

# **Development of a Network Model for Identification and Regulation of Systemic Risk in the Financial System**

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# DEVELOPMENT OF A NETWORK MODEL FOR IDENTIFICATION AND REGULATION OF SYSTEMIC RISK IN THE FINANCIAL SYSTEM

*Han Chen      Zhou Fang      Tony Ha      Shaun Wang<sup>1</sup>      Xin Wang      Bruce Yang*

## **Abstract**

Since the 2007 Financial Crisis, regulators have been very interested in modeling and measuring systemic risk in the financial system. In this study, a network approach is taken to characterize the systemic risk of two nontraditional insurance industries: the bond insurer industry and the CDS market. These industries were chosen since traditional insurance industries do not generate significant systemic risk. The network model for bond insurers demonstrates that after an exogenous shock (a fall in the housing market), bond insurers become insolvent not because of the cross holding of assets but because of the drastic increase in their liabilities. A second, structurally different network model of the CDS market shows how certain parameters of a network can affect the expected loss of the system relative to the initial loss caused by a default. This model also demonstrates how a clearinghouse stymies loss propagation and highlights the usefulness of important data such as counterparty exposures that are not publicly available. If regulators collected counterparty exposure data, they could use it in this kind of model to identify systemically important institutions and better monitor the financial system.

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## INTRODUCTION

The 2007 Financial Crisis illustrated the severity of losses resulting from systemic risk. Top European and U.S. banks lost over \$1.3 trillion on toxic assets and bad loans from 2007-2010.<sup>2</sup> Bank bailouts cost the U.S. government in excess of \$200 billion.<sup>3</sup> With the bailouts being financed by the tax-paying public, the Dodd-Frank Wall Street Reform and Consumer Protection Act was passed in 2010 to address consumer protection, executive pay, and bank capital requirements. The act also expanded regulation on the shadow banking system and financial derivatives and enhanced authority of the Federal Reserve to safely wind-down systemically important institutions. As part of the Act, The Financial Stability Oversight Council and the Office of Financial Research were created.<sup>4</sup> Since its creation, the new Financial Stability Oversight Council has been charged with identifying and regulating threats to financial stability with systemic risk being the key focus.

Systemic risk is the risk that the failure of one significant financial institution can cause or significantly contribute to the failure of other significant financial institutions as a result of their linkages to each other.<sup>5</sup> Systemic risk can also be defined to include the possibility that one exogenous shock may simultaneously cause or contribute to the failure of multiple significant financial institutions in an economy.

Systemic failure can arise from four different sources: direct bilateral interbank exposures, common asset exposure among banks, net settlement systems for large payments, and imitative runs fueled by information contagion.<sup>6</sup> Direct bilateral exposures between institutions represent one of the most common sources of systemic risk. Failures can occur when one bank holds deposits from several other banks, and the failure in the first bank results in either distress or failure that spreads to other firms that are connected to the distressed institution. Similarly, systemic failure can occur from the counterparty exposure risk in derivative transactions. The most common and recognized of these activities are credit default swaps (CDS). Systemic risk arises from CDS when one institution fails to settle its derivative position with another institution – the end result being that both institutions fail. If the second institution fails to settle its obligations with its other counterparties, the contagion<sup>7</sup> of failures continues through the exposed institutions until only institutions with adequate capital remain or the system itself fails. Systemic failure also occurs when a commonly-held asset or a class of assets such as a mortgage-

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<sup>2</sup> Reuters, Factbox – European, U.S. Bank Writedowns and Credit Losses, February 24, 2011.

<sup>3</sup> CNN Money, “Special Report: Bailed Out Banks,”

<http://money.cnn.com/news/specials/storysupplement/bankbailout>, November 19, 2010.

<sup>4</sup> The Financial Stability Oversight Committee was established by the Dodd-Frank Wall Street Reform and Consumer Protection Act in 2010 to coordinate across agencies in monitoring risks and emerging threats to U.S. financial stability.

<sup>5</sup> ECB Workshop, “Recent Advances in Modelling Systemic Risk Using Network Analysis,” January 2010.

<sup>6</sup> Hal Scott, “Reduction of Systemic Risk in the United States Financial System,” Harvard Journal of Law & Public Policy, Vol.33, 2009.

<sup>7</sup> Contagion is a mechanism describing how systemic failures can occur. It can be thought of as a “domino effect,” a failure of one institution leading to failures of more institutions.

backed security (MBS) suddenly drops in value, and the resulting devaluation leads to distress in or failure of a large portion of the industry. Systemic failure also occurs when one bank fails to settle a very large position in a clearinghouse, or some other version of a net settlement system. Then, other counterparty banks do not get paid and as a result, fail themselves. In addition, chain reaction failures result from imitative runs when one bank fails, and depositors in other banks withdraw funds in a panic. The ensuing liquidity crisis in these banks ultimately leads to failure.

The severity of direct bilateral exposure failure is dependent on the degree of interconnectivity among the financial institutions involved in the derivative transactions. The lack of existing information on the degree of connectedness among the institutions remains a concern to the Financial Stability Oversight Council and other governmental regulatory bodies.

This paper proposes two network models to identify and measure the systemic risk in a financial system which may have a high degree of interconnectedness and whose failures may result in further distress or breakdowns in the system. Networks are particularly useful for modeling risk in a financial system due to their handling of contagion, resulting in either losses propagating through a financial system in crisis or the absorption of shocks in a resilient, well capitalized financial system. Network models have been applied in other areas, notably in communications, transportation, and electric power distribution. In each of these areas, some item flows from point to point through a network that involves connections between points. In the financial system there is interest in the flow of cash and credit between financial institutions.

While networks have been used to model systemic risk in financial institutions since 2003, only modest research exists for systemic risk in the insurance industry. A study by the Geneva Association in 2010 suggests a reason for this lack of research.<sup>8</sup> The Geneva Association paper states that traditional insurance and reinsurance businesses are relatively small sources of systemic risk compared to banks and other financial institutions. The Association posits that the structure of the traditional insurance model – upfront premiums, relative lack of interconnectedness, and “substitutability”<sup>9</sup> -- reduces the systemic impact of the insurance industry. The group also states systemic risk does exist for two specific groups that are involved in more non-traditional activities in insurance. These two groups include firms involved in credit derivative security activities such as AIG and bond insurers such as FSA, AMBAC, and MBIA.

This research paper adds to the existing research in systemic risk by specifically applying network models to two non-traditional insurance industries that experienced disaster stemming from systemic risk during the 2007 Financial Crisis: the monoline bond insurance industry and the credit default swap (CDS) security industry. The bond insurance model focuses only on a particular segment of the financial system while the CDS model incorporates a broader portion of the financial system, covering multiple industry segments.

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<sup>8</sup> Geneva Association Systemic Risk Working Group, “Systemic Risk in Insurance – An Analysis of Insurance and Financial Stability,” March 2010.

<sup>9</sup> In this context, “substitutability” means the opposite of “Too big to fail.”

## RELATED LITERATURE

Current research on systemic risk in the financial system can be divided into two camps: an econometric approach based on high frequency data and a network approach based on modeling the inherent structures and links in the system. While the two approaches are different in methodology, the general conclusions from both research camps are somewhat similar.

### Econometric Approach

Billio et al (2011) propose several econometric measures of connectedness based on principal component analysis and Grainger-Causality networks. They apply this structure using high frequency data including the monthly returns of hedge funds, banks, brokers, and insurance companies to capture the causal relationships among the largest financial institutions across these sectors. The authors find that all four sectors have become highly interrelated over the past decade, likely increasing the level of systemic risk through a complex and time-varying network of relationships. They find that these measures can identify and quantify financial crisis periods and contain some predictive power in out-of-sample tests. Finally, their results show that an asymmetry exists in the degree of connectedness among the four sectors indicating that banks and insurance companies are greater sources of systemic risk compared to hedge funds and brokers.<sup>10</sup> The authors cite the Geneva Association study mentioned above in suggesting that the insurance industry contributes to systemic risk not from its traditional activities but rather from “non-core activities such as insuring financial products, credit-default swaps, derivatives trading, and investment management.”

Chen et al (2012) use high frequency market value data on credit default swap spreads and intra-day stock prices to measure systemic risk in the insurance sector. The authors extend non-linear causality tests, finding evidence of significant bi-directional causality between insurers and banks. After correcting for conditional heteroskedasticity, the researchers find however that the impact of banks on insurers is stronger and of longer duration than the impact of insurers on banks. They also use stress tests to confirm that banks create significant systemic risk for insurers but not vice versa.

Brownlees and Engle (2011) define the systemic risk of a financial institution as its expected contribution to the total capital shortfall of the system in a future crisis. They propose a systemic risk measure that captures a firm’s capital shortage by incorporating the firm’s leverage and Marginal Expected Shortfall, the tail expectation of the firm equity returns conditional on a substantial loss in the market. In order to estimate Marginal Expected Shortfall, they build a dynamic model for the market and firm returns characterized by time varying volatility and correlations. Their sample consists of data from 94 top U.S. financial firms between July 2000 and July 2010. The results indicate that this methodology may provide useful rankings of systemically risky firms at various stages of a financial crisis.

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<sup>10</sup> One of Billio’s explanations for the asymmetry is that banks lend capital to other institutions, so the nature of their relationships with counterparties is not symmetric. Also, by competing with other financial institutions in non-traditional businesses, banks and insurers take on risks more appropriate for hedge funds, resulting in a “shadow hedge fund system” that cannot be managed by traditional regulatory instruments.

### ***Econometric Approach Drawbacks***

Current econometric methods as shown in Billio et al (2011) take advantage of the large amount of the more readily available capital market data to perform modeling. In hindsight, some of the econometric models are able to confirm insights from real-world observations. However, econometric models have some disadvantages. First, market prices are subject to daily swings of market sentiment, resulting in significant noise when connecting market instrument values to the systemic effects. Second, most econometric methods fail to take into account the structural connections of various institutions through common or cross holdings in the balance sheet. Econometric models can indicate correlation among important variables in the financial system but fail to define the actual cause-and-effect in the system. Without this type of understanding, the capability to incorporate drivers of market valuations, such as the Case-Schiller housing price index for the housing sector, is difficult.

The econometric, high-frequency data modeling of systemic risk further suffers from a lack of flexibility in its framework. Most econometric methods lack the capability to perform sensitivity testing on the effect of the structure of the financial network, and this inability makes it difficult for regulators to determine the effect of firm heterogeneity and concentration on risk to the system. Similarly, an inability of the econometric framework to model the structural effects of a system makes the assessment of the inclusion of a national clearinghouse for credit default swaps impossible. Furthermore, econometric methods generally lack the ability to characterize the effect of common asset holdings on risk propagation. Consequently, such methods fail to demonstrate the mechanism of how loss is propagated among various institutions and do not provide the insight into how insolvency of a single or several institutions can disrupt the whole financial system. This limitation of econometric models invites other approaches to assess systemic risk.

