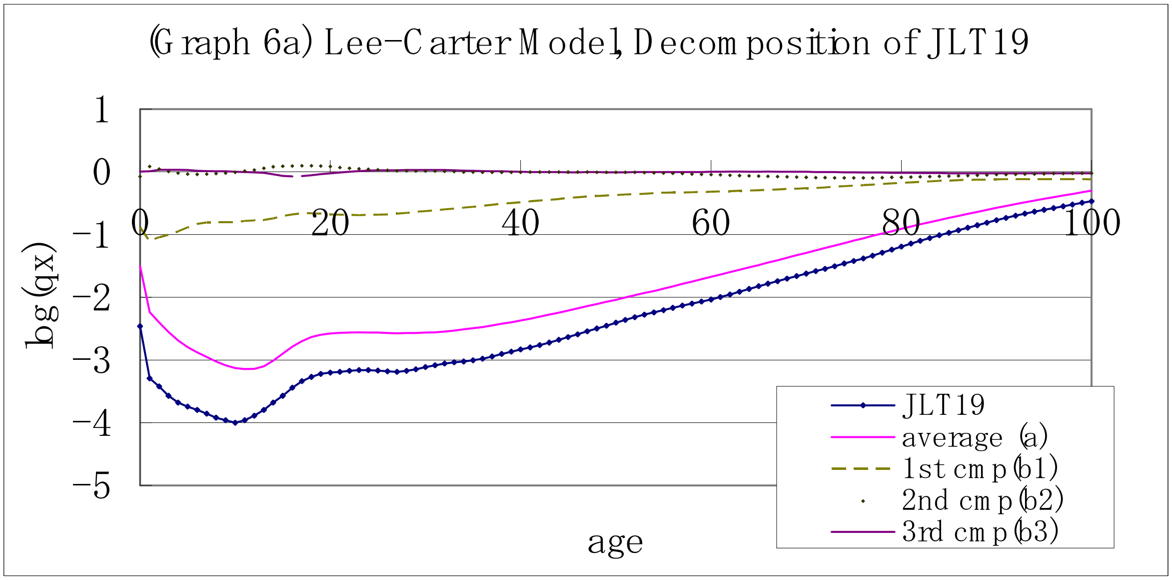

(Graph 5c) Weibull plot -1st and 2nd comp eliminated
ln(-ln(s(x)))
ln(x)

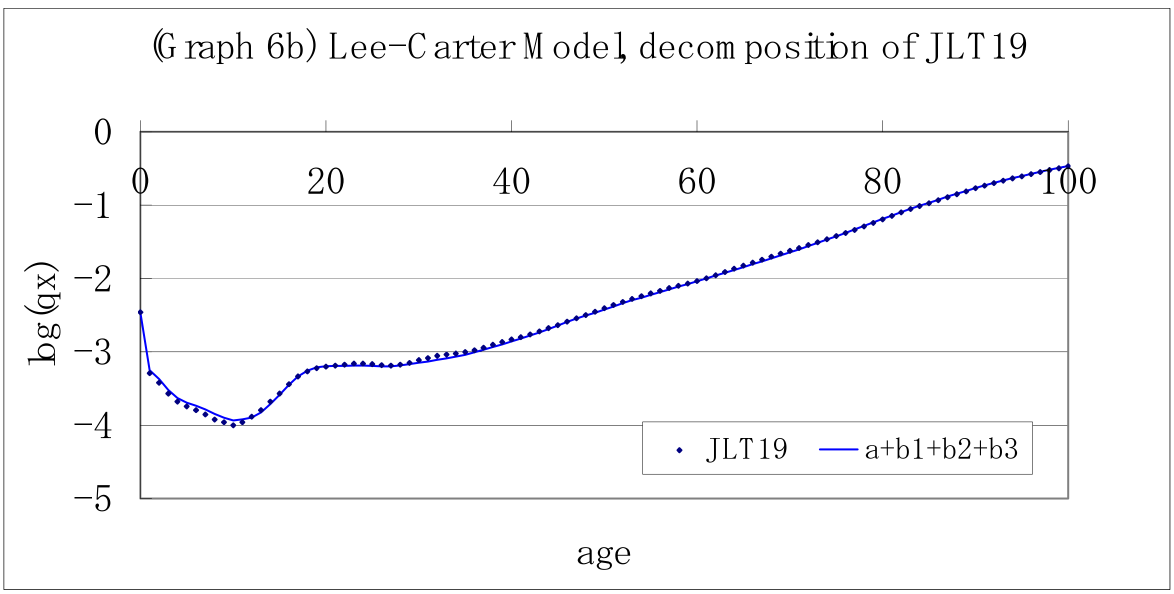

(Graph 5d) Weibull plot - 1st to 3rd comp eliminated
ln(-ln(s(x)))
ln(x)

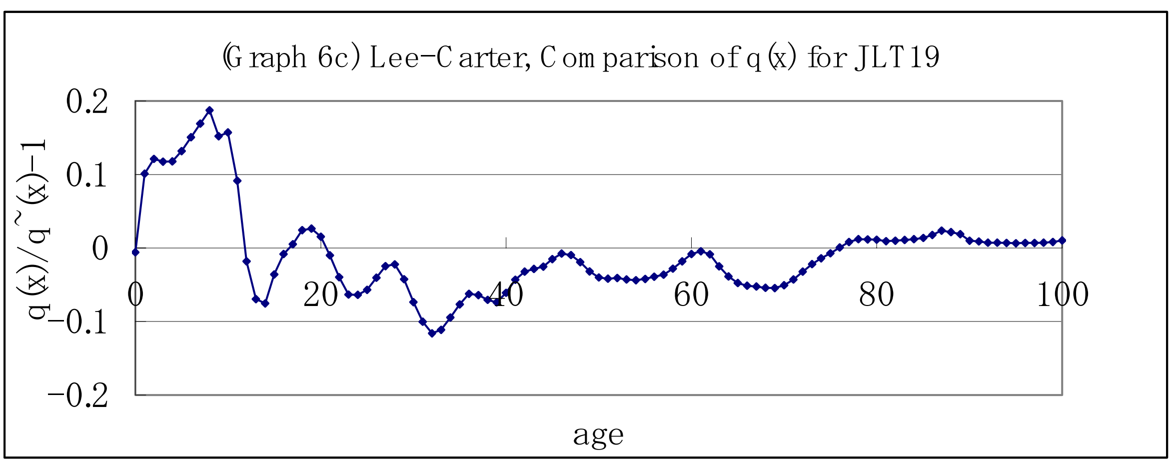
(Table 7) Determinaton of Mixed Weibull Parameters for Projection

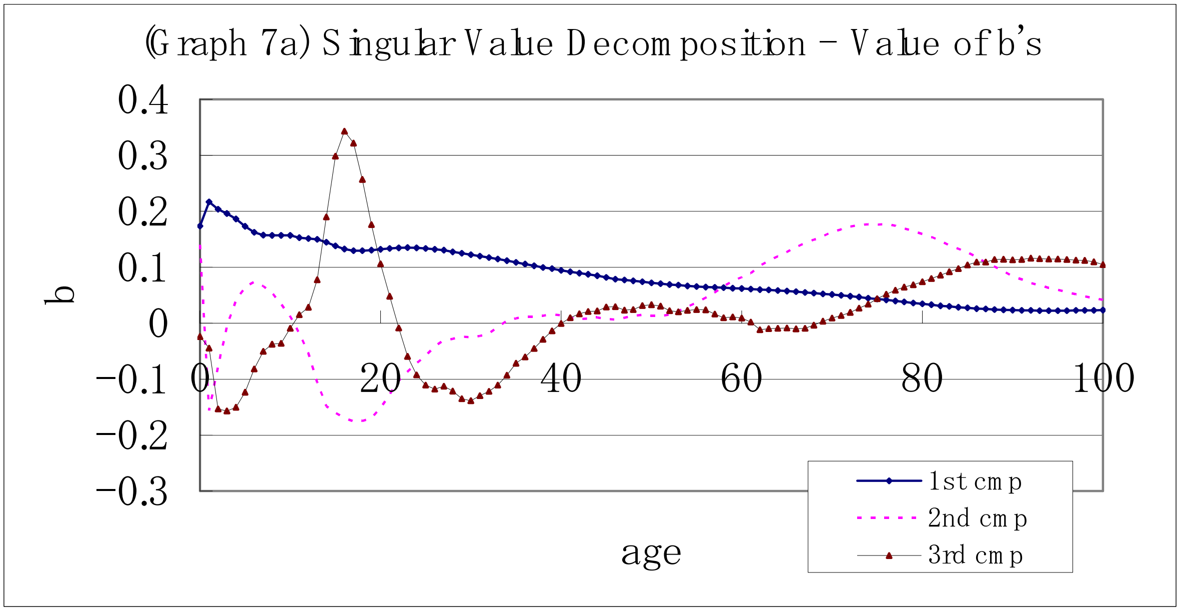
	JLT15	JLT17	JLT19		Y2025
YEAR	1980	1990	2000		
P1	0.0202 *	0.0126 *	0.0088 *		0.0088
P2	0.0131 *	0.0118 *	0.0107 *		0.0107
P3	0.3956 *	0.2833 *	0.3826 *		0.3826
P4	0.5712	0.6923	0.5979		0.5979
m1	0.3217	0.3217	0.3217	0.3217 *	0.3217
m2	2.4152	2.4152	2.4152	2.4152 *	2.4152
m3	4.9390	4.9390	4.9390	4.9390 *	4.9390
m4	4.5308	4.5308	4.5308	4.5308 *	4.5308
t1	9.3670	9.3670	9.3670	9.3670 *	9.3670
t2	16.1416	16.1416	16.1416	16.1416 *	16.1416
t3	59.3891	59.3891	59.3891	59.3891 *	59.3891
t4	33.4673	33.4673	33.4673	33.4673 *	33.4673
γ1	0.0000	0.0000	0.0000		0.0000
γ2	11.0722 *	10.7380 *	11.7634 *		11.7634
γ3	12.5527 *	10.7380 *	14.9981 *		18.0549 #
γ4	50.0153 *	51.7264 *	54.2350 *		59.5096 #

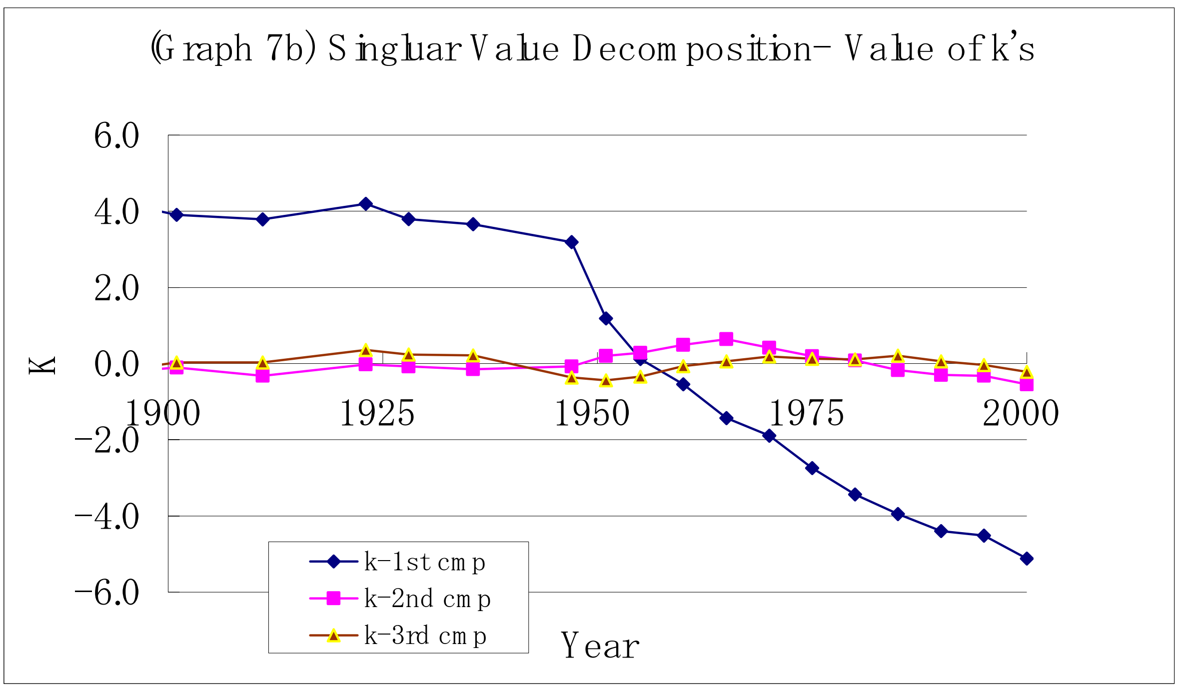
* indicate estimated parameters using LSS.
# projected by linear extrapolation