### ***Network Approach***

Researchers have recently proposed that network models can help model the systemic risk in financial systems which display complex degrees of connectedness. Network models have been used in many fields such as communications, transportation, and utility distribution where the intricacies of the connections make optimization of the system flow analytically challenging. The application of networks to model systemic risk in financial systems has seen significant progress since the 2007 sub-prime mortgage initiated crisis. Most current research in the area of financial network models uses institution-level financial firms or banks as the nodes in the system and their bilateral exposures as the arcs or connections. Within this framework, the existing literature can be further divided by the types of data used to populate the model. Nier et al (2008), Gai (2009), and Georg (2010) use simulated data to capture insights into the network system. Castren (2009), Markose (2010), and Cont (2010) use empirical data to model their system.

Nier et al (2008) use simulated data in their network model to investigate the effect of the financial system's structure on systemic risk. In their simulated framework, banks serve as the nodes, and their interbank exposures act as the connections or the arcs in the network. The authors determine the effects of capitalization, connectivity, and concentration on contagious default in this simulated framework. They find that better capitalized banks are more resilient to contagious defaults, but the effect is non-linear. The researchers also determine that connectivity's effect on systemic risk depends

on the level of connectivity. At low levels, an increase in connectivity acts as a shock transmitter, increasing the contagion effect, whereas, at sufficiently high levels, the shock absorption effect dominates, and the initial shock is spread over more and more of the bank nodes. Finally, the authors show that everything else equal, more concentrated banks are prone to larger systemic risk.

Gai and Kapadia (2009) also investigate the dual nature of connectivity in their paper. In their model, banks are again the nodes, and the interbank exposures are the arcs. Then, they assume a random (Poisson) probability that each node is linked. From this model, the authors find that the complex financial networks exhibit a “robust-yet-fragile” nature; greater connectivity helps lower the probability of contagion but increases its spread in the event that problems do occur. Furthermore, they find that illiquid markets for key financial assets compound the contagion problem, amplifying both the likelihood and the severity of the risk. Finally, they argue that credit derivatives create far-reaching inter-linkages that reduce the probability of contagion with greater use under some plausible scenarios, but the resultant exposure leads to greater financial impact in a crisis.

Georg and Poschmann (2010) continue the research in financial network models by using numerical simulations to examine the effect of a central bank in the network model of the financial system. In their model, bank nodes including a central bank are connected via their balance sheet exposures and incorporate a constant relative risk aversion utility function in determining their portfolios. The authors find that the presence of a central bank has a stabilizing effect on the financial system, and this stability effect may arise from the enhanced liquidity allocation provided by the central bank. From their model, the researchers find that systemic risk increases with credit “lumpiness,” defined as fewer, large credit counterparties. The authors define two types of shocks, one resulting from the insolvency of a large bank and resulting in contagion effects throughout the network and another in which a shock affects all the banks in a network via commonly held assets. They posit that the destabilizing effect of common shocks poses a greater threat to systemic stability than the direct contagion effect.

Castren and Kavonius (2009) use historical Euro Area Accounts data to calibrate a sector-level network model to help identify the potential key triggers to instability, to detect the contagion mechanisms in the system, and to determine the effects of leverage on a system’s resistance to shock and contagion in a multi-period setting. In their model, the sectors include households, banks, non-financial corporations, insurance and pension fund companies, other financial intermediaries, general government, and the rest of the world. They extend the accounting based bilateral exposure connected network to a risk-based network by applying a contingent claims analysis approach developed by Moody’s KMV. The authors find that in the 10 years since the creation of the European Monetary Union, the interlinking arcs represented by the bilateral financial accounts have grown significantly with the banking sector playing a key role in the system. They determine in their simulations that local cash-flow shocks can spread quickly via the bilateral exposures and even without the presence of defaults in the process. The authors also find that sectors with highest leverage are the most vulnerable ones to shocks.

Markose et al (2009) apply a complex agent-based computational variant of the financial network model to assess systemic risk. The authors use FDIC data and market share data of 26 banks to create a U.S.



credit default swap (CDS) market-based network to investigate the consequences of the fact that the top 5 banks are responsible for 92% of the activity in the \$16 trillion U.S. CDS market. Their network model uses the major banks as the main nodes in the system and incorporates a “non-U.S. bank” node to include monolines<sup>11</sup>, hedge funds, and other insurers. The links are the bilateral obligations of the CDS. The authors argue that the implied incentives of the credit risk transfer scheme included in Basel II may have contributed to the 2007 Financial Crisis in two ways. First, the use of risk transfer mechanism allows a decrease in the actual regulatory reserve requirements which may have stopped the contagion from spreading. Second, the growth and popularity of the synthetic securitization of these risk transfers concentrates the risk among a few large dominant players. The authors determine that the intervention of the Federal Reserve to bail out certain “too large to fail” institutions could not be averted because their large number of links to other institutions could have resulted in the failure of the whole CDS market and possibly the whole financial system. Furthermore, they identify these “super-spreaders” and propose a “system risk ratio” which quantifies how much capital is lost collectively when one of these firms fail.

Cont et al (2010) examine the financial network approach to modeling system risk in the Brazilian financial system and to measuring the systemic importance of a single institution in the system. Their model incorporates Brazilian interbank exposure data including fixed income instruments, borrowing and lending, derivatives, foreign exchange, and instruments linked to exchange-traded equity risk. The authors stress test the model by applying correlated market shocks to the balance sheets of all the banks in the network in various default scenarios. They find that connectivity and concentration of exposures as measured by counterparty susceptibility and local network fragility are highly correlated to the systemic importance of an institution. The researchers also show that a minimum capital ratio reduces the effect of large institution defaults and that a similar effect can occur by requiring minimum capital reserves on only those systemically important firms and those who are exposed to them. Finally, they introduce a “Contagion Index” which measures the expected loss to the network triggered by the default of the institution subjected to a market shock.

### *Common Findings in the Network Model Literature*

There seems to be some consensus in the Network Model research of systemic risk that structural parameters of a network, such as connectivity and concentration, matter as much as size when assessing the systemic importance of an institution. Size alone cannot be used to determine a firm’s systemic importance. The Cont study is unique in that it studied the effect of *local* measures of connectivity and concentration on systemic risk. Most studies, like the Nier study, focus on *aggregate* measures of connectivity and concentration. The Cont study found that their two local measures, counterparty susceptibility and local network frailty, can significantly explain default contagion.

There is some disagreement over the relationship between the connectivity of a network and contagion risk. Some authors including Babus found that greater connectivity reduces contagion risk in interbank markets, and if a certain connectivity threshold is reached, contagion risk is practically nonexistent. Gai,

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<sup>11</sup> Monolines in this study refer to bond insurance companies such as AMBAC, MBIA, and FSA who provided guarantees to financial assets.

Georg and Nier came to different conclusions. The Gai and Georg studies found that financial networks, especially interbank networks, exhibited a “robust-yet-fragile” property. Greater connectivity of a network does help to lower the probability of contagion but actually *increases* its spread in the event that contagion breaks out. In the network constructed by Nier, greater connectivity led to a more resilient system after a certain connectivity threshold was reached, but for a low degree of connectivity, greater connectivity actually led to an increased contagion effect. Thus, Gai, George and Nier observed that connectivity has a contingent nature: depending on the actual level of connectivity, greater connectivity can either increase or decrease contagion risk.

Most papers focus almost exclusively on systemic risk through *contagion* effects, but the Georg and Cont papers argue that common asset shocks which affect all institutions of a network via commonly held assets may pose an even greater threat to systemic stability. The Cont study, in particular, shows that systemic risk is understated when common shocks are not considered.

This paper introduces a network model to characterize the systemic risk in the financial system as performed in some of the literature above. The study differs from the other papers by specifically testing the structure of the network applied to two existing financial systems: the monoline bond insurer network and the credit default swap based network. Furthermore, the paper identifies information that may not be publicly available but would be vital for regulators in monitoring systemic risk.

## MODELS

### Bond Insurer Network

#### Introduction

Monoline bond insurers provide insurance coverage for various types of securities such as municipal bonds, residential mortgage-backed securities (RMBS), asset-backed securities (ABS), and collateralized debt obligations (CDO). The nature of the bond insurance business is to provide credit enhancement to relatively low-rated bonds in order to reduce the borrowing costs of the bond issuer. Consequently, a strong credit rating is critical for the bond insurer’s business.

At the start of the 2007 Financial Crisis, numerous insured financial assets such as RMBS, CDO, and other securities were heavily downgraded by rating agencies. Subsequently, the corresponding insurers were forced to book large loss reserves. Moreover, since bond insurers’ assets include their own insured bonds as well as bonds insured by others, the devaluation of insured bonds resulted in the deterioration of the insurers’ balance sheet as well. As a result, most of the insurers became financially distressed, and their credit ratings were downgraded. Ultimately, the rating downgrades of the bond insurers triggered large-scale downgrades of all the relevant insured bonds, which ultimately depreciated the value of the entire bond market. In the end, many bond insurers either declared bankruptcy or merged

with other insurers due to the dramatically increased liabilities and the deterioration of assets. Since the crisis, the bond insurer market and its business model have undergone drastic changes.

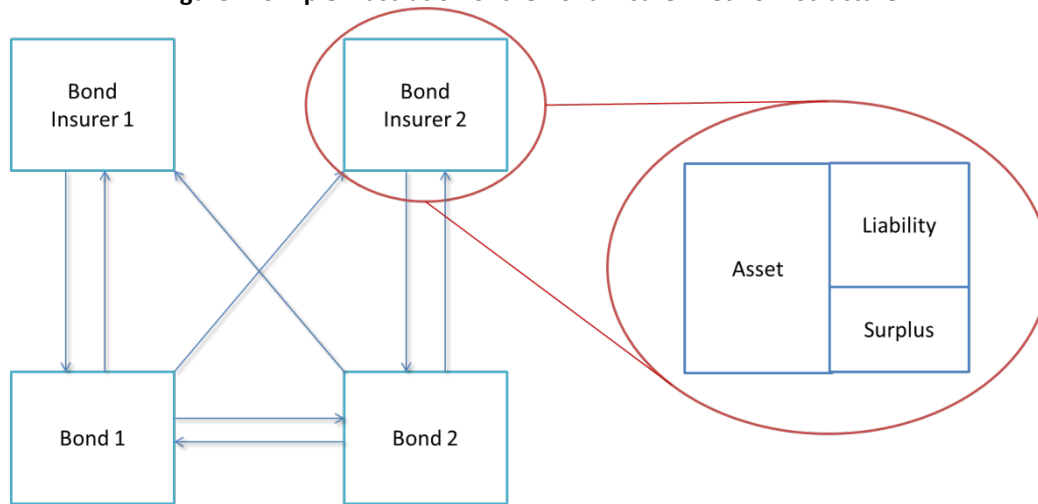
### **Bond Insurer Market Structure**

In the bond insurer network, a change in an insurer's credit rating is transmitted directly to the insured asset's rating. Thus, credit downgrades of insurers affect the value of insured bonds. Rather unique to the industry, bond insurers hold the same bonds that they insure as a part of their asset portfolio. Hence, the value of the bonds impacts the asset value of the insurers. Additionally, some bonds such as ABS and RMBS represent a pool of securitized mortgage loans or debts which may be backed by similar assets. This structure implies that a change in the value of one underlying asset may result in the change in value of multiple bonds or securities. For example, a drop in one home's value results in a drop in the value of the homes in the surrounding neighborhood. Consequently, values of those bonds whose underlying assets are the surrounding homes drop.

### **Network Model of the Bond Insurer Market**

The structure of the bond insurance market can be viewed as a network where the bond insurers and the insured securities are the nodes, and the bilateral balance sheet exposures are the arcs. The nodes are entities with their own balance sheets. For bond insurers, a percentage of the assets is invested in the bonds that they insure; a percentage of the liabilities is shown as the loss reserve; and a certain amount of surplus is reflected as capital on the balance sheet. The insured bonds are also treated as balance sheet nodes in which the assets are represented by the marked-to-market (MtM) value of the underlying homes, and the liabilities reflect the face value of the loan that needs to be repaid. The arcs in the model are bilateral balance sheet exposures. The asset side of an insurer is linked to the liability side of each of the insured bonds it owns. This model assumes that each bond is insured by only one insurer so that the liability side of an insurer is linked only to the bonds it insures. An illustration of the bond insurer network structure is shown in Figure 1.

**Figure 1. Simple Illustration of the Bond Insurer Network Structure**



## Model Assumptions

The following assumptions are used in setting up the network model to analyze the bond insurer structure and to reveal the systemic risk factors. First, the model includes only those bonds that are residential mortgage-backed securities (RMBS). Second, this model assumes that there are only six bond insurers and six RMBS in the world, and that each insurer proportionally invests in all the RMBS, including the bond it insures. Third, any one bond is insured by only one insurer, implicitly specifying a one-to-one bilateral linkage. Finally, the connection between different RMBS is measured by the compound effect of the Asset Closeness Factor which is defined in Appendix 1.

In the bond insurer network model, a downgrading cycle mechanism causes the contagion effect within the network, propagating the losses and initiating systemic risk. The downgrading cycle can be described as follows:

1. Downgrades to the RMBS deteriorate the financial status of bond insurers and indirectly cause the downgrade of the insurers.
2. Downgrades of the insurers trigger further downgrades of all the insured bonds.
3. Downgrades of the insured bonds lead to further downgrades of the insurers since the insurers hold the insured bonds as their assets.

Each time there is a devaluation or ratings downgrade, additional losses occur. In the model, an initial shock means a sudden large-scale price decline in the housing market along with a rising default probability, which is exogenous to the model. When the network system suffers a shock, the losses can propagate through the arcs until the network reaches equilibrium. This paper assumes that bankruptcy does not occur while the network achieves equilibrium.

The complexity of the bond insurer network arises from the fact that a bond insurer holds the same bond that it insures. Since insurers hold the insured RMBS in the market as an asset, the price of the

mortgage backed security affects the value of the insurer's total assets. Additionally, since the underlying asset value and default probability of RMBS affect the loss reserves of the insurers, the RMBS asset can also affect the insurer's liabilities.

## Model Mechanism

The bond insurer network model, at its heart, determines whether the system will survive after a given exogenous shock, i.e. a significant drop in the housing market. The ultimate objective of this model is to determine the asset and liability value of each insurer after the exogenous shock and compare them to the pre-shock asset and liability values, which are found empirically. The insurer's assets are composed of RMBS, other insured bonds, treasury bonds, and other securities. Its liabilities are composed of a reserve for the insured RMBS and a reserve for other insured bonds. In this model, since the shock hits the housing market, this paper is particularly concerned with valuing RMBS and the insurer's loss reserves for the RMBS. This valuation method is inspired by the Black-Scholes framework and Moody's KMV methodology.

### *Pre-Shock*

This paper assumes that the underlying assets of the RMBS follow a geometric Brownian motion. Based on the observed yields to maturity, the risk free rate and the insurer's balance sheet information, this paper derives the implied volatility of RMBS underlying assets before the shock. This is done by using a manipulation of the Black-Scholes option pricing formula. With this implied volatility, the underlying asset distribution can be simulated.<sup>12</sup>

This paper uses this underlying asset distribution to obtain the implied liability. When the price of an underlying asset falls below its mortgage face value, or implied liability, default occurs. Consequently, the insurer then has to cover the difference. Given an observed default probability  $p_0$ , an implied liability can be calculated as the  $VaR(p_0)$  of the underlying asset distribution.

### *Post-shock*

Once a shock hits the housing market, the distribution of the implied asset value shifts. The mean of the distribution decreases while the volatility increases, as seen in Figure 3 (note that the distributions here are normal, not lognormal, for illustrative purposes). Once the underlying asset value falls below the liability of the RMBS, the difference between the underlying asset value and liability of the RMBS is the insurer's loss. As a result, the probability of default for the RMBS rises as well as the loss reserves of the bond insurer. The loss reserves are represented by the shaded area to the left of the implied liability of the RMBS. These loss reserves are found via simulation. As a result, the liabilities of the insurer increase.

When the liabilities of the insurer increase, the insurer becomes less solvent, and the firm's rating falls. This leads to a drop in the RMBS value, and because the insurer holds the same RMBS that it insures, the

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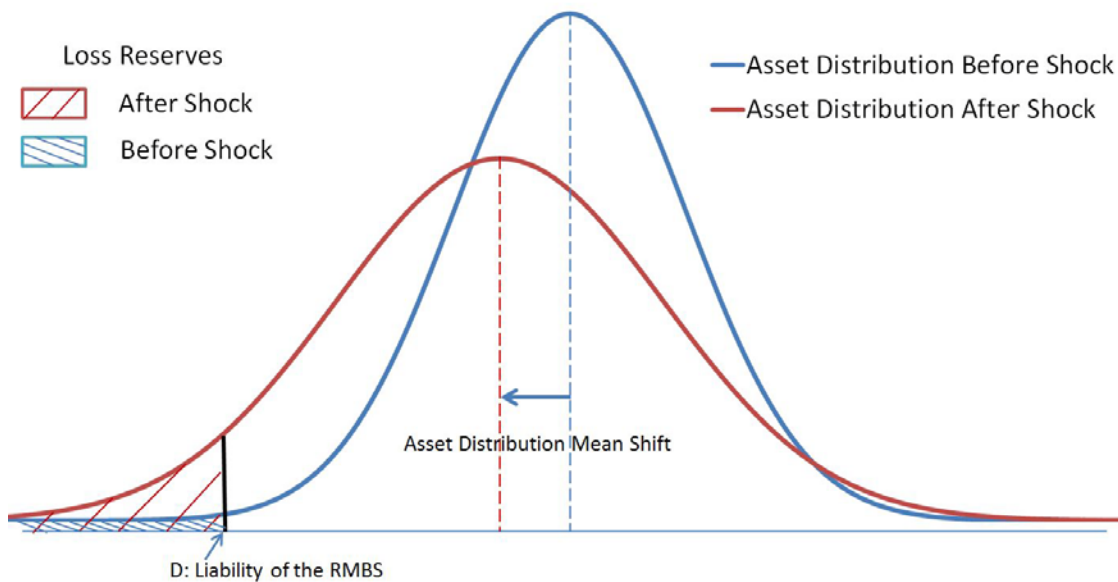
<sup>12</sup> The final loss reserve calculations require a comparison of the distributions before and after the shock. Under the Black-Scholes framework, these distributions are assumed to be lognormal.

value of the insurer's assets decreases too. This cycle continues until equilibrium is reached and at such a point, one can solve for the equilibrium value of the insurer's assets. The equilibrium value of an insurer's assets can be written as:

$$I_i^*(T) = \sum_{n=1}^6 \beta_{i,n} V_n^{ins*}(T) + A_i, i = 1, 2, \dots, 6, n = 1, 2, \dots, 6$$

where  $\beta_{i,n}$  is the holding weight of  $V_n^{ins*}(T)$  for insurer  $i$ ,  $V_n^{ins*}$  is the equilibrium value of insured RMBS, and  $A_i$  represents all other assets of insurer  $i$ . Both  $\beta_{i,n}$  and  $A_i$  are assumed to be constant in the bond insurer model.

**Figure 3. The Loss Reserves for RMBS before and after Shock**



The equilibrium value of each insured RMBS that the insurer holds is as follows:

$$\begin{aligned} V_n^{ins*} &= E[\min(B_n(t_0) + I_i(V_n^{ins*}), D_n)] \\ &= e^{-rT} * D_n - Put(B_n(t_0), D_n - I_i(V_n^{ins*}), r_0, T, \sigma_n^*) \end{aligned}$$

where  $B_n(t)$  is the underlying asset value,  $D_n$  the face value or the legal liability of the RMBS (debt),  $I_i$  is the asset value of insurer  $i$  before the shock and is a constant,  $\sigma_n^*$  is the post-shock implied volatility of RMBS  $n$ , and  $Put(B_n(t), D_n - I_i, r_0, T, \sigma_n^*)$  represents a European put option with the risk-free rate  $r_0$ , time to maturity  $T$ , and implied volatility of  $\sigma_n^*$ . This equation, where the RMBS is decomposed into a discounted bond and a put option, is derived from the Black-Scholes framework and Moody's KMV methodology.

After equilibrium has been reached and the insurer's capital, defined as the assets less the liabilities, is not positive, the insurer defaults.

## Data

In order to populate the balance sheet structure of the six bond insurers in the bond insurer model, the historical balance sheet data of six insurers are imported as of Q4, 2006. The six insurers are Ambac, ACA, CIFG, MBIA, FGIC and Syncora. These six bond insurers dominated the market before the 2007 Financial Crisis. For simplicity in modeling, it is assumed that there are only six bond insurers in the bond insurance market. Additionally, the model utilizes the commercial mortgage-backed security yield spread data in 2006<sup>13</sup> and the RMBS default probability<sup>14</sup> data to calculate the loss reserve and the marked-to-market value of the six RMBS.