(Graph 6a) Lee-Carter Model, Decomposition of JLT19
log(qx)
age
JLT19
average (a)
1st cmp (b1)
2nd cmp (b2)
3rd cmp (b3)


(Graph 6b) Lee-Carter Model, decomposition of JLT19
log(qx)
age
JLT19
a+b1+b2+b3


(Graph 6c) Lee-Carter, Comparison of q(x) for JLT19
q(x)/q~(x)-1
age

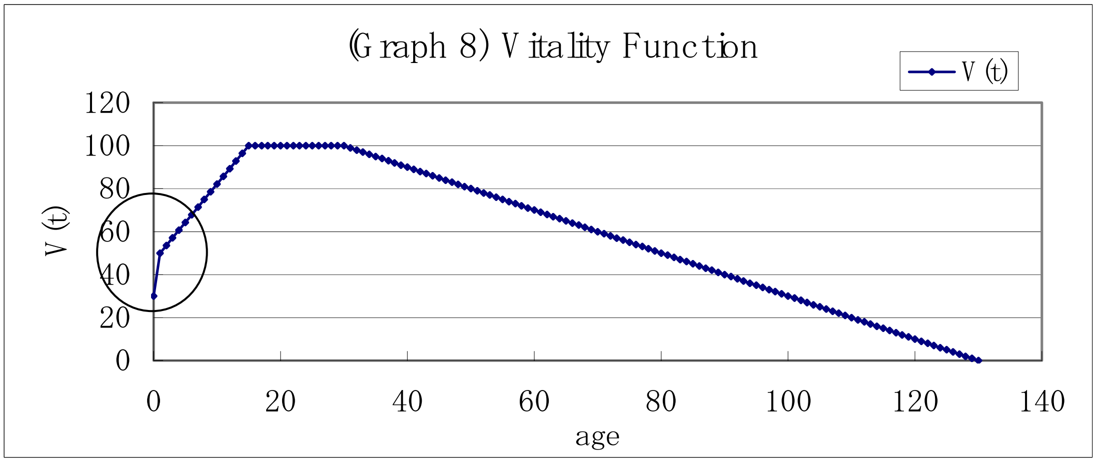

(Graph 7a) Singular Value Decomposition - Value of b's
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
b
0
20
40
60
80
100
age
1st cmp
2nd cmp
3rd cmp

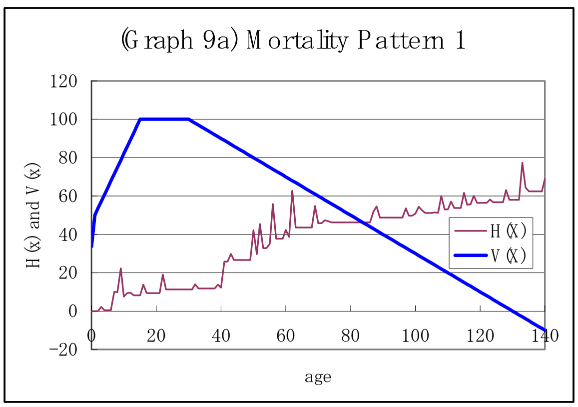

(Graph 7b) Singular Value Decomposition- Value of k's
6.0
4.0
2.0
0.0
-2.0
-4.0
-6.0
K
1900
1925
1950
1975
2000
k-1st cmp
k-2nd cmp
k-3rd cmp
Year

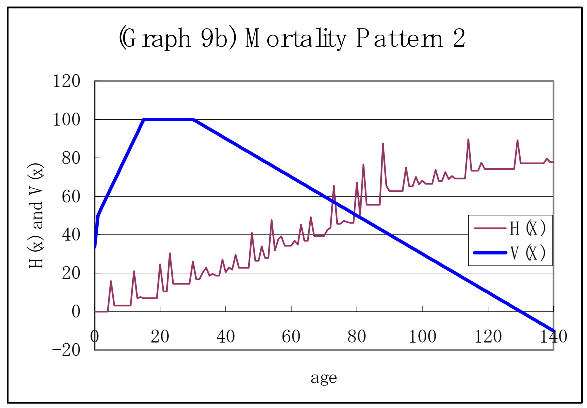
(Table 8) Singular Value Decomposition – trend of k

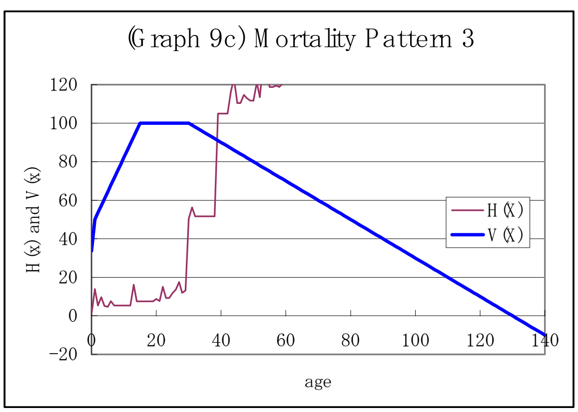
TABLE	Year (average)	k-1st cmp	k-2nd cmp	k-3rd cmp	
JLT01	1895	4.2153	-0.2265	-0.1759	JLT01(1891—98)
JLT02	1901	3.9075	-0.1021	0.0317	JLT02(1899—03)
JLT03	1911	3.7832	-0.3169	0.0331	JLT03(1909—13)
JLT04	1923	4.1952	-0.0241	0.3585	JLT04(1921—25)
JLT05	1928	3.7921	-0.0778	0.2416	JLT05(1926—30)
JLT06	1936	3.6572	-0.1477	0.2142	JLT06(1935—36)
JLT08	1947	3.1910	-0.0761	-0.3637	JLT08(1947)
JLT09	1951	1.1878	0.2036	-0.4380	JLT09(1950—52)
JLT10	1955	0.1175	0.2837	-0.3462	JLT10(1955)
JLT11	1960	-0.5455	0.4889	-0.0663	JLT11(1960)
JLT12	1965	-1.4313	0.6412	0.0596	JLT12(1965)
JLT13	1970	-1.8914	0.4141	0.1876	JLT13(1970)
JLT14	1975	-2.7457	0.1941	0.1284	JLT14(1975)
JLT15	1980	-3.4371	0.0836	0.1131	JLT15(1980)
JLT16	1985	-3.9540	-0.1681	0.2103	JLT16(1985)
JLT17	1990	-4.3997	-0.3002	0.0614	JLT17(1990)
JLT18ee	1995	-4.5198	-0.3235	-0.0338	JLT18ee(1995)
JLT19	2000	-5.1223	-0.5460	-0.2155	JLT19(2000)
projected*	2025	-8.6703	-0.5460	-0.2155	projected for 2025

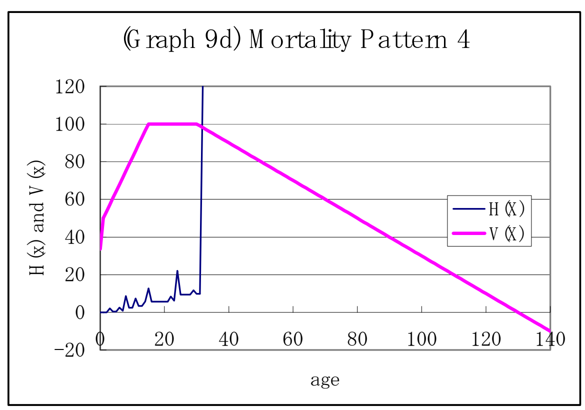
* Linear regression for first component. Same as of year 2000 for second and third components


(Graph 8) Vitality Function
V(t)
V (t)
age
0
20
40
60
80
100
120
140


(Graph 9a) Mortality Pattern 1
H(x) and V(x)
age
H(X)
V(X)


(Graph 9b) Mortality Pattern 2
H(x) and V(x)
age
H(X)
V(X)


(Graph 9c) Mortality Pattern 3
H(x) and V(x)
age
H(X)
V(X)

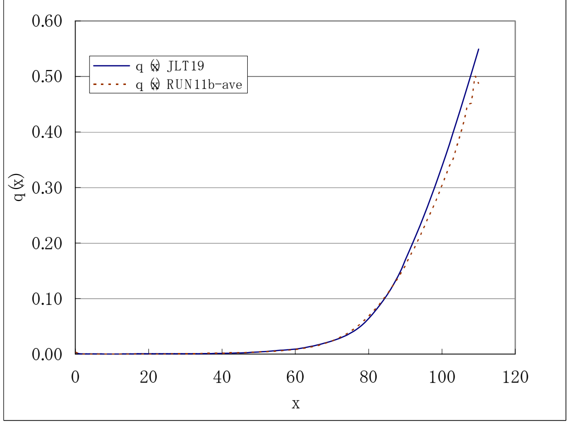

(Graph 9d) Mortality Pattern 4
H(x) and V(x)
age
H(X)
V(X)

(Table 9) Simulation Model, Summary of Parameters- JLT19

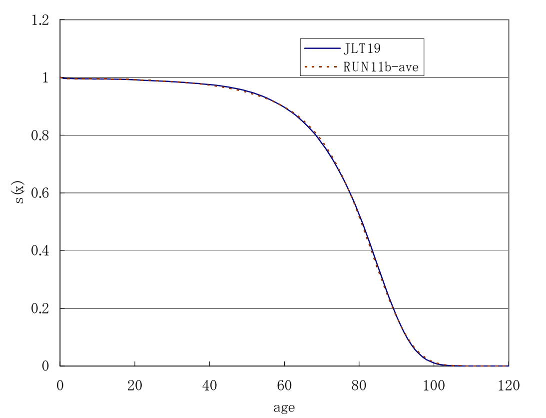
u	33.6417
v	49.5028
p1	0.3320
1/k1	1.4639
I	5.0000
p2	0.0057
1/k2	30.0000
p3	0.00010

e(0)	77.68
JLT19-e(0)	77.73

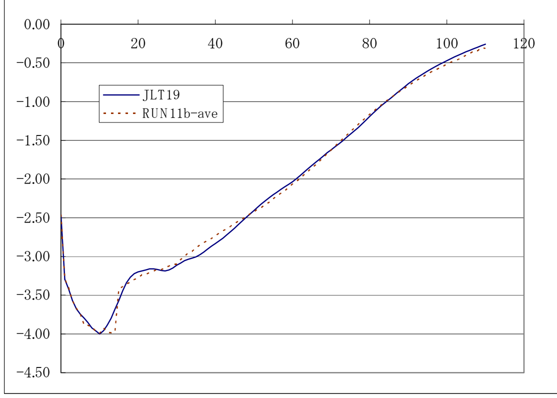
(Graph 10c) Simulation model, q(x)-JLT19