## Scenario Test and Sensitivity Test

### 1. Scenario Test

Scenario tests are performed for the bond insurer network model to explore the sources of systemic risk and to examine the impact of initial shocks of differing scales. Additionally, sensitivity tests are performed to examine the influence of the Asset Closeness Factor  $r$ .

Some basic assumptions and settings are addressed below. There are six bond insurers in the market, denoted as C1, C2,..., C6. There are various kinds of bonds in the market in this model world, however, only six of them are the resident mortgage-backed securities (RMBS), denoted as B1, B2,..., B6. Each RMBS is insured by only one insurer, which means a one-to-one relationship. A shock is defined as an exogenous event that causes significant devaluation of one or more RMBS in the market. The historical balance sheet data of Ambac, ACA, CIFG, MBIA, FGIC and Syncora as of Q4, 2006, are adopted to mimic the historical balance sheet structure before the shock. Each insurer's liability is only composed of two kinds of loss reserves, where 50% is used to cover the RMBS it insures, and the rest is for "other reserves" that will not be affected by a shock in the housing market. The market shares of the six RMBS are determined by the percentage of its loss reserve to the total loss reserve for all six RMBS. For the asset side of an insurer, 25% of the total assets represent the investments in the six RMBS, where the portfolio composition matches the market share of the six RMBS. Another 25% is the investments in other insured bonds in the market. Although the loss reserves for other insured bonds will not be affected by the shock (the default probability stays constant), the market price of these bonds will decline since the deterioration of an insurer's financial status has a negative impact on all the bonds it insures. The remaining 50% of the total assets is an investment in Treasury bonds and will not be influenced by the shock. Before starting the tests, the value of basic parameters are derived from empirical data including the risk free rate, the RMBS market price, and default probability. Based on the empirical data and the formulas, the pre-shock regime implied volatility for each RMBS is determined.

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<sup>13</sup> Credit Suisse, "CMBS Market Watch Weekly", Fixed Income Research, <http://www.credit-suisse.com/researchandanalytics>, January 19, 2012

<sup>14</sup> Donchev, T. "Modeling Defaults in Residential Mortgage Backed Securities: An Intensity Based Approach.", Unpublished Paper, August 2008

In the real housing market, it can be observed that there is a simultaneous movement of housing prices. For instance, if the price of homes in one region goes up, it is highly likely that the price of other homes in surrounding regions will rise too. Moreover, the housing price of other regions may also increase due to the positive market trend. Since the RMBS' underlying asset is the house, different RMBS are also correlated. In this model, one factor is introduced to specifically manage the correlation between different RMBS. This is called Asset Closeness Factor  $r$ . It is related but not equivalent to the housing sector correlation. The range of  $r$  is between 0 and 1, with the default value of  $r$  set to 0.7 in the model, and larger  $r$  values represent closer correlation. A more detailed explanation is presented in Appendix 1.

By incorporating the Asset Closeness Factor into the bond insurer network model, the underlying assets of RMBS are linked together to create a more reasonable network structure that may assist regulators in verifying the source of systemic risk. In order to demonstrate how the model reacts to shocks, and to provide more insight on the bond insurer network, six different scenarios are tested:

1. The underlying asset of B6 drops 10%, where the market share of B6 is less than 1%.
2. The underlying asset of B1 drops 10%, where the market share of B1 is around 38%.
3. The underlying assets of B1 and B2 both drop 10%, where the total market share of two bonds is around 80%.
4. The underlying asset of B6 drops 20%, where the market share of B6 is less than 1%.
5. The underlying asset of B1 drops 20%, where the market share of B1 is around 38%.
6. The underlying assets of B1 and B2 both drop 20%, where the total market share of two bonds is around 80%.

**Table 1. The Market Share of Six RMBS in the Bond Insurer Model**

Bond Name	B1	B2	B3	B4	B5	B6
<b>Market Share (6 bond)</b>	38.07%	41.28%	16.59%	2.21%	1.33%	0.53%

The scenario tests are designed to define a shock from two perspectives: the amount of bond devaluation it causes, and the percentage of the market it initially affects. Table 2 illustrates the results of the scenario tests. For a bond insurer, the Asset Change represents the change of the asset value compared with the total asset value before shock, as a percentage. Similarly, the Liability Change portion measures the change in the insurers' liabilities.

Starting with the Asset Change scenarios from Table 2, it is clear that shock has little impact on the value of the insurers' assets. Due to the common asset holding in RMBS, the decline in asset value is similar for all six insurers, no matter the size of the market that the shock impacts.



**Table 2. The Scenario Test Results for the Bond Insurer Model**

Scenario	Asset Change					
	C1	C2	C3	C4	C5	C6
1	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%
2	-1.53%	-1.53%	-1.53%	-1.53%	-1.53%	-1.53%
3	-2.04%	-2.04%	-2.04%	-2.04%	-2.04%	-2.04%
4	-0.04%	-0.04%	-0.04%	-0.04%	-0.04%	-0.04%
5	-3.55%	-3.55%	-3.55%	-3.55%	-3.55%	-3.55%
6	-15.26%	-15.26%	-15.26%	-15.26%	-15.26%	-15.26%
Scenario	Liability Change					
	C1	C2	C3	C4	C5	C6
1	0.3%	0.0%	0.2%	0.1%	0.4%	34.3%
2	186.7%	88.9%	138.4%	39.9%	38.3%	23.1%
3	227.5%	219.9%	243.3%	67.9%	64.9%	39.2%
4	2.19%	0.81%	0.66%	0.24%	1.23%	91.77%
5	738.81%	331.05%	519.17%	133.98%	141.36%	75.04%
6	3394.29%	3322.66%	4022.54%	1669.54%	1592.73%	988.66%

For example, in Scenario 2, the asset value of the bonds falls 1.53% for all six insurers when the underlying asset value falls 10% while the asset value falls 3.55% when housing values fall 20% in scenario 5. This relationship indicates that inter-connections may amplify the initial shock.

The liabilities of the bond insurers are significantly influenced by a shock. In scenarios 1 and 4, only C6 experiences notable liability increases since C6 has a tiny market share and has fewer connections to the other insurers. When a shock hits C1, it can be seen that the liabilities of all six insurers rise significantly. In scenario 2, the three largest insurers become bankrupt, while in scenario 5 all insurers except C6 become insolvent after the shock. In the worst case scenario where a very severe shock hits a large portion (80%) of the market (as in scenario 6), all six insurers become insolvent and the whole bond insurance system collapses. Moreover, there is a non-linear relationship between the shock severity and a change in insurer liability. In scenario 3, the liability increase associated with a less severe shock (10% value devaluation) is less than 250%, while a relatively large shock (20% value devaluation) in scenario 6 leads to an extraordinary increase in all six insurers' liabilities.

Above all, an exogenous shock has a significant impact on the insurers' liabilities and may lead to a collapse of the entire system if it causes a sufficient devaluation of the insured security that represents a portion of the market. The results also reveal there is a non-linear relationship between the shock and the liability, which implies that the existing bond insurer network structure acts as a shock amplifier rather than a shock absorber.

## 2. Sensitivity Test

The sensitivity tests are designed to examine the effect of the Asset Closeness Factor. The Asset Correlation Factor acts as a proxy for the implied correlation among the insured securities but is not

equivalent to the correlation itself. The Asset Closeness Factor  $r$  reflects only one parameter in the correlation relationship with  $\alpha$  the other parameter given weights  $w_i$  and  $w_j$  in the correlation equation:

$$\hat{\rho}_{i,j} = r^{\alpha(\frac{1}{w_i} + \frac{1}{w_j})}$$

The results are shown in Table 3. The value of  $\alpha$  is fixed as 0.17, and three values of  $r$  are specified as 0.66, 0.70, and 0.74. All the tests are completed under the assumption that a shock causes the underlying asset of B1 to drop 10%, and other assumptions are consistent with those in the scenario tests.

The test results in Table 3 imply that the Asset Closeness Factor  $r$  has little effect on the asset value of insurers, whereas it significantly impacts the insurers' liabilities. As the value of  $r$  increases, the liability of every insurer also increases, and thus the aggregate liability of the bond insurance industry becomes larger. Initially, when  $r$  is no greater than 0.70, C1, C2, and C3 become insolvent after a shock. When  $r$  reaches 0.74, C4 in addition to C1 through C3 becomes insolvent, suggesting that larger  $r$  leads to greater systemic risk. Additionally, as the value of  $r$  becomes larger, the marginal effect of  $r$  on the liability side of insurers also becomes greater, implying that a non-linear relationship between  $r$  and insurers' liabilities exists. For instance, as  $r$  goes up to 0.70 from 0.66 (6% change), C1's Liability increases by around 0.3%. As  $r$  rises to 0.74 from 0.70 (6% change), C1's Liability increases 8.67%. This non-linear effect is valid for all six insurers.

**Table 3. Sensitivity Test on  $r$  (Asset Closeness Factor)**

Parameter		Asset Change						Liability Change					
$r$	$\alpha$	C1	C2	C3	C4	C5	C6	C1	C2	C3	C4	C5	C6
0.66	0.17	-1.5%	-1.5%	-1.5%	-1.5%	-1.5%	-1.5%	186.2%	82.1%	128.6%	36.4%	34.8%	21.1%
0.70	0.17	-1.5%	-1.5%	-1.5%	-1.5%	-1.5%	-1.5%	186.7%	88.9%	138.4%	39.9%	38.3%	23.1%
0.74	0.17	-1.7%	-1.7%	-1.7%	-1.7%	-1.7%	-1.7%	202.9%	109.0%	170.5%	48.2%	46.7%	27.9%

## Credit Derivative Securities Based Network

### Introduction

Credit default swaps have been commonly blamed for the 2007 Financial Crisis. Nevertheless, CDS still dominate the credit derivatives market and are at the center of the global financial system.<sup>15</sup> The U.S., Europe, as well as other global financial institutions possess large exposures to CDS markets. The 2007 Financial Crisis underscored the challenge of measuring, monitoring and pricing credit risk.