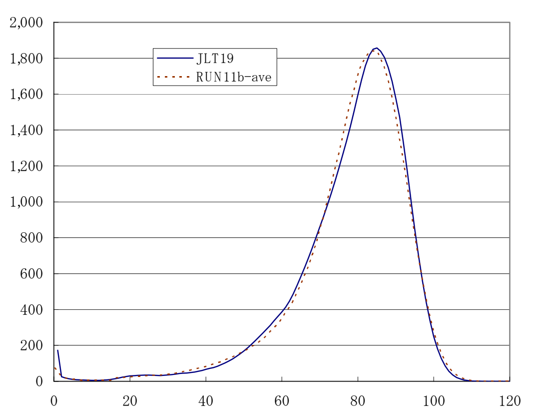
(Graph 10a) Simulation model, s(x)-JLT19



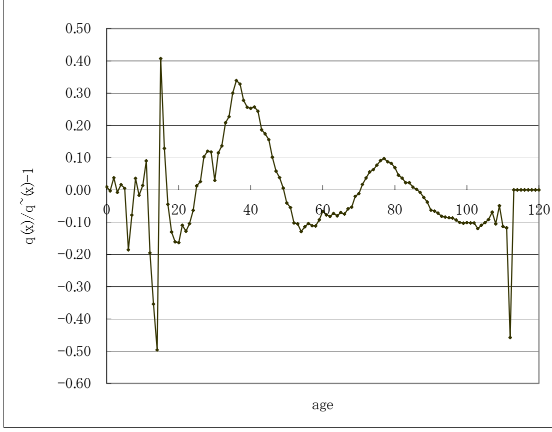
(Graph 10d) Simulation model, log(q(x))-JLT19



(Graph 10b) Simulation model, d(x)-JLT19



(Graph 10e) Simulation model, ratio of q(x)



(Table 10) Simulation Model – Parameter Estimation for Year 2025

	JLT15 1980	JLT17 1990	JLT19 2000	Projected 2025	
u	28.70	31.51	33.64	33.64	(set equal to JLT19)
v	46.13	46.85	49.50	49.50	(set equal to JLT19)
p1	0.3700	0.3500	0.3320	0.2899	(trend*)
I	5	5	5	5	(set equal to JLT19)
1/k1	1.5000	1.4700	1.4639	1.4199	(trend*)
p2	0.0057	0.0057	0.0057	0.0057	(set equal to JLT19)
1/k2	30	30	30	30	(set equal to JLT19)
p3	0.00020	0.00014	0.00010	0.00010	(set equal to JLT19)

* calculated as JLT19*(JLT19/JLT15)^1.25

(Table 11a) Summary of Projected result - q(x)

HP=Heligman-Pollard
MW=Mixed Weibull
LC= Lee-Carter
Sim=Simulation
SP=Simple Projection

SP=Simple Projection
-(a) last 25 years of improvement used
-(b) last 10 years of improvement used

	1975	1990	2000	2025(projected)					
age	JLT14	JLT17	JLT19	HP	MW	LC	Sim	SP-a	SP-b
0	0.01110	0.00495	0.00345	0.00344	0.00242	0.00085	0.00256	0.00107	0.00140
1	0.00140	0.00078	0.00051	0.00050	0.00032	0.00004	0.00040	0.00019	0.00018
2	0.00101	0.00057	0.00038	0.00037	0.00026	0.00003	0.00025	0.00014	0.00014
3	0.00076	0.00042	0.00027	0.00026	0.00018	0.00003	0.00021	0.00010	0.00009
4	0.00062	0.00033	0.00021	0.00020	0.00014	0.00003	0.00016	0.00007	0.00007
5	0.00055	0.00029	0.00018	0.00016	0.00012	0.00003	0.00015	0.00006	0.00005
6	0.00050	0.00025	0.00016	0.00014	0.00011	0.00002	0.00015	0.00005	0.00005
7	0.00044	0.00022	0.00014	0.00012	0.00010	0.00002	0.00012	0.00004	0.00005
8	0.00037	0.00018	0.00012	0.00010	0.00008	0.00002	0.00011	0.00004	0.00004
9	0.00032	0.00015	0.00011	0.00009	0.00008	0.00002	0.00011	0.00004	0.00005
10	0.00028	0.00014	0.00010	0.00007	0.00007	0.00002	0.00010	0.00004	0.00004
11	0.00028	0.00015	0.00011	0.00008	0.00008	0.00002	0.00010	0.00004	0.00005
12	0.00027	0.00016	0.00013	0.00010	0.00010	0.00004	0.00013	0.00006	0.00008
13	0.00029	0.00018	0.00016	0.00013	0.00013	0.00005	0.00016	0.00009	0.00012
14	0.00039	0.00023	0.00021	0.00017	0.00018	0.00008	0.00022	0.00011	0.00017
15	0.00056	0.00034	0.00027	0.00023	0.00023	0.00009	0.00025	0.00013	0.00015
16	0.00075	0.00050	0.00036	0.00031	0.00032	0.00012	0.00035	0.00017	0.00016
17	0.00092	0.00066	0.00046	0.00041	0.00041	0.00016	0.00046	0.00023	0.00019
18	0.00102	0.00078	0.00054	0.00048	0.00048	0.00018	0.00050	0.00029	0.00022
19	0.00105	0.00084	0.00060	0.00054	0.00054	0.00020	0.00056	0.00034	0.00026
20	0.00105	0.00083	0.00063	0.00056	0.00056	0.00021	0.00061	0.00038	0.00032
21	0.00106	0.00081	0.00065	0.00057	0.00057	0.00022	0.00061	0.00040	0.00037
22	0.00107	0.00078	0.00067	0.00059	0.00059	0.00024	0.00061	0.00042	0.00046
23	0.00107	0.00076	0.00069	0.00060	0.00060	0.00026	0.00063	0.00044	0.00054
24	0.00106	0.00075	0.00069	0.00059	0.00060	0.00026	0.00063	0.00045	0.00056
25	0.00105	0.00073	0.00068	0.00056	0.00058	0.00025	0.00061	0.00044	0.00057
26	0.00104	0.00071	0.00066	0.00053	0.00055	0.00024	0.00058	0.00042	0.00055
27	0.00104	0.00071	0.00065	0.00051	0.00053	0.00023	0.00057	0.00041	0.00052
28	0.00106	0.00073	0.00067	0.00051	0.00054	0.00025	0.00059	0.00042	0.00054
29	0.00110	0.00075	0.00071	0.00054	0.00057	0.00028	0.00061	0.00046	0.00062
30	0.00116	0.00078	0.00077	0.00058	0.00062	0.00032	0.00066	0.00051	0.00075
31	0.00122	0.00080	0.00082	0.00061	0.00065	0.00036	0.00072	0.00055	0.00087
32	0.00131	0.00083	0.00088	0.00065	0.00070	0.00040	0.00077	0.00059	0.00102
33	0.00139	0.00087	0.00092	0.00066	0.00071	0.00042	0.00079	0.00061	0.00106
34	0.00147	0.00092	0.00095	0.00066	0.00072	0.00044	0.00084	0.00061	0.00103
35	0.00159	0.00099	0.00099	0.00067	0.00073	0.00045	0.00082	0.00062	0.00099
36	0.00174	0.00107	0.00105	0.00070	0.00076	0.00048	0.00086	0.00063	0.00100
37	0.00192	0.00117	0.00114	0.00075	0.00081	0.00053	0.00096	0.00068	0.00107
38	0.00212	0.00129	0.00125	0.00082	0.00088	0.00060	0.00106	0.00074	0.00116
39	0.00234	0.00140	0.00136	0.00088	0.00094	0.00067	0.00113	0.00079	0.00126
40	0.00259	0.00155	0.00147	0.00094	0.00100	0.00073	0.00122	0.00083	0.00129
41	0.00287	0.00172	0.00159	0.00100	0.00107	0.00079	0.00130	0.00088	0.00131
42	0.00318	0.00188	0.00173	0.00108	0.00115	0.00086	0.00140	0.00094	0.00141
43	0.00349	0.00206	0.00190	0.00118	0.00126	0.00096	0.00159	0.00103	0.00155
44	0.00378	0.00228	0.00210	0.00131	0.00139	0.00108	0.00174	0.00117	0.00171
45	0.00404	0.00254	0.00232	0.00144	0.00154	0.00121	0.00192	0.00133	0.00185
46	0.00431	0.00284	0.00258	0.00161	0.00173	0.00136	0.00212	0.00154	0.00203
47	0.00460	0.00315	0.00287	0.00179	0.00194	0.00154	0.00234	0.00179	0.00227
48	0.00491	0.00344	0.00318	0.00199	0.00217	0.00174	0.00255	0.00206	0.00261
49	0.00529	0.00372	0.00352	0.00220	0.00243	0.00197	0.00285	0.00234	0.00307
50	0.00573	0.00405	0.00392	0.00246	0.00275	0.00224	0.00323	0.00268	0.00361
51	0.00618	0.00446	0.00435	0.00274	0.00309	0.00252	0.00354	0.00306	0.00409
52	0.00667	0.00495	0.00480	0.00302	0.00345	0.00281	0.00399	0.00345	0.00444
53	0.00723	0.00557	0.00527	0.00330	0.00384	0.00312	0.00433	0.00384	0.00459
54	0.00790	0.00630	0.00575	0.00357	0.00424	0.00344	0.00469	0.00419	0.00458
55	0.00872	0.00710	0.00625	0.00383	0.00465	0.00377	0.00500	0.00448	0.00454
56	0.00971	0.00791	0.00678	0.00411	0.00510	0.00411	0.00536	0.00473	0.00461
57	0.01074	0.00873	0.00737	0.00442	0.00560	0.00448	0.00577	0.00506	0.00483
58	0.01174	0.00954	0.00795	0.00468	0.00607	0.00482	0.00606	0.00538	0.00504
59	0.01275	0.01040	0.00854	0.00493	0.00654	0.00518	0.00642	0.00572	0.00522