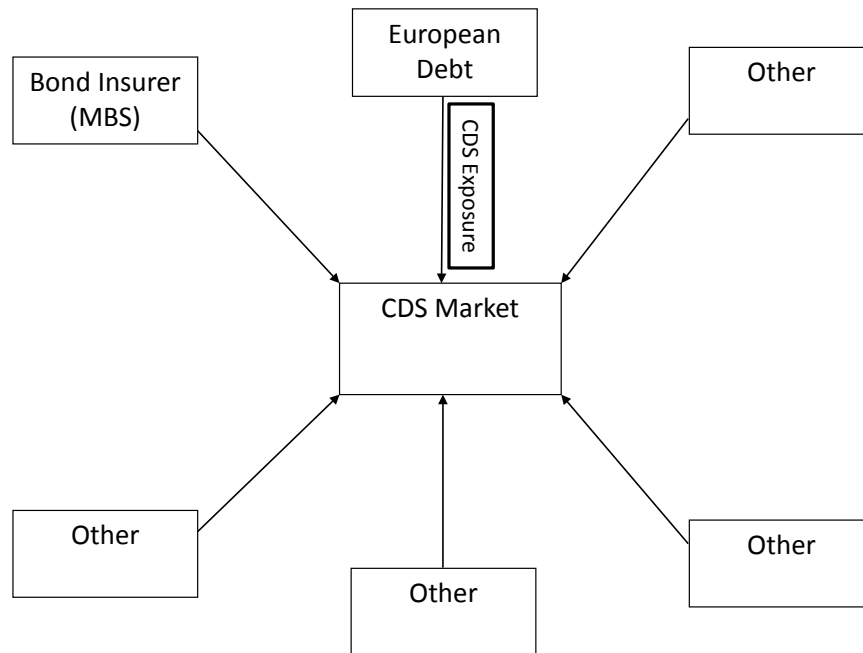
The CDS market can be recognized as a financial network structure that connects various financial institutions through complex CDS bilateral exposures and cross holdings. Some major players in the

<sup>15</sup> <http://www.gailfosler.com/featured/credit-default-swaps-and-the-financial-system-an-interview-with-marti-subrahmanyam>

center of the CDS market, such as AIG, have become “too interconnected to fail” since the failure of one of these institutions can bring down the entire financial system. Asymmetric and insufficient information disclosure imposes even more credit risk on financial institutions. Therefore, it is critical to develop a network model specifically for the CDS market, so that regulators or other market participants are able to identify those systemically important financial institutions, as well as assess the systemic risk under a certain set of circumstances. This paper proposes a CDS network model to assist regulators in monitoring the CDS network system and identifying those “hot spots”<sup>16</sup> which may result in total systemic failure.

Currently, the CDS market can be viewed as a capstone in the financial system. The CDS market is a network consisting of major banks, insurance companies, hedge funds and other institutions, all connected via CDS exposure, as shown in Figure 1. The various sectors of the CDS market may be viewed as large nodes, such as the mortgage-backed securities (MBS) market or the European sovereign debt market. The arcs between those markets and the CDS market are CDS exposures that cover the underlying securities from those markets. If one market sector encounters a crisis, the loss shock can propagate to the banks and insurance companies through the exposure links.

Figure 1. CDS Market in Financial System



In the CDS network, the large nodes, which represent certain market sectors, contain many small nodes which are banks and other major financial institutions. Each of the smaller nodes may have exposure to

<sup>16</sup> “Hot Spots” are nodes where the firm is “too big to fail”; where failure could have a devastating impact on the entire network.

more than one market sector, and each smaller node has its own balance sheet. The arcs linking different nodes are the various types of CDS exposures from different sectors.

Figure 2. CDS Network Structure

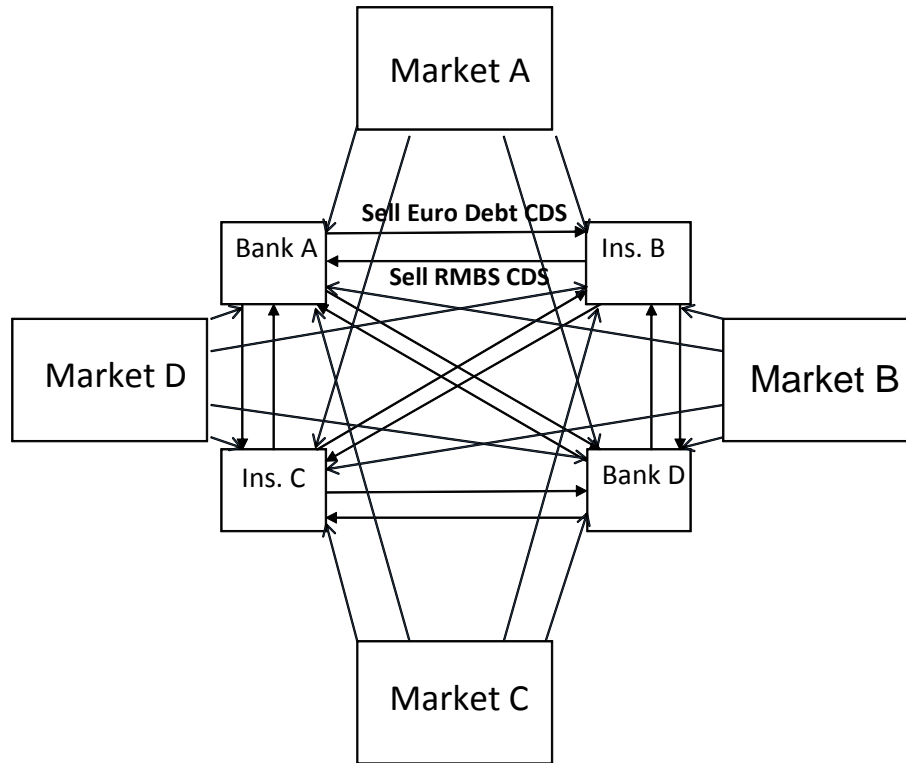


Figure 2 illustrates a plausible CDS network structure. Suppose a financial institution defaults as it suffers losses exceeding a certain threshold of its core capital. If a European debt market crisis causes *Bank A* and *Insurance Company C* to become bankrupt, the losses due to their defaults can propagate to other nodes who have bought Euro Debt CDS from *Bank A* and *Insurance Company C*. Given this condition, the **Systemic Risk Ratio of a sector** is defined as the ratio of the expected final loss of the total system to the expected initial loss of a sector:

$$\text{Sector Systemic Risk Ratio} = E(\text{Final Total System Loss})/E(\text{Initial Sector Loss})$$

Similarly, the **Systemic Risk Ratio of a financial institution** is defined as the ratio of the expected final loss of the total system to the expected initial loss caused by a specific institutional default:

$$\text{Institutional Systemic Risk Ratio} = E(\text{Final Total System Loss})/E(\text{Initial Loss caused by a Default})$$

The network structure of different market sectors may vary considerably. A big CDS seller in the RMBS market may be a big CDS buyer in the European debt market. Separate modules can be established to analyze the shock from a specific market sector (e.g. European debt crisis, sharp decline in housing prices), and to verify those “hot spots” in different market sectors. This paper proposes that the following five factors have the greatest impact on the systemic risk ratio:

- Weight of a sector in the CDS market
- Capital level
- Recovery rate (salvage ratio)
- Default criterion
- Degree of bilateral exposure within a sector

The impact of adding a national clearinghouse into the financial system is discussed later in the paper. This paper illustrates how a clearinghouse can restrain loss propagation and mitigate the systemic risk ratio.

In summary, this paper establishes a framework eligible for further development and expansion through building modules for different market sectors and collecting bilateral exposure data from banks, insurance companies, and other financial institutions. This network model aims to help regulators identify the companies that are too big or too interconnected to fail during any specific market crisis.

## Network Model for CDS Market

### Network Structure

CDS contracts are off balance sheet items, and therefore, “neither the SEC nor any regulator has authority over the CDS market, even to require minimal disclosure to the market.”<sup>17</sup> Hence, there is no explicit one-to-one bilateral CDS exposure data currently available. However, for each FDIC registered bank, the gross CDS purchase or sell data can be acquired from the FDIC database.

In order to construct a network structure for the CDS market in this study, an algorithm has been developed in which a bilateral connection matrix is generated stochastically in order to simulate a plausible CDS network reflecting the real market.<sup>18</sup> The bilateral connection matrix is generated in a manner that replicates the gross buy (sell) totals for each bank, but with connections to other banks that are randomly generated portions of the totals. In this way, the larger CDS market participants tend to have more connections and larger connections in the generated bilateral connection matrix.

It is important to understand that the network model requires only one bilateral connection matrix for input in order to produce its results. However, since the authors do not have access to complete data, they generate a large number of “plausible” matrices for the input, and run the model for each one. This generates a large number of “plausible” results that can be averaged or analyzed in other ways. This stochastic element of the model process would not be required if complete data were available.

The details of the algorithm used to generate a “plausible” bilateral connection matrix are presented below.

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<sup>17</sup> “Testimony Concerning Turmoil in U.S. Credit Markets: Recent Actions Regarding Government Sponsored Entities, Investment Banks and Other Financial Institutions”, <http://www.sec.gov/news/testimony/2008/ts092308cc.htm>

<sup>18</sup> This simulation of the one-to-one bilateral connections is performed as current data are not available.

Suppose there are  $N$  FDIC banks participating in the CDS market (Assume the gross CDS buy or sell amount is greater than zero), which are indexed as  $i=1, 2, \dots, N$ . The  $N+1$ th agent represents an external node that includes all other CDS trading entities except FDIC banks and is named "Other Entities." Based on the gross CDS buy (sell) data, the market share for each bank can be obtained in the following way:

$$S_i^B = \frac{B_i}{B} : \text{Bank}_i \text{ market share on the buy side of CDS}$$

$$S_i^S = \frac{S_i}{S} : \text{Bank}_i \text{ market share on the sell side of CDS}$$

where,

$B_i$  is the amount of CDS which  $\text{Bank}_i$  buys.

$S_i$  is the amount of CDS which  $\text{Bank}_i$  sells.

$B$  is the total amount of CDS bought within all banks.

$S$  is the total amount of CDS sold within all banks.

For  $\text{Bank}_i$ , the number of banks from which it buys CDS is calculated:

$$N_i^B = S_i^B \cdot N^S$$

where  $N^S$  is the total number of banks that have written CDS as guarantor (i.e. the number of banks for which  $S(i,s) > 0$ ).

Also, an  $N \times N$  bilateral trading probability matrix  $X$  is derived from CDS market share data of  $N$  banks:

$$X = \begin{bmatrix} 0 & x_{1,2} & \dots & x_{1,N} \\ x_{2,1} & 0 & & \cdot \\ \cdot & & 0 & x_{N-1,N} \\ x_{N,1} & \dots & x_{N,N-1} & 0 \end{bmatrix}$$

where  $x_{i,j}$  represents the likelihood of  $\text{Bank}_j$  buys CDS from  $\text{Bank}_i$ , equal to  $S_i^S / (1 - S_j^S)$  when  $i \neq j$ , and is zero when  $i = j$ . Since the bank index is following the order from the largest to the smallest CDS market share,  $x_{m,j} > x_{n,j}, \forall m < n, m \neq j$  and  $n \neq j$ .

A vector of random numbers is introduced to establish the bilateral connections of a plausible network structure. For example, when establishing the bilateral connections for  $\text{Bank}_j$ , the number of banks it

buys CDS from is represented by  $N_j^B$ . Hence,  $N_j^B$  random numbers that are uniformly distributed from 0 to 1 are generated, denoted as  $r_{j,k}^B, k = 1, 2, \dots, N_j^B$ .