	1975	1990	2000	2025(projected)					
age	JLT14	JLT17	JLT19	HP	MW	LC	Sim	SP-a	SP-b
60	0.01393	0.01132	0.00923	0.00524	0.00706	0.00560	0.00674	0.00612	0.00554
61	0.01534	0.01228	0.01007	0.00566	0.00770	0.00614	0.00739	0.00661	0.00613
62	0.01701	0.01323	0.01106	0.00618	0.00843	0.00681	0.00805	0.00719	0.00707
63	0.01889	0.01424	0.01226	0.00687	0.00931	0.00768	0.00857	0.00796	0.00843
64	0.02098	0.01536	0.01359	0.00764	0.01025	0.00864	0.00944	0.00880	0.01001
65	0.02351	0.01664	0.01498	0.00840	0.01117	0.00965	0.01020	0.00954	0.01152
66	0.02649	0.01805	0.01646	0.00920	0.01210	0.01072	0.01105	0.01023	0.01307
67	0.02975	0.01968	0.01808	0.01006	0.01306	0.01189	0.01194	0.01099	0.01463
68	0.03310	0.02160	0.01988	0.01103	0.01410	0.01320	0.01294	0.01194	0.01616
69	0.03655	0.02382	0.02181	0.01205	0.01514	0.01462	0.01358	0.01301	0.01750
70	0.04024	0.02641	0.02384	0.01308	0.01615	0.01609	0.01456	0.01412	0.01846
71	0.04453	0.02946	0.02605	0.01419	0.01720	0.01770	0.01561	0.01524	0.01915
72	0.04958	0.03292	0.02850	0.01543	0.01834	0.01952	0.01640	0.01638	0.01988
73	0.05507	0.03679	0.03126	0.01686	0.01963	0.02159	0.01780	0.01774	0.02080
74	0.06094	0.04095	0.03437	0.01852	0.02109	0.02398	0.01940	0.01938	0.02218
75	0.06700	0.04542	0.03784	0.02041	0.02273	0.02665	0.02123	0.02137	0.02397
76	0.07365	0.05055	0.04162	0.02245	0.02449	0.02963	0.02318	0.02352	0.02560
77	0.08137	0.05661	0.04606	0.02500	0.02673	0.03317	0.02525	0.02607	0.02750
78	0.09010	0.06367	0.05127	0.02816	0.02954	0.03742	0.02853	0.02917	0.02983
79	0.09980	0.07148	0.05731	0.03196	0.03298	0.04244	0.03216	0.03291	0.03299
80	0.11039	0.08039	0.06401	0.03623	0.03690	0.04811	0.03691	0.03712	0.03621
81	0.12161	0.08997	0.07156	0.04115	0.04148	0.05461	0.04227	0.04211	0.04037
82	0.13272	0.09997	0.07962	0.04637	0.04640	0.06156	0.04714	0.04776	0.04507
83	0.14407	0.11032	0.08813	0.05181	0.05161	0.06893	0.05388	0.05391	0.05027
84	0.15618	0.12106	0.09699	0.05738	0.05702	0.07665	0.05999	0.06023	0.05572
85	0.16957	0.13240	0.10640	0.06326	0.06284	0.08491	0.06717	0.06676	0.06160
86	0.18384	0.14472	0.11678	0.06988	0.06954	0.09395	0.07419	0.07418	0.06831
87	0.19851	0.15847	0.12806	0.07715	0.07704	0.10358	0.08316	0.08261	0.07518
88	0.21345	0.17309	0.14042	0.08526	0.08558	0.11442	0.09372	0.09238	0.08324
89	0.22841	0.18816	0.15378	0.09414	0.09447	0.12592	0.10477	0.10353	0.09286
90	0.24225	0.20323	0.17013	0.10578	0.10680	0.14041	0.11853	0.11948	0.10908
91	0.25905	0.21957	0.18465	0.11539	0.11730	0.15300	0.12980	0.13162	0.11975
92	0.27513	0.23665	0.19968	0.12531	0.12834	0.16565	0.14252	0.14492	0.13059
93	0.29154	0.25449	0.21523	0.13560	0.13995	0.17892	0.15625	0.15889	0.14157
94	0.30829	0.27310	0.23129	0.14628	0.15216	0.19223	0.17064	0.17352	0.15267
95	0.32537	0.29247	0.24787	0.15739	0.16501	0.20622	0.18426	0.18883	0.16390
96	0.34275	0.31259	0.26496	0.16897	0.17854	0.22027	0.19953	0.20483	0.17526
97	0.36042	0.33347	0.28255	0.18107	0.19278	0.23444	0.21613	0.22150	0.18672
98	0.37838	0.35507	0.30065	0.19375	0.20778	0.24915	0.23514	0.23889	0.19835
99	0.39660	0.37738	0.31923	0.20707	0.22357	0.26416	0.25173	0.25695	0.21010
100	0.41505	0.40037	0.33828	0.22107	0.24021	0.27895	0.26779	0.27571	0.22198
101	0.43373	0.42399	0.35778	0.23580	0.25775	0.29502	0.28647	0.29513	0.23403
102	0.45259	0.44821	0.37772	0.25134	0.27628	0.31147	0.30857	0.31524	0.24626
103	0.47163	0.47296	0.39808	0.26773	0.29592	0.32826	0.33277	0.33600	0.25872
104	0.49081	0.49819	0.41881	0.28499	0.31675	0.34535	0.35045	0.35737	0.27138
105	0.51009	0.52381	0.43989	0.30317	0.33897	0.36273	0.36516	0.37935	0.28430
106	0.52946	0.54974	0.46129	0.32228	0.36280	0.38038	0.38229	0.40190	0.29752
107	0.54887	0.57589	0.48297	0.34234	0.38854	0.39826	0.38872	0.42498	0.31108
108	0.56829	0.60215	0.50487	0.36331	0.41652	0.41631	0.43348	0.44853	0.32499
109	1.00000	0.62842	0.52696	0.38518	0.44717	0.43453	0.42112	0.46815	0.33921
110		1.00000	0.54918	0.40790	0.48084	0.45285	0.48116	0.48789	0.35351
111			0.57147	0.43139	0.51774	0.47123	0.48638	0.50770	0.36786
112			0.59378	0.45559	0.55768	0.48963	0.50269	0.52752	0.38222
113			1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

* To calculate e(x) for LC, mortality at age 101 and over are extrapolated by using the ratio q(x+1)/q(x) of JLT19
Same extrapolation is applied for SP-a and SP-b at age 109 and over.

	1975	1990	2000	2025(projected)					
	JLT14	JLT17	JLT19	HP	MW	LC	Sim	SP-a	SP-b
e(0)	71.73	75.92	77.73	83.33	82.31	82.17	82.00	82.95	82.22
improvement*			6.00	5.60	4.58	4.44	4.27	5.23	4.49
e(50)	25.56	28.40	29.91	35.18	34.07	33.13	34.00	34.28	33.98
improvement*			4.35	5.26	4.16	3.22	4.09	4.37	4.07
e(70)	10.53	12.66	13.97	18.07	17.66	15.99	17.51	17.47	17.59
improvement*			3.44	4.10	3.69	2.02	3.54	3.50	3.62
e(80)	6.09	7.36	8.48	11.58	11.47	9.81	11.05	11.01	11.59
improvement*			2.39	3.11	2.99	1.33	2.57	2.53	3.11

* The difference between year 2025 and year 2000 (or between 2000 and year 1975 for JLT19)

(Table 11b) Summary of Model Result-q(x)

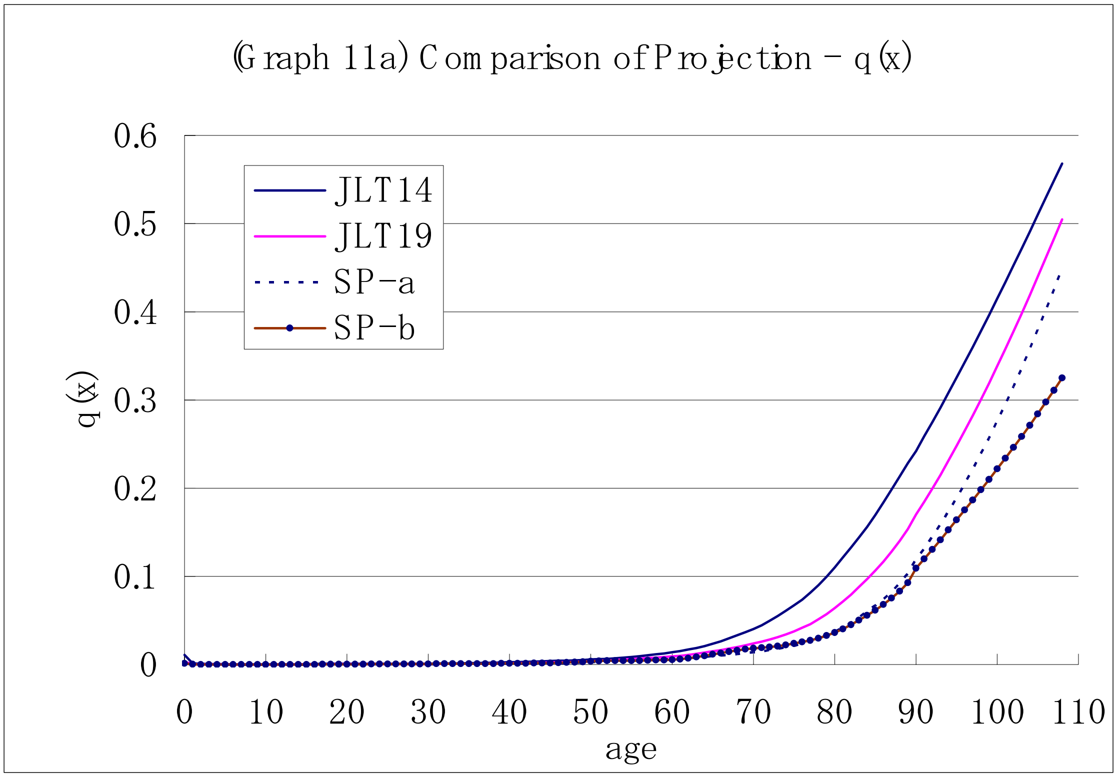
	Model Parameters are found in the follows
HP	(Table 5)
MW	(Table 7)
LC	(Table 8)
Sim	(Table 10)

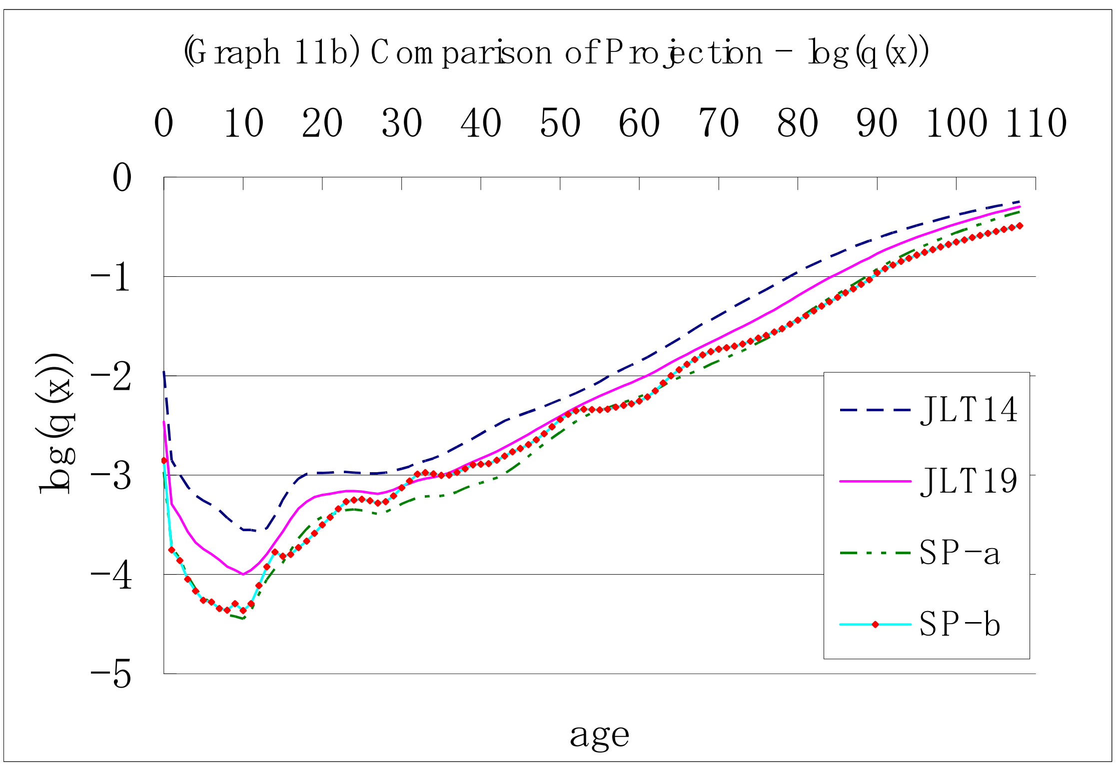
Model result, year 2000				
age	HP	MW	LC	Sim
0	0.00338	0.00338	0.00343	0.00348
1	0.00061	0.00062	0.00056	0.00051
2	0.00034	0.00039	0.00043	0.00039
3	0.00024	0.00029	0.00030	0.00027
4	0.00019	0.00023	0.00023	0.00021
5	0.00016	0.00019	0.00020	0.00018
6	0.00014	0.00016	0.00018	0.00013
7	0.00013	0.00014	0.00016	0.00013
8	0.00012	0.00012	0.00014	0.00012
9	0.00012	0.00011	0.00013	0.00011
10	0.00012	0.00010	0.00012	0.00010
11	0.00013	0.00009	0.00012	0.00012
12	0.00015	0.00010	0.00013	0.00010
13	0.00017	0.00015	0.00015	0.00010
14	0.00022	0.00020	0.00019	0.00011
15	0.00027	0.00026	0.00026	0.00038
16	0.00033	0.00033	0.00036	0.00041
17	0.00040	0.00040	0.00046	0.00044
18	0.00046	0.00047	0.00055	0.00047
19	0.00053	0.00053	0.00062	0.00050
20	0.00059	0.00059	0.00064	0.00053
21	0.00063	0.00064	0.00064	0.00058
22	0.00067	0.00068	0.00064	0.00058
23	0.00070	0.00071	0.00065	0.00062
24	0.00072	0.00072	0.00065	0.00065
25	0.00074	0.00073	0.00064	0.00069
26	0.00074	0.00073	0.00063	0.00068
27	0.00075	0.00072	0.00063	0.00072
28	0.00075	0.00070	0.00066	0.00075
29	0.00076	0.00069	0.00068	0.00079
30	0.00077	0.00067	0.00071	0.00079
31	0.00078	0.00066	0.00074	0.00091
32	0.00080	0.00066	0.00078	0.00100
33	0.00083	0.00067	0.00082	0.00111
34	0.00087	0.00070	0.00086	0.00117
35	0.00091	0.00074	0.00091	0.00129
36	0.00097	0.00080	0.00098	0.00141
37	0.00103	0.00087	0.00107	0.00151
38	0.00111	0.00097	0.00116	0.00160
39	0.00120	0.00109	0.00126	0.00171
40	0.00130	0.00124	0.00138	0.00184
41	0.00142	0.00141	0.00152	0.00200
42	0.00155	0.00160	0.00167	0.00215
43	0.00170	0.00181	0.00185	0.00225
44	0.00187	0.00206	0.00205	0.00247
45	0.00206	0.00233	0.00228	0.00268
46	0.00227	0.00262	0.00256	0.00284
47	0.00251	0.00294	0.00284	0.00304
48	0.00277	0.00329	0.00312	0.00330
49	0.00306	0.00367	0.00341	0.00354
50	0.00338	0.00408	0.00376	0.00376
51	0.00373	0.00451	0.00417	0.00411
52	0.00413	0.00497	0.00460	0.00431
53	0.00456	0.00546	0.00504	0.00472
54	0.00505	0.00598	0.00550	0.00501
55	0.00558	0.00652	0.00599	0.00553
56	0.00617	0.00709	0.00651	0.00607
57	0.00683	0.00770	0.00710	0.00655
58	0.00755	0.00834	0.00773	0.00706
59	0.00835	0.00903	0.00839	0.00775

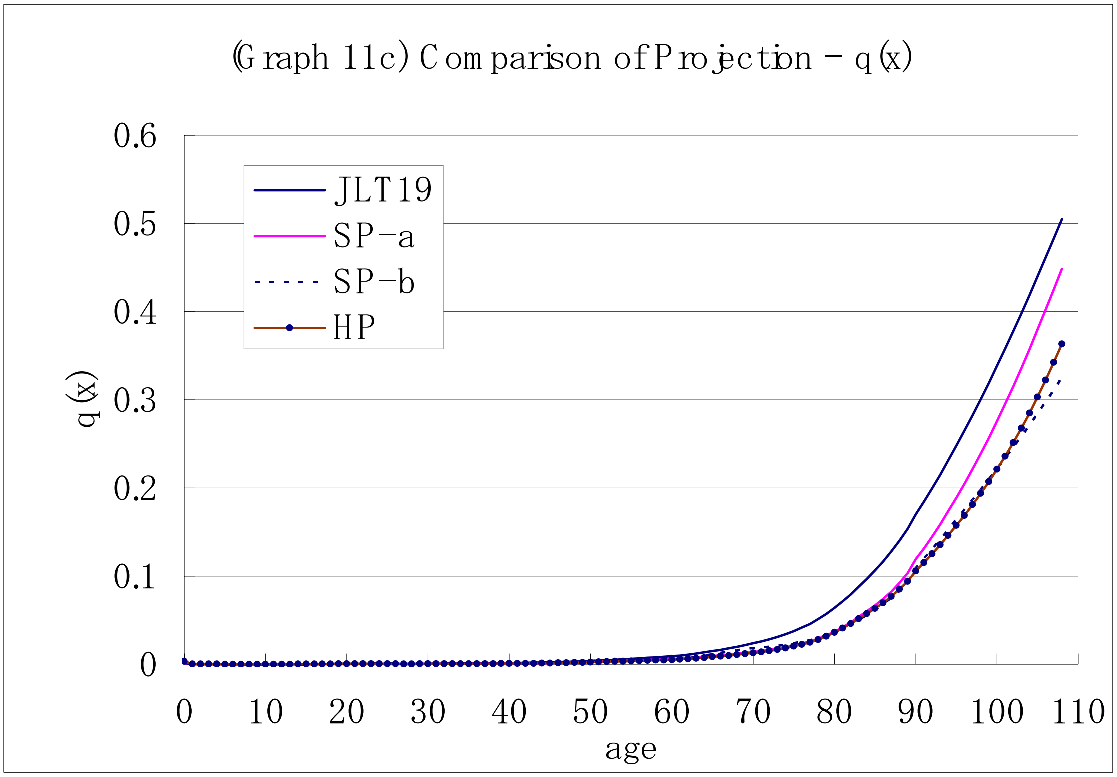
Model result, year 2025				
age	HP	MW	LC	Sim
0	0.00337	0.00235	0.00083	0.00259
1	0.00060	0.00043	0.00010	0.00040
2	0.00033	0.00027	0.00008	0.00026
3	0.00023	0.00020	0.00006	0.00020
4	0.00018	0.00016	0.00005	0.00016
5	0.00014	0.00013	0.00005	0.00015
6	0.00012	0.00011	0.00005	0.00012
7	0.00011	0.00010	0.00005	0.00011
8	0.00010	0.00009	0.00004	0.00011
9	0.00009	0.00008	0.00004	0.00010
10	0.00009	0.00007	0.00003	0.00010
11	0.00010	0.00006	0.00003	0.00011
12	0.00011	0.00008	0.00004	0.00010
13	0.00014	0.00012	0.00004	0.00010
14	0.00018	0.00017	0.00006	0.00011
15	0.00023	0.00022	0.00008	0.00036
16	0.00028	0.00029	0.00012	0.00039
17	0.00035	0.00035	0.00016	0.00044
18	0.00041	0.00041	0.00019	0.00043
19	0.00047	0.00047	0.00021	0.00046
20	0.00052	0.00052	0.00022	0.00051
21	0.00056	0.00057	0.00022	0.00053
22	0.00059	0.00060	0.00021	0.00052
23	0.00061	0.00062	0.00021	0.00056
24	0.00062	0.00063	0.00022	0.00059
25	0.00062	0.00063	0.00022	0.00062
26	0.00062	0.00062	0.00022	0.00060
27	0.00061	0.00060	0.00022	0.00064
28	0.00060	0.00058	0.00023	0.00067
29	0.00059	0.00055	0.00024	0.00070
30	0.00058	0.00052	0.00026	0.00069
31	0.00057	0.00050	0.00028	0.00082
32	0.00057	0.00048	0.00030	0.00089
33	0.00057	0.00047	0.00032	0.00098
34	0.00058	0.00047	0.00035	0.00105
35	0.00059	0.00048	0.00038	0.00111
36	0.00061	0.00050	0.00042	0.00122
37	0.00064	0.00054	0.00046	0.00134
38	0.00068	0.00060	0.00051	0.00141
39	0.00072	0.00068	0.00057	0.00148
40	0.00077	0.00077	0.00064	0.00159
41	0.00084	0.00088	0.00072	0.00170
42	0.00091	0.00102	0.00081	0.00182
43	0.00099	0.00117	0.00090	0.00195
44	0.00108	0.00135	0.00102	0.00210
45	0.00118	0.00155	0.00117	0.00228
46	0.00130	0.00177	0.00134	0.00238
47	0.00143	0.00202	0.00152	0.00250
48	0.00158	0.00229	0.00168	0.00267
49	0.00174	0.00258	0.00186	0.00287
50	0.00192	0.00290	0.00208	0.00307
51	0.00212	0.00325	0.00234	0.00331
52	0.00234	0.00363	0.00262	0.00350
53	0.00259	0.00403	0.00290	0.00378
54	0.00286	0.00447	0.00319	0.00394
55	0.00316	0.00492	0.00351	0.00429
56	0.00350	0.00541	0.00384	0.00466
57	0.00387	0.00592	0.00421	0.00495
58	0.00428	0.00646	0.00460	0.00517
59	0.00474	0.00702	0.00502	0.00562

Model result, year 2000				
age	HP	MW	LC	Sim
60	0.00923	0.00977	0.00916	0.00861
61	0.01021	0.01058	0.01002	0.00929
62	0.01129	0.01147	0.01096	0.01015
63	0.01248	0.01245	0.01196	0.01137
64	0.01380	0.01353	0.01306	0.01250
65	0.01525	0.01474	0.01427	0.01393
66	0.01686	0.01609	0.01561	0.01523
67	0.01863	0.01759	0.01713	0.01702
68	0.02058	0.01929	0.01880	0.01882
69	0.02273	0.02119	0.02062	0.02137
70	0.02510	0.02333	0.02263	0.02356
71	0.02771	0.02573	0.02494	0.02649
72	0.03058	0.02843	0.02758	0.02956
73	0.03375	0.03146	0.03057	0.03301
74	0.03722	0.03485	0.03389	0.03653
75	0.04104	0.03865	0.03756	0.04074
76	0.04524	0.04288	0.04165	0.04540
77	0.04984	0.04759	0.04644	0.05054
78	0.05488	0.05281	0.05188	0.05573
79	0.06039	0.05858	0.05798	0.06203
80	0.06643	0.06494	0.06474	0.06842
81	0.07302	0.07191	0.07223	0.07483
82	0.08020	0.07953	0.08040	0.08252
83	0.08803	0.08783	0.08909	0.09009
84	0.09654	0.09682	0.09815	0.09914
85	0.10578	0.10654	0.10787	0.10732
86	0.11579	0.11699	0.11887	0.11702
87	0.12661	0.12819	0.13109	0.12706
88	0.13829	0.14012	0.14345	0.13713
89	0.15086	0.15358	0.15668	0.14803
90	0.16435	0.16720	0.17182	0.15942
91	0.17879	0.18159	0.18631	0.17255
92	0.19421	0.19674	0.20115	0.18536
93	0.21061	0.21261	0.21678	0.19755
94	0.22801	0.22920	0.23289	0.21181
95	0.24640	0.24646	0.24949	0.22650
96	0.26577	0.26435	0.26678	0.24185
97	0.28607	0.28282	0.28445	0.25611
98	0.30728	0.30180	0.30284	0.27024
99	0.32933	0.32122	0.32181	0.28617
100	0.35217	0.34099	0.34174	0.30399
101	0.37569	0.36099		0.32117
102	0.39982	0.38110		0.33898
103	0.42445	0.40113		0.35039
104	0.44946	0.42090		0.37298
105	0.47473	0.44013		0.39535
106	0.50012	0.45850		0.41893
107	0.52552	0.47560		0.44971
108	0.55078	0.49095		0.45165
109	0.57579	0.50400		0.50108
110	0.60041	0.51420		0.48707
111	0.62453	0.52114		0.50420
112	0.64806	0.52478		0.54237