In order to determine the first counterparty of  $Bank_j$ , a vector of trading probabilities  $y_{i,j}, i = 0, 1, \dots, N$  is obtained, where  $y_{i,j} = x_{i,j} / \sum_{i=1}^N x_{i,j}$ . The trading relationships between  $Bank_j$  and other banks are noted as  $I_{i,j}^B, i = 1, 2, \dots, N$ , and where  $I_{i,j}^B = 1$  indicates that  $Bank_j$  has bought CDS from  $Bank_i$ , whereas  $I_{i,j}^B = 0$  means there is no CDS bilateral exposure between  $Bank_j$  and  $Bank_i$ . The bank index number of the first counterparty is  $n^*$ , where  $n^* = \inf\{n | \sum_{i=1}^n y_{i,j} > r_{j,1}^B, n = 1, 2, \dots, N\}$ , and the counterparty is noted as  $Bank_{n^*}$ . The basic idea of this algorithm is to split the total probability space into  $N_j^B$  sections that reflect the corresponding trading probabilities, and the random number is used to anchor a bilateral connection within the total probability space and to determine the counterparty accordingly.

After the first counterparty is determined, a similar random process is performed for selecting the second one. However, the trading probability  $y_{i,j}$  is modified as  $y_{i,j} = x_{i,j} / \sum_{i=1, i \neq n^*}^N x_{i,j}, i \neq n^*$ , to exclude  $Bank_{n^*}$  from the bank list. Then,  $r_{j,2}^B$  is used to determine the index number of the second counterparty. By repeating this stochastic selection procedure,  $N_j^B$  counterparties of  $Bank_j$  are set. A stochastic bilateral connection matrix can be established by adopting a similar stochastic process for  $Bank_i, i = 1, 2, \dots, N$ . However, when  $I_{i,j}^B = I_{j,i}^B = 1$ , a random number (0~1) is generated. If it is less than 0.5,  $I_{i,j}^B=0$ ; otherwise,  $I_{j,i}^B=0$ . In the end, a stochastic bilateral connection matrix can be written:

$$I = \begin{bmatrix} 0 & I_{1,2}^B & \dots & I_{1,N}^B \\ I_{2,1}^B & 0 & & \cdot \\ \cdot & & 0 & I_{N-1,N}^B \\ I_{N,1}^B & \dots & I_{N,N-1}^B & 0 \end{bmatrix}$$

Based on the stochastic bilateral connection matrix  $I$ , the matrix of CDS trading amounts is obtained:

$$T = \begin{bmatrix} 0 & T_{1,2} & \dots & T_{1,N} \\ T_{2,1} & 0 & & \cdot \\ \cdot & & 0 & T_{N-1,N} \\ T_{N,1} & \dots & T_{N,N-1} & 0 \end{bmatrix}$$

where  $T_{i,j} = I_{i,j}^B \cdot B_i \cdot S_j^S \cdot S/B, i, j = 1, 2, \dots, N$ .

The upper limit for the number of banks from which  $Bank_j$  can buy CDS is  $N_j^B$ . If  $\sum_{j=1}^N T_{i,j} < B_i$ , the unallocated CDS purchasing amount is linked to the Other Entities.

In addition, the user of the CDS network model is allowed to input predetermined CDS trading connections and trading amounts with the model simulating the rest of the network structure. Users can accordingly test the systemic risk ratios under different network structures.

## Contagion Mechanism

To simplify the model, naked CDS positions are prohibited. A naked CDS position means that a bank buys CDS but does not hold the underlying debt or sells corresponding CDS to a third party.

In this paper, the primary reason that a bank buys CDS is to hedge its CDS selling position. After the selling position is fully covered, the remaining CDS long position aims to hedge the credit risk of the debt it holds. When  $Bank_i$  suffers losses exceeding 20% of its Tier One Capital<sup>19</sup>, it enters bankruptcy. If  $Bank_i$  becomes bankrupt, all the CDS it has written become worthless. Suppose that  $Bank_j$  has bought CDS from  $Bank_i$ , and therefore  $Bank_j$  needs additional capital to cover the emerged credit risk due to losing the CDS coverage. It is assumed that  $Bank_j$  is not able to inject sufficient capital in time, hence  $Bank_j$  has to write down its capital, and the loss amount equals the notional amount of debt that CDS covered.

Within a financial network,  $Bank_j$  may suffer losses caused by a specific bank default or multiple bank defaults due to a common shock. If the aggregate loss  $Bank_j$  suffers becomes greater than 20% of its Tier One Capital, it defaults too. The insolvency of  $Bank_j$  triggers further losses, and these losses propagate to other banks. This domino effect stops only when banks no longer become bankrupt. The ultimate loss that the system suffers may be a multiple of the initial shock.

## Measure of the Systemic Risk

### *Company Failure*

The Company Failure scenario is designed to estimate the systemic risk that a specific bank (noted as  $Bank A$ ) may pose to a certain market sector. In this scenario, the initial loss to the system equals the total CDS amount that  $Bank A$  sells for that specific sector. Remember this is still a scenario based stress test, which means that only one sector of the CDS market fails. If any bank fails due to  $Bank A$ 's default, the losses spread to other banks. The system becomes stable when banks stop failing.

The **Systemic Risk Ratio of a financial institution** is defined as the ratio of the expected final loss of the total system to the expected initial loss due to the institution's default:

$$\text{Institutional Systemic Risk Ratio} = E(\text{Final Total System Loss})/E(\text{Initial Loss caused by a default})$$

### *Sector Failure*

The Sector Failure scenario is designed to assess the systemic risk associated with a market sector. If a market sector collapses and the related debt defaults, all the banks that have written CDS in this market suffer losses. The model sets the initial loss suffered by each bank equal to a proportion of the initial sector loss, where the proportion is the CDS market share of that bank. The losses start to spread if any of the banks become insolvent after suffering their initial losses.

The **Systemic Risk Ratio of a sector** is defined as the ratio of the expected final loss of the total system to the expected initial loss from a sector failure:

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<sup>19</sup> The 20% threshold of Tier One Capital is commonly proposed as a critical point in literature



$$\text{Sector Systemic Risk Ratio} = E(\text{Final Total System Loss})/E(\text{Initial Sector Loss})$$

## Clearinghouse

In this model, a clearinghouse is set up as an intermediate between CDS buyers and sellers. By embedding a clearinghouse into the financial network model, loss shocks will not spread from an insolvent bank to other parts of the system. The clearinghouse guarantees that CDS are still valid even though the original writer defaults. The capital that is needed to support a clearinghouse can be estimated subject to a certain network structure, where accurate CDS bilateral exposure data is crucial for estimation. Because bilateral exposure data is unavailable, the authors do not analyze the capital adequacy of the clearinghouse in this paper.

By comparing the expected final loss of the system with the expected first round loss, the effect of a clearinghouse's contribution to the system robustness is demonstrated.

## Sensitivity Testing

There are five major factors that are incorporated into this network model including Segment Weight, Recovery Ratio, Capital Level, Default Criterion and Bilateral Exposure. When performing a sensitivity test for one specific factor, the values of the other factors are held constant. However, since the true bilateral exposure structure is not known, only two major CDS players (JP Morgan Chase and Citibank) are chosen for investigating the impact of bilateral exposure between certain nodes on the whole CDS system. The common network structure is derived from FDIC 2008 Q4 CDS aggregate exposure data.

All the sensitivity tests are based on the Sector Failure scenario. The sensitivity test results are summarized for each of the five factors' effects on systemic risk ratio and expected system loss, with and without a clearinghouse. Table 1 shows a list of default values of major factors (except degree of bilateral exposure), as well as the sensitivity test range.

**Table 1. The Default Value and Test Range of Four Major Factors**

Factor	Default Value	Lower Level	Upper Level
Segment Weight as of total CDS market	5%	0%	50%
Capital level	100%	0%	500%
Recovery Ratio (Salvage Ratio)	50%	0%	90%
Default Criterion (as % of Tier One Capital)	20%	0%	100%

### 1. Segment weight as a percentage of total CDS market

First, this paper examines the effect of segment weight on the systemic risk ratio and expected system loss. A higher segment weight implies that a sector represents more of the CDS market share, and therefore is more important. The failure of a dominant sector is more likely to trigger a systemic failure.

**Figure 3. Impact of Segment Weight**

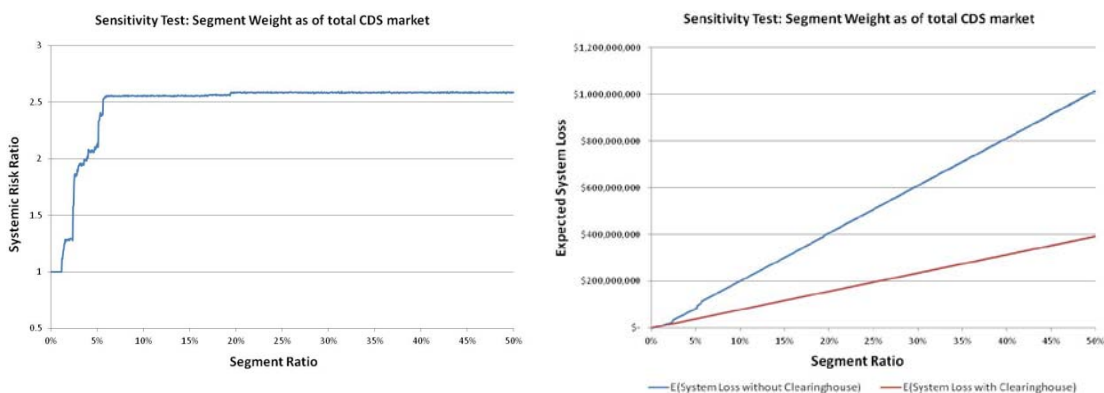
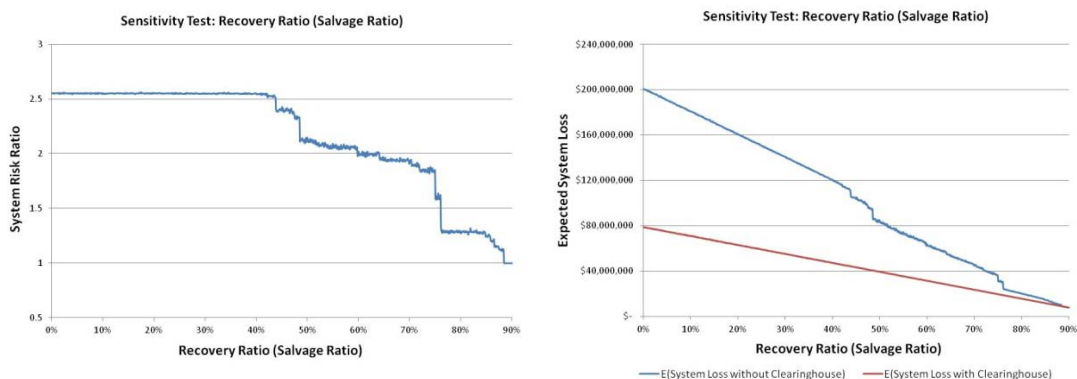


Figure 3 shows these results. The segment weight shows a positive but non-linear relationship with the systemic risk ratio. With all other factors held constant, the systemic risk ratio increases sharply as segment weight rises from 0% to about 8%, and then plateaus at a high level. The relationship between segment weight and expected system loss without a clearinghouse seems linear but shows some jumps in the lower range. In contrast, there is a clear linear relationship between segment weight and expected system loss with a clearinghouse. It is observed that the expected system loss with a clearinghouse is always smaller than without a clearinghouse.