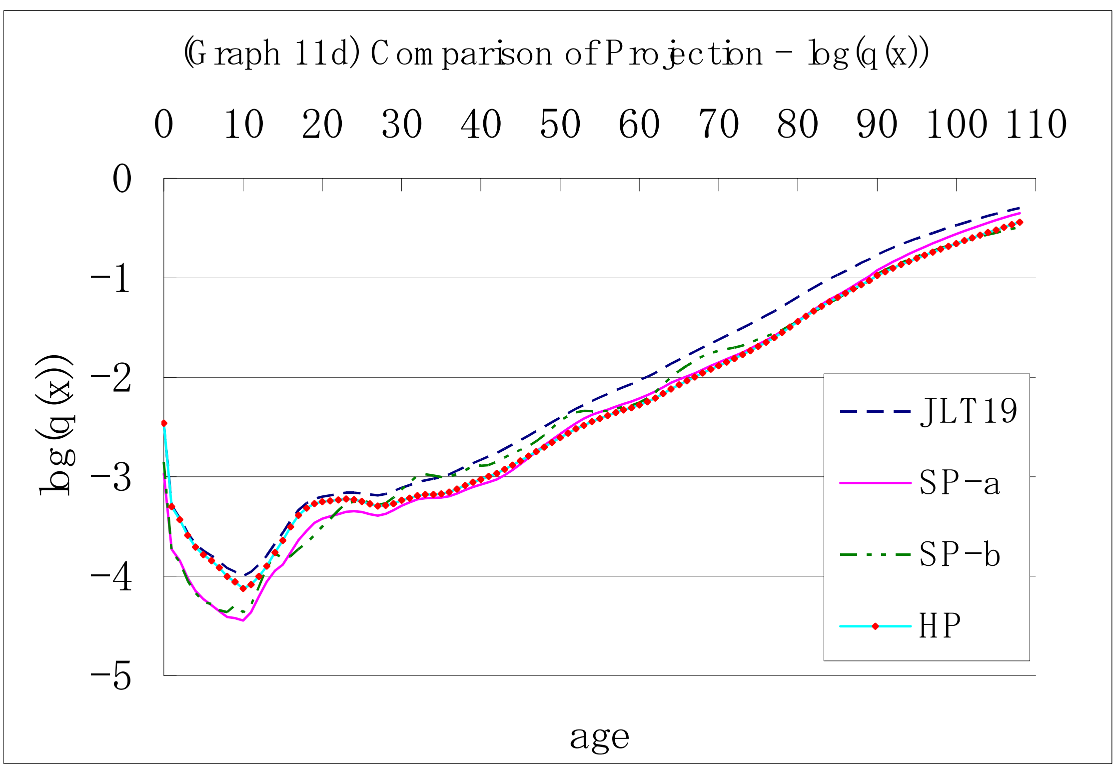
Model result, year 2025				
age	HP	MW	LC	Sim
60	0.00524	0.00761	0.00553	0.00613
61	0.00580	0.00821	0.00610	0.00661
62	0.00641	0.00884	0.00672	0.00714
63	0.00709	0.00950	0.00737	0.00767
64	0.00784	0.01019	0.00812	0.00834
65	0.00868	0.01093	0.00894	0.00915
66	0.00959	0.01172	0.00987	0.00982
67	0.01061	0.01258	0.01094	0.01088
68	0.01173	0.01350	0.01213	0.01188
69	0.01297	0.01452	0.01343	0.01314
70	0.01434	0.01564	0.01489	0.01429
71	0.01585	0.01688	0.01659	0.01605
72	0.01751	0.01827	0.01860	0.01747
73	0.01935	0.01983	0.02091	0.01955
74	0.02138	0.02157	0.02350	0.02156
75	0.02361	0.02354	0.02638	0.02413
76	0.02607	0.02576	0.02966	0.02696
77	0.02878	0.02826	0.03356	0.02974
78	0.03176	0.03108	0.03803	0.03299
79	0.03504	0.03425	0.04311	0.03688
80	0.03865	0.03783	0.04884	0.04132
81	0.04260	0.04183	0.05527	0.04554
82	0.04695	0.04631	0.06234	0.05004
83	0.05171	0.05130	0.06989	0.05583
84	0.05693	0.05685	0.07781	0.06214
85	0.06264	0.06299	0.08639	0.06809
86	0.06889	0.06975	0.09604	0.07444
87	0.07570	0.07717	0.10661	0.08216
88	0.08313	0.08528	0.11745	0.09043
89	0.09121	0.09427	0.12882	0.09902
90	0.10000	0.10388	0.14210	0.10782
91	0.10953	0.11424	0.15466	0.11770
92	0.11984	0.12539	0.16712	0.12820
93	0.13099	0.13733	0.18047	0.13857
94	0.14300	0.15007	0.19383	0.15116
95	0.15592	0.16361	0.20783	0.16289
96	0.16977	0.17793	0.22208	0.17643
97	0.18459	0.19305	0.23633	0.18970
98	0.20038	0.20893	0.25135	0.20472
99	0.21717	0.22556	0.26675	0.21867
100	0.23495	0.24291	0.28240	0.23350
101	0.25372	0.26096		0.24986
102	0.27344	0.27966		0.26984
103	0.29410	0.29897		0.28508
104	0.31564	0.31884		0.30462
105	0.33800	0.33921		0.32062
106	0.36111	0.36001		0.33993
107	0.38488	0.38117		0.35546
108	0.40922	0.40261		0.38026
109	0.43400	0.42421		0.39524
110	0.45913	0.44586		0.41904
111	0.48446	0.46741		0.41911
112	0.50987	0.48868		0.45129


(Graph 11a) Comparison of Projection - q(x)
0.6
0.5
0.4
0.3
0.2
0.1
0
q(x)
JLT14
JLT19
SP-a
SP-b
0 10 20 30 40 50 60 70 80 90 100 110
age

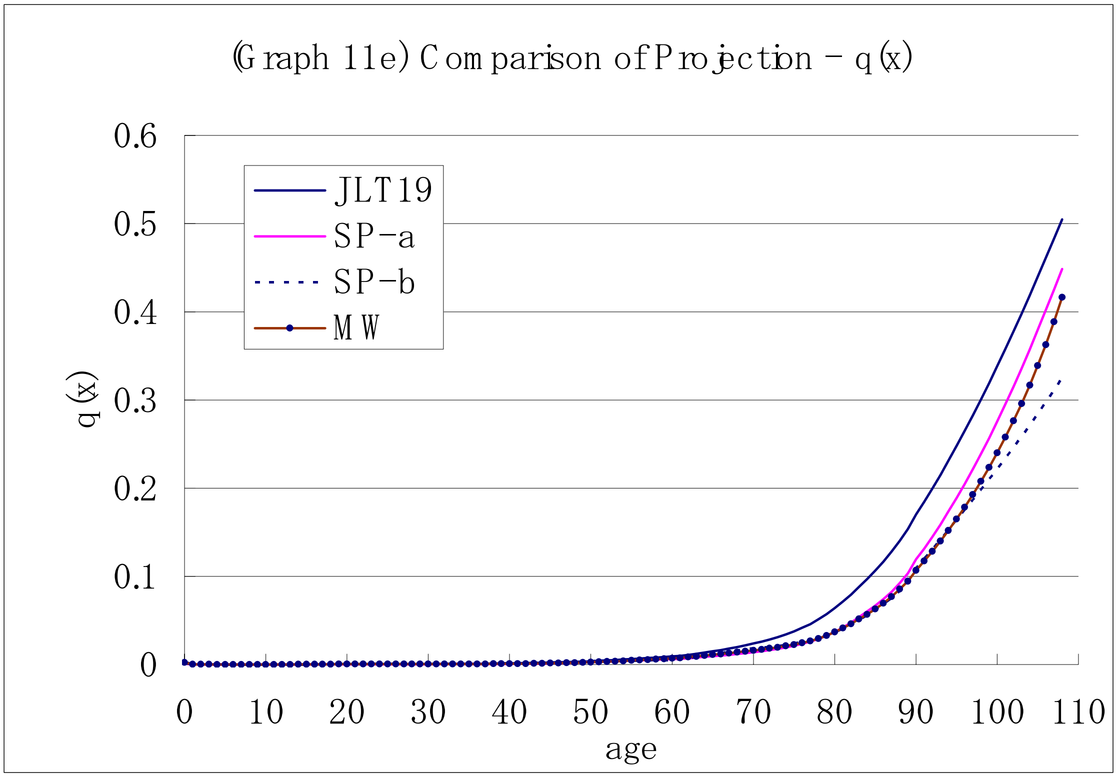

(Graph 11b) Comparison of Projection - log(q(x))
0 10 20 30 40 50 60 70 80 90 100 110
0
-1
-2
-3
-4
-5
log(q(x))
JLT14
JLT19
SP-a
SP-b
age

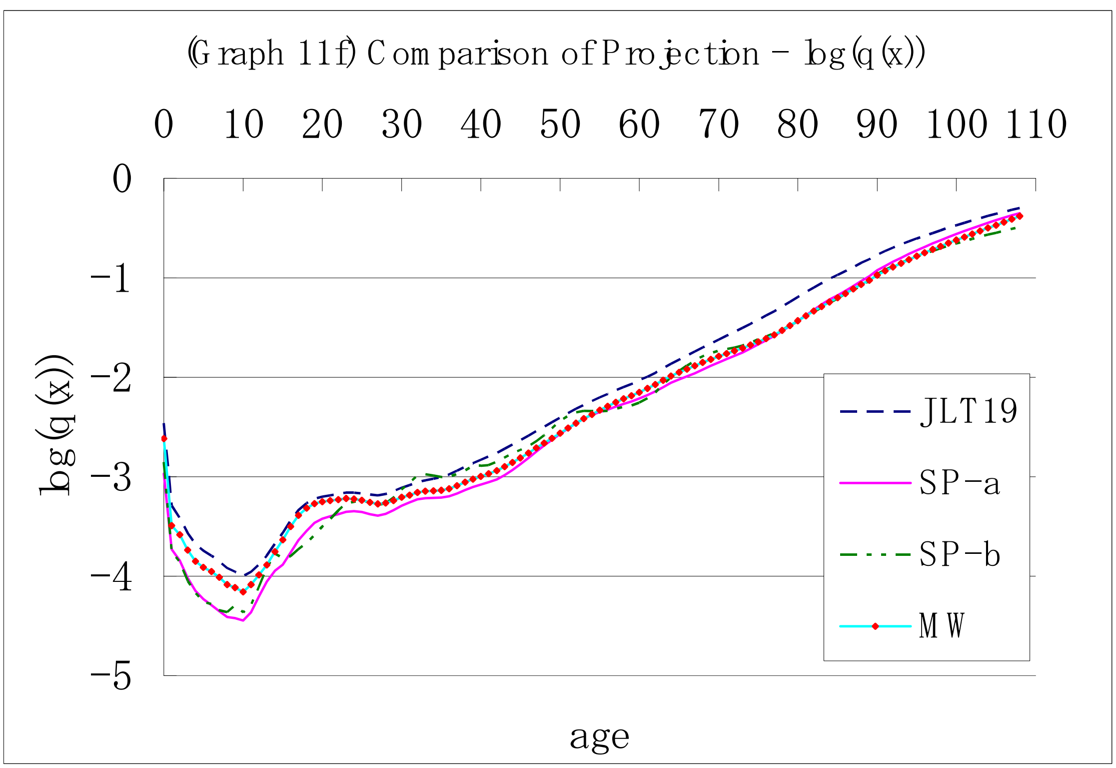


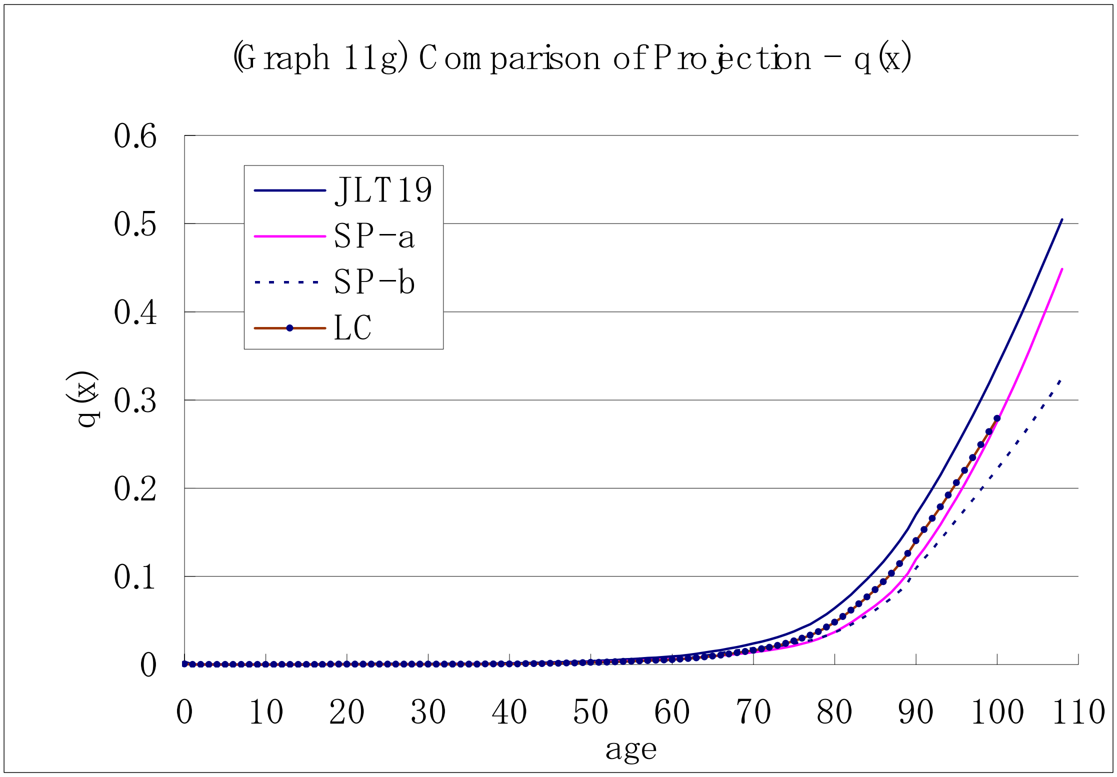
(Graph 11c) Comparison of Projection – q(x)
0.6
0.5
0.4
0.3
0.2
0.1
0
q(x)
JLT19
SP-a
SP-b
HP
0 10 20 30 40 50 60 70 80 90 100 110
age

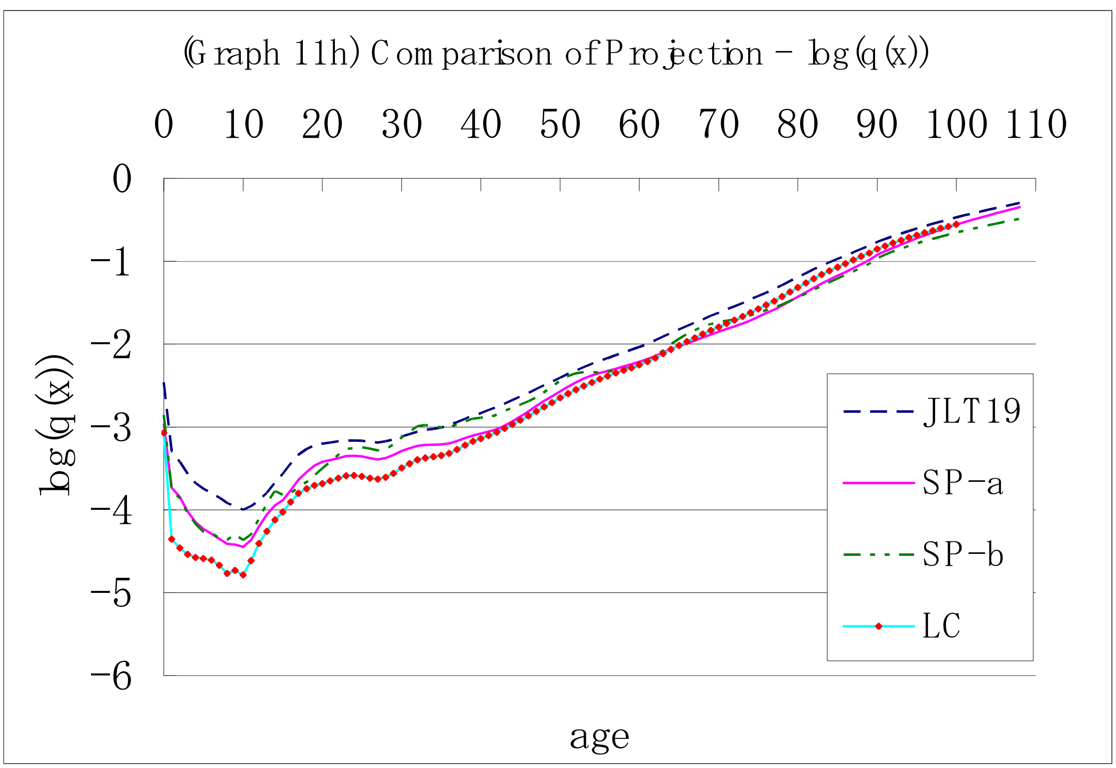


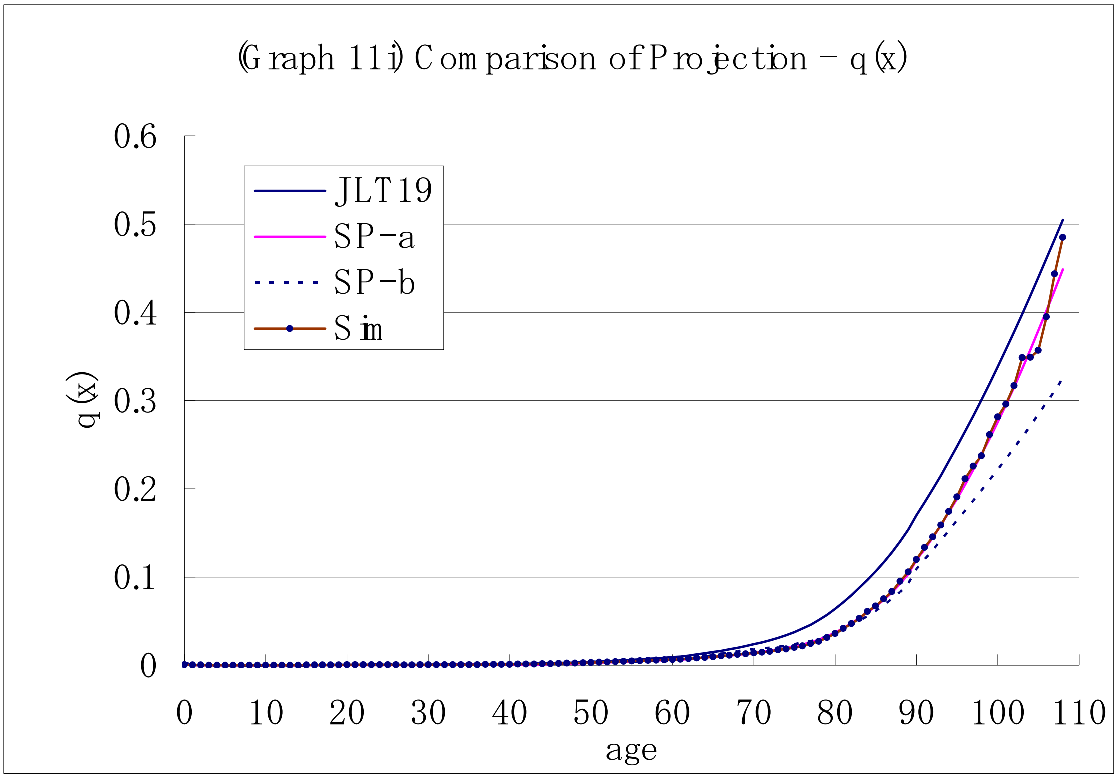
(Graph 11d) Comparison of Projection – log(q(x))
0 10 20 30 40 50 60 70 80 90 100 110
0
-1
-2
-3
-4
-5
log(q(x))
JLT19
SP-a
SP-b
HP
age

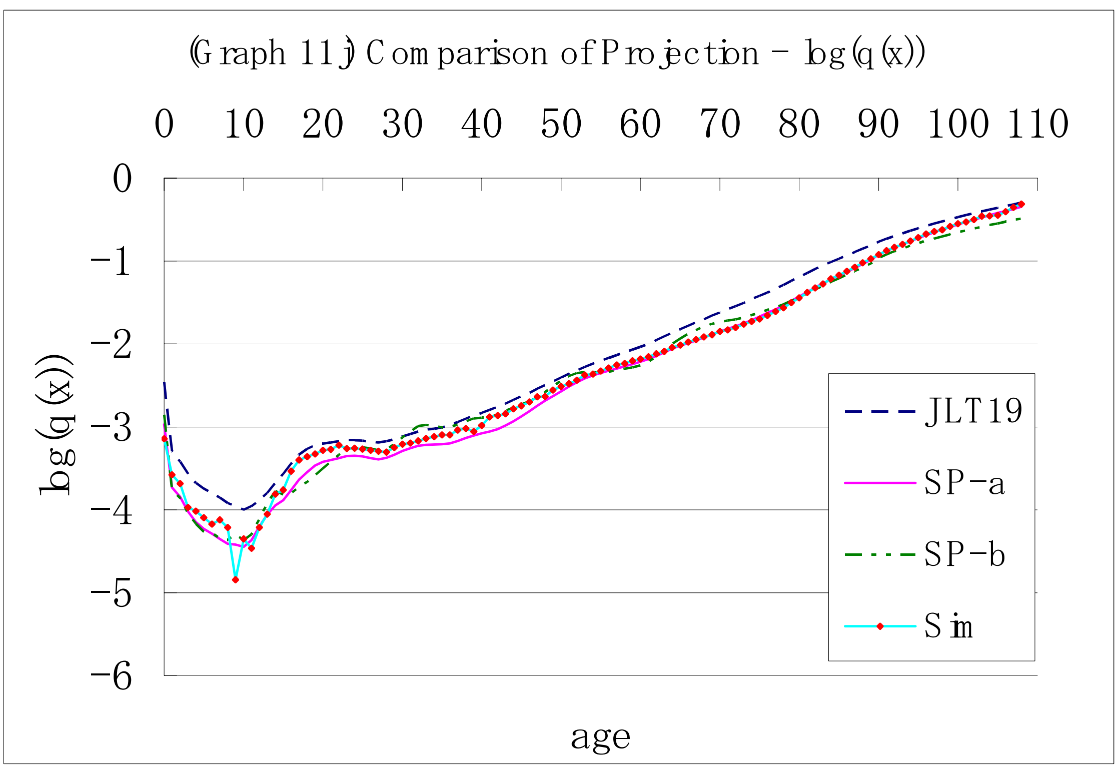

(Graph 11e) Comparison of Projection - q(x)
q(x)
0
0.1
0.2
0.3
0.4
0.5
0.6
0
10
20
30
40
50
60
70
80
90
100
110
age
JLT19
SP-a
SP-b
MW