**2. Recovery ratio (salvage ratio)**

In a second sensitivity test, this paper investigates the effect of the recovery ratio (salvage ratio) on the systemic risk ratio as well as the expected system loss. High recovery ratios mitigate losses from defaulting banks and bolster the system’s robustness. Keeping other factors the same, recovery ratios between 0% and 90% are tested.

**Figure 4. Impact of Recovery Ratio (Salvage Ratio)**



From Figure 4, it is apparent that up to a threshold of 40%, the recovery ratio does not affect the systemic risk ratio. Once the recovery ratio exceeds that threshold, the systemic risk ratio starts to decline and eventually converges to 1, which means there is no more contagion effect in the network. Figure 4 also implies an effect of the recovery ratio on the expected loss which is a mixture of linearity and jumps, but this effect is in the opposite direction of the segment weight effect.

### 3. Bank capital level and default criterion

Capital level measures capital sufficiency. Default criterion establishes the benchmark level of capital loss that a bank can bear while remaining solvent. With a fixed default criterion, higher capital levels decrease the default probability. If the capital level remains constant, a higher default criterion reduces the default probability. The sensitivity test results of banks' capital levels and default criteria (as % of Tier One Capital) in one sector are presented together because the effects of these two factors are essentially the same but on different scales.

Figure 5. Impact of Capital Level

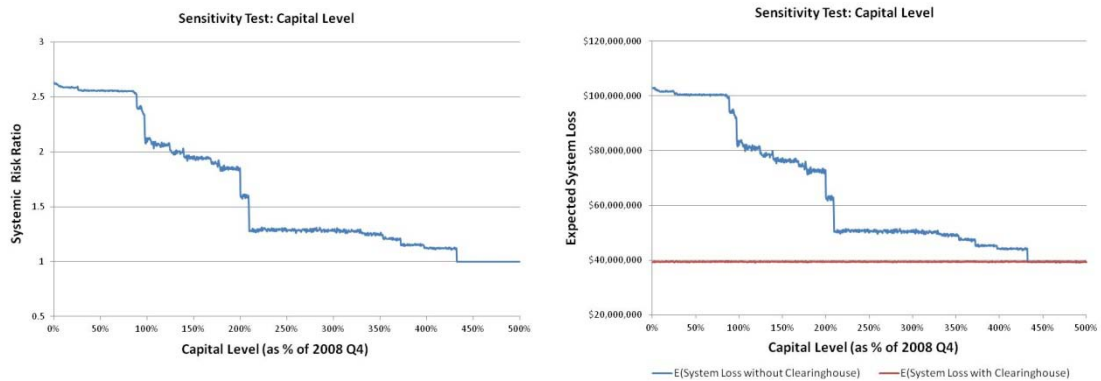
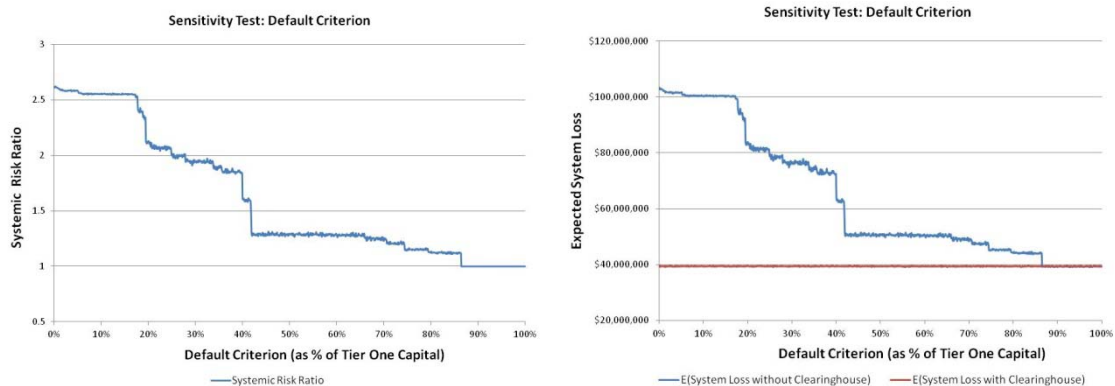


Figure 6. Impact of Default Criterion (as % of Tier One Capital)



Figures 5 and 6 indicate that capital level and default criterion have almost identical effects on the systemic risk ratio if viewed on the same horizontal axes. When these two factors increase, the systemic risk ratio gradually decreases to 1. Similarly, the impact of these two factors on the expected system loss seems to be identical. Particularly for the network with a clearinghouse, the expected loss is constant. This implies that the clearinghouse absorbs the first round losses, and thus capital level and default criterion can be viewed as independent of expected system loss.

#### **4. Degree of bilateral exposure**

Degree of bilateral exposure is expressed as the bilateral exposure amount, which has two opposing effects. Bilateral exposures may propagate losses to other institutions and then to the whole system, or the losses may be absorbed into the network via the bilateral exposure.

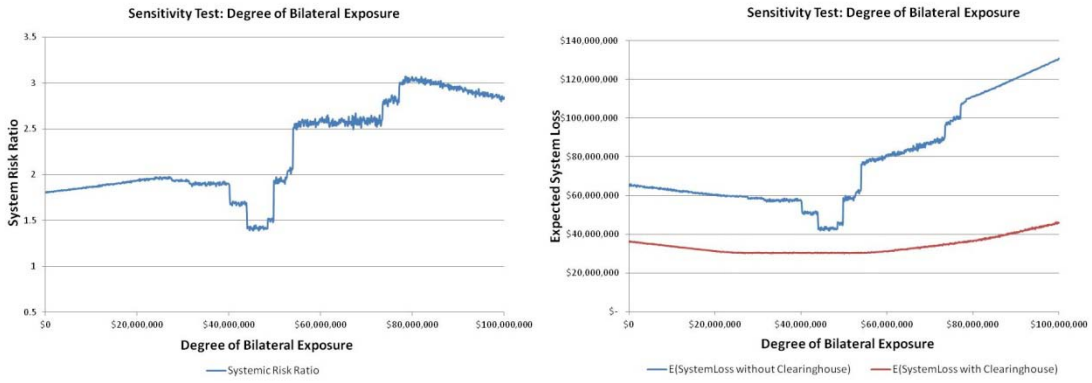
In a specific bilateral exposure sensitivity test, the authors establish a series of nominal CDS exposure amounts that JP Morgan Chase buys from Citibank, ranging from \$0 to \$100 million. The authors initially investigated the effect of bilateral exposure on the systemic risk ratio. From figure 7, two opposing effects can be seen over different ranges, generating a non-linear and non-monotonous curve. First, for low levels of exposure, the systemic risk ratio rises along with increases in bilateral exposure until a threshold is reached, and then it starts declining. As the exposure increases further, the systemic risk ratio rises again. But when exposure is sufficiently large, further increases in bilateral exposure decrease the systemic risk ratio.

Figure 7 also shows how the degree of bilateral exposure may affect the expected system loss, where a similar, non-linear relationship is observed in the system without a clearinghouse. However, when the exposure becomes sufficiently large, the expected system loss continues to rise as exposure increases, which is the opposite effect of the systemic risk ratio. This divergence implies that the systemic risk ratio may underestimate the systemic risk in some conditions. Consequently, using the systemic risk ratio is not enough to assess systemic risk. For the system with a clearinghouse, the system loss curve emulates a strangle<sup>20</sup> payoff curve.

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<sup>20</sup> A long strangle is an investment strategy implemented by buying both a call option and a put option of the same underlying security.

**Figure 7. Impact of bilateral exposure (JP Morgan Chase and Citibank)**



## CONCLUSION

Network models can be applied to different segments of the financial system in order to characterize systemic risk. In this study, network models of systemic risk were applied to a specific section of the financial market, the monoline bond insurance industry, as well as a broader section, the CDS market.

For the bond insurer network model, this study indicated that a dramatic drop in prices in the U.S. housing market resulted in significant systemic risk to the bond insurer system. Specifically, home price decreases in the U.S. housing market (the exogenous shock) did not reduce the value of the bond insurers' assets significantly, but the shock did dramatically increase the bond insurers' liabilities, particularly the reserves for the bonds. Thus, bond insurers became insolvent not because of the cross holding of assets but because of drastic increases in their liabilities.

For the CDS network model, the study indicated that higher segment weights, lower recovery ratios, lower capital levels and a lower default criterion resulted in higher systemic risk ratios, and these relationships were non-linear in nature. In contrast, the impact of bilateral exposure on the systemic risk ratio was more complicated, non-linear and non-monotonous.

Besides the systemic risk ratio, the other measure of systemic risk tested was the expected system loss. Under the scenarios without a clearinghouse, the sensitivity tests of the five factors on the expected system loss showed two general types of impacts. For segment weight and recovery ratio, the relationship was a mixture of linearity and jumps. Segment weight exhibited a positive relationship with expected system loss while recovery ratio exhibits a negative relationship. Capital level and default criterion sensitivity tests revealed negative, non-linear relationships while bilateral exposure had a more complicated, non-linear relationship with the expected system loss.

In scenarios where a clearinghouse was introduced to the system, the study showed that capital level and default criterion no longer affected the expected losses. Not surprisingly, the expected loss with a clearinghouse was always lower than the expected loss without a clearinghouse. This difference in expected system loss in the two scenarios was most notable when the default criterion, the capital level, or the recovery ratio was low or when the segment weight was high.

As seen in the bilateral exposure sensitivity tests, solely using the systemic risk ratio to assess systemic importance may be misleading because the expected system loss can still rise while the systemic risk ratio is declining. Thus, the authors propose incorporating both the systemic risk ratio and expected loss to assess systemic risk more comprehensively.