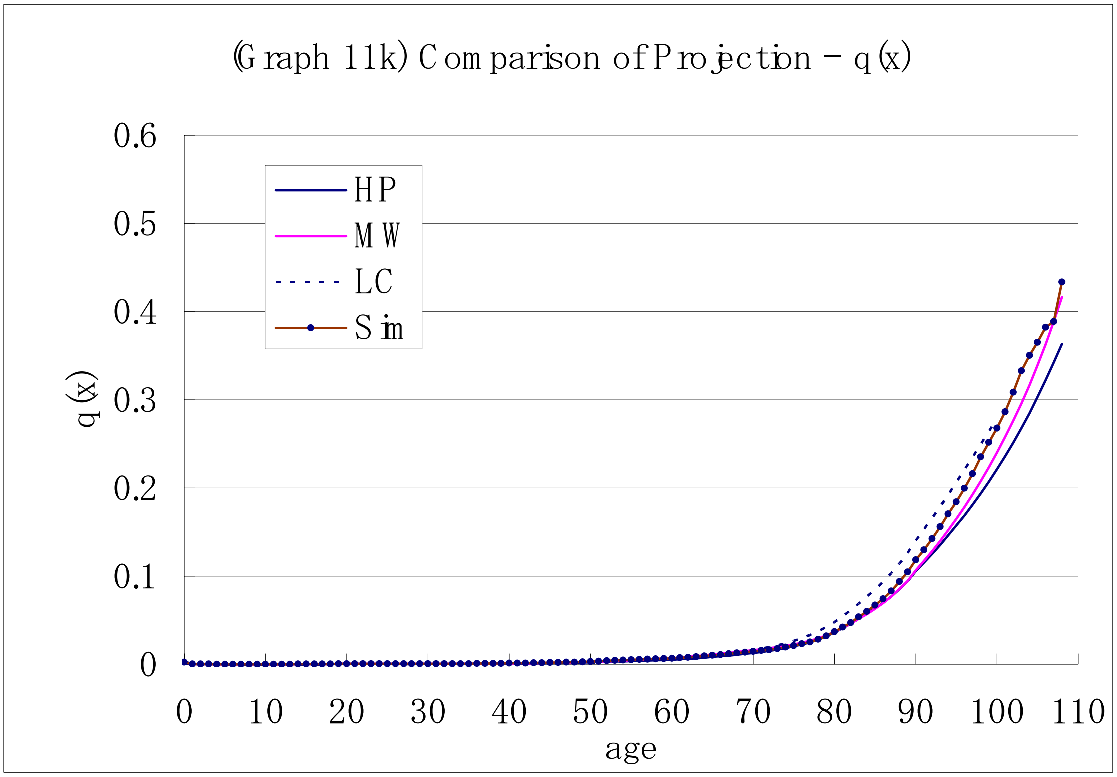

(Graph 11f) Comparison of Projection - log(q(x))
0
10
20
30
40
50
60
70
80
90
100
110
0
-1
-2
-3
-4
-5
log(q(x))
age
JLT19
SP-a
SP-b
MW

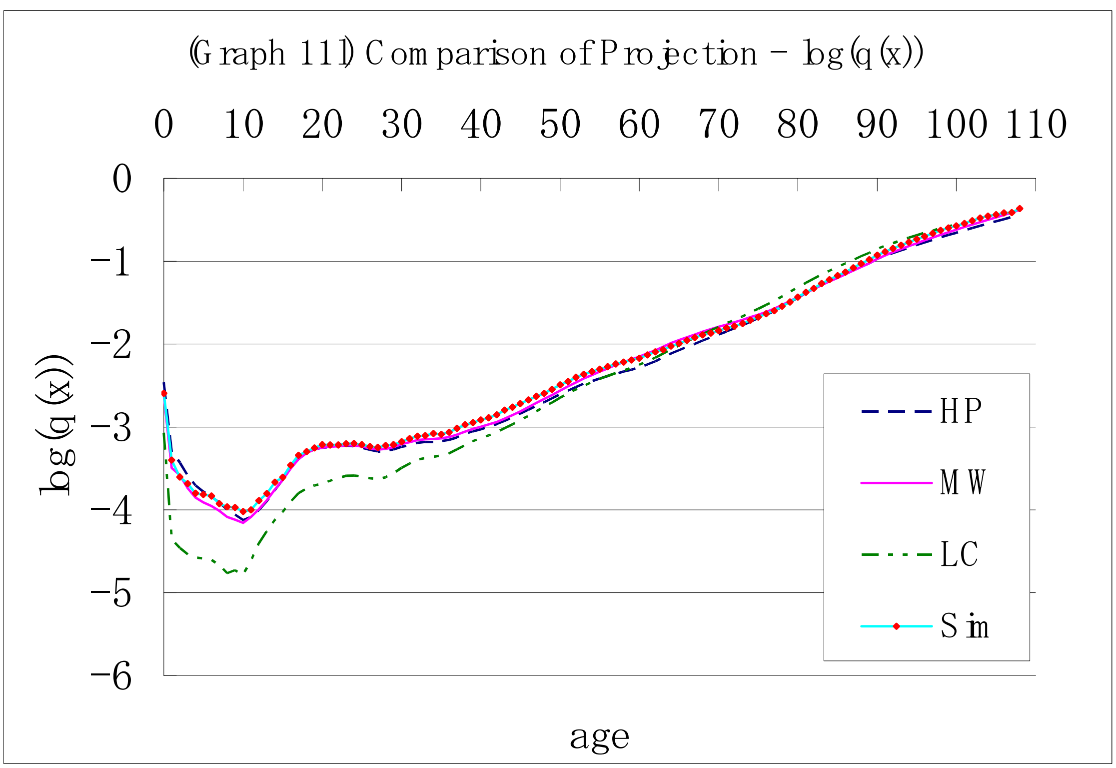

(Graph 11g) Comparison of Projection - q(x)
0.6
0.5
0.4
0.3
0.2
0.1
0
q(x)
JLT19
SP-a
SP-b
LC
0 10 20 30 40 50 60 70 80 90 100 110
age

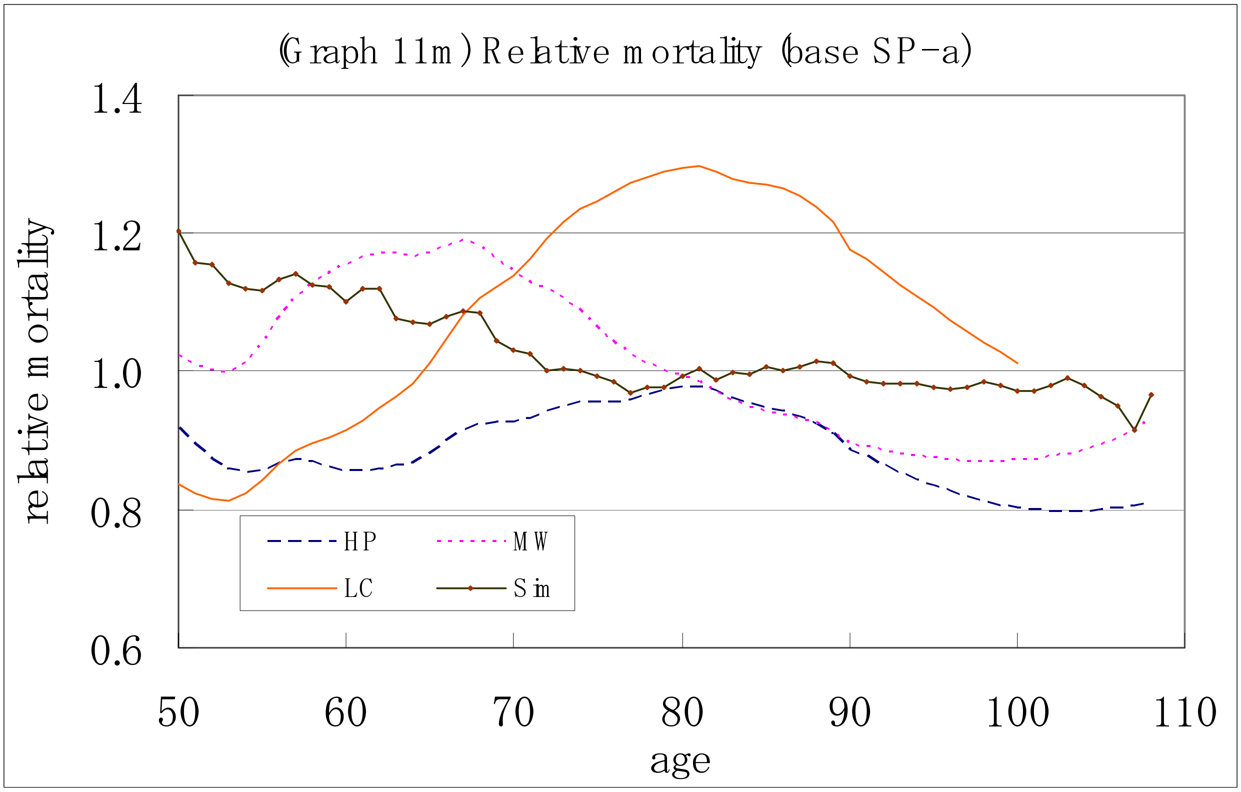

(Graph 11h) Comparison of Projection - log(q(x))
0 10 20 30 40 50 60 70 80 90 100 110
0
-1
-2
-3
-4
-5
-6
log(q(x))
JLT19
SP-a
SP-b
LC
age


(Graph 11i) Comparison of Projection – q(x)
0.6
0.5
0.4
0.3
0.2
0.1
0
q(x)
JLT19
SP-a
SP-b
Sim
0
10
20
30
40
50
60
70
80
90
100
110
age


(Graph 11j) Comparison of Projection – log(q(x))
0
10
20
30
40
50
60
70
80
90
100
110
0
-1
-2
-3
-4
-5
-6
log(q(x))
JLT19
SP-a
SP-b
Sim
age


(Graph 11k) Comparison of Projection - q(x)
0.6
0.5
0.4
0.3
0.2
0.1
0
q(x)
HP
MW
LC
Sim
0
10
20
30
40
50
60
70
80
90
100
110
age


(Graph 11l) Comparison of Projection - log(q(x))
0
10
20
30
40
50
60
70
80
90
100
110
0
-1
-2
-3
-4
-5
-6
log(q(x))
HP
MW
LC
Sim
age


(Graph 11m) Relative mortality (base SP-a)
1.4
1.2
1.0
0.8
0.6
relative mortality
50
60
70
80
90
100
110
age
HP
MW
LC
Sim