For both models, data limitations exist. Currently, there are no public data on one-to-one bilateral CDS exposures nor the individual bond liability associated with each secured bond in the system. Stochastic algorithms were performed to simulate the one-to-one exposures in the CDS market as well as the individual bond liability. A bilateral connection matrix was generated stochastically to simulate a plausible CDS network.

This study points out to the Financial Stability Oversight Committee critical data that are not available publicly or currently missing. By structuring the models so that the stochastically generated data can be replaced with data that may be obtained by the Financial Stability Oversight Committee, flexible models can be created to measure systemic risk in the financial system.

## APPENDICES

### Appendix 1 Asset Closeness Factor

The RMBS is backed by residential real estate. In the real world, the prices in the housing market are correlated. For instance, if the price of one house in a neighborhood goes up, the price of other houses in surrounding neighborhoods will probably rise too. Moreover, the housing prices in other regions may also increase. The co-movement of housing prices leads to correlation between different RMBS.

However, the RMBS correlation is implicitly incorporated in the market so it is not observable. In order to capture the implied RMBS correlation, an Asset Closeness Factor  $r$  is introduced to estimate the RMBS correlation. However,  $r$  is not equivalent to the correlation itself. The Asset Closeness Factor  $r$  reflects only one parameter in the correlation relationship.

In the model, it is assumed that the marked-to-market underlying asset values of different RMBS follow a correlated geometric Brownian motion.

For illustrative purposes, suppose there are two correlated RMBS underlying assets in the market, whose prices follow a correlated geometric Brownian motion:

$$dB_1(t)/B_1(t) = r_0dt + \sigma_1dX_1(t)$$

$$dB_2(t)/B_2(t) = r_0dt + \sigma_2dX_2(t)$$

where  $X_1(t)$  and  $X_2(t)$  are two correlated Brownian motions,

$X_1(t)$  and  $X_2(t)$  can be expressed as  $X_1(t) = \rho_1W_0 + \sqrt{1 - \rho_1^2}W_1$ , and  $X_2(t) = \rho_2W_0 + \sqrt{1 - \rho_2^2}W_2$ , where  $W_0$ ,  $W_1$  and  $W_2$  are three independent Brownian motions and  $\rho_1$  and  $\rho_2$  are the coefficients of correlation.

When considering  $n$  assets, the correlation between RMBS  $i$  and RMBS  $j$  can be written as  $\rho_{i,j}$ , which is unobservable in the market. The authors simulate different pairs of bond market share, and from these results, determine that the coefficient of correlation in the correlated Brownian motion is  $\hat{\rho}_{i,j} =$

$r^{\alpha(\frac{1}{w_i} + \frac{1}{w_j})}$ , where  $r$  is the Asset Closeness Factor and  $w_i$  represents the market share of RMBS  $i$ . Based on the change of the monoline bond insurance industry's liabilities from before the 2007 Financial Crisis to after the crisis (obtained from Bloomberg), the default value of  $r$  is calibrated as 0.7 and the default value of  $\alpha$  is 0.17.



## Appendix 2 – Asset and Liability Pricing Module Mechanism

### Model Construction

The bond insurer model is inspired by both the Black-Scholes framework and Moody's KMV Credit methodology. The insured RMBS price is derived from the option pricing theory, and the marked-to-market values of the insured RMBS's underlying assets follow a log-normal distribution.

The Asset Pricing Module is designed to estimate the market value of RMBS as well as the asset value of insurers. In the Liability Pricing Module, the marked-to-market RMBS's underlying asset distribution is simulated by applying a two-layer compound model. Subsequently, the loss reserves for RMBS are calibrated based on the asset distribution, the liability of the RMBS, and the default probability. The liabilities of insurers are obtained accordingly. These two modules are connected by cross referencing some key parameters, including the implied volatility. Ultimately, the balance sheet structures of insurers before and after shock are calculated, revealing the impact of a shock on the bond insurer network.

#### 1. Asset Pricing Module

Suppose in the bond insurance market there are  $i$  insurers,  $i = 1, 2, \dots, 6$ , denoted by C1, C2, ..., C6. Meanwhile, there are  $n$  RMBS,  $n = 1, 2, \dots, 6$ , denoted by B1, B2, ..., B6. B1 is insured by C1, B2 is insured by C2, etc. For modeling simplification, an assumption is made that all RMBS are zero coupon bonds.

The total assets of insurer  $i$  at time  $t$  can be written as:

$$I_i(t) = \sum_{n=1}^6 \beta_{i,n} V_n^{ins}(t) + A_i \quad (1)$$

where  $V_n^{ins}(t)$  represents the marked-to-market value of insured RMBS and  $\beta_{i,n}$  is the holding weight of  $V_n^{ins}(t)$  for insurer  $i$ . Additionally,  $A_i$  represents all other assets of the insurer  $i$ . Both  $\beta_{i,n}$  and  $A_i$  are assumed to be constant in the bond insurer model.

In order to obtain  $I_i(t)$ , the marked-to-market value of insured RMBS  $V_n^{ins}(t)$  has to be captured first. Starting with pricing an uninsured RMBS, suppose that a zero coupon RMBS  $n$  has marked-to-market underlying asset value  $B_n$  and face value  $D_n$ .  $B_n(t)$  is assumed to follow a geometric Brownian motion process:

$$dB_n(t)/B_n(t) = rdt + \sigma_n dX_n \quad (2)$$

The marked-to-market value of RMBS  $n$  without any insurance can be expressed as:

$$\begin{aligned} V_n(t) &= E^Q[\min(B_n(t), D_n) | F_t] \\ &= e^{-rT} * D_n - E^Q[(D_n - B_n(T))_+ | F_t] \\ &= e^{-rT} * D_n - Put(B_n(t), D_n, r_0, T, \sigma_n) \end{aligned} \quad (3)$$

where  $E^Q[\ ]$  is the expectation in the risk neutral probability space. The  $Put(B_n(t), D_n, r_0, T, \sigma_n)$  represents a European put option with the risk-free rate  $r_0$ , the time to maturity  $T$ , and the pre-shock regime implied volatility  $\sigma_n$ .<sup>21</sup>

The uninsured RMBS marked-to-market price is equivalent to the difference between the discounted face value of its liability and a put option of its underlying asset with strike price equal to  $D_n$ . The value of the put option can be calculated using the Black-Sholes formula.

Based on this preliminary equation (2), the marked-to-market price of the insured RMBS  $n$  can be developed, as:

$$\begin{aligned} V_n^{ins}(t) &= E^Q[\min(B_n(t) + I_i(t), D_n) | F_t] \\ &= e^{-rT} * D_n - E^Q[(D_n - I_i(t) - B_n(T))_+ | F_t] \\ &= e^{-rT} * D_n - Put(B_n(t), D_n - I_i(t), r_0, T, \sigma_n) \end{aligned} \quad (4)$$

where  $I_i(t)$  is the asset value of the insurer  $i$  at time  $t$ . In order to get a closed-form solution of equation (3), a simplifying assumption is made: the value of the insurer's assets stays constant, i.e.  $I_i(t) = I_i$ , where  $I_i$  is the asset value of the insurer  $i$  as of Q4, 2006. Hence  $V_n^{ins}(t)$  can be calculated as:

$$V_n^{ins}(t) = e^{-rT} * D_n - Put(B_n(t), D_n - I_i, r_0, T, \sigma_n) \quad (5)$$

The insured RMBS marked-to-market price is equal to the difference between the discounted face value of the bond and the value of a put option with a strike price  $(D_n - I_i)$ . The insured RMBS  $n$  defaults when  $B_n(t) + I_i < D_n$ , which means the total value of the bond's underlying asset plus the insurer's marked-to-market assets is less than the face value of its liability.

Before a shock strikes the bond insurer network,  $B_n(t)$  is the marked-to-market implied value of the RMBS underlying asset and is unobservable. There is no explicit value for  $\sigma_n$  either. In order to calibrate these two parameters, the insured and uninsured RMBS prices are addressed as present values of future cash flows.

The value of an uninsured RMBS can be expressed as the present value discounted at the yield rate  $y_n$ :

$$V_n(t) = D_n / (1 + y_n)^T \quad (6)$$

where  $y_n$  is derived from historical data.

Similarly, the value of an insured RMBS can be calculated as the present value discounted at the yield rate  $y_n^{ins}$ :

$$V_n^{ins}(t) = D_n / (1 + y_n^{ins})^T \quad (7)$$

<sup>21</sup> The pre-shock regime implied volatility  $\sigma_n$  is the implied volatility valid before the network suffers a shock. In the bond insurer network model, if a shock occurs, the implied volatility value will be changed to a so-called regime equilibrium volatility  $\sigma_n^*$  whose value is different from  $\sigma_n$ .

where  $y_n^{ins}$  comes from historical data and typically,  $y_n^{ins} < y_n$ .

However, in this model, it is assumed the loss reserve of RMBS, denoted as  $R_n$  is known before the shock. The loss reserve can be expressed as:

$$\begin{aligned} R_n(t) &= E^Q[(D_n - B_n(T))_+ | F_t] \\ &= e^{-rT} * D_n - V_n(t) \end{aligned} \quad (8)$$

Re-arrange equation (8) to obtain:

$$D_n = (R_n(t) + V_n(t))/e^{-rT} \quad (9)$$

Suppose a shock occurs at  $t_0$ . Combining equations (3), (6), and (9) results in equation (10) and combining equations (5), (7), and (9) results in equation (11):

$$(R_n(t) + V_n(t))/(e^{-rT}(1 + y_n)^T) = R_n(t) + V_n(t) - Put(B_n(t_0), D_n, r_0, T, \sigma_n) \quad (10)$$

$$(R_n(t) + V_n(t))/(e^{-rT}(1 + y_n^{ins})^T) = R_n(t) + V_n(t) - Put(B_n(t_0), D_n - I_i, r_0, T, \sigma_n) \quad (11)$$

Only parameters  $B_n(t_0)$  and  $\sigma_n$  need to be calculated since all other parameters can be acquired from empirical data. Moreover,  $B_n(t_0)$  and  $\sigma_n$  are used as initial values in the Liability Pricing Module.

As already mentioned, once the bond insurer network suffers a shock, the losses can spread through the network following the downgrading cycle mechanism until equilibrium is achieved. Usually, it takes several rounds of interactions before the network converges to a steady state. Due to the assumption that no insurer claims bankruptcy until the network reaches equilibrium, the asset value of the insurers after shock can be captured by analyzing the equilibrium value of RMBS.

Equation (1) shows that the asset value of the insurer can be expressed as a linear function of the marked-to-market insured RMBS. Thus, equation (5) can be modified as below:

$$V_n^{ins}(t) = e^{-rT} * D_n - Put(B_n(t_0), D_n - I_i(V_n^{ins}(t)), r_0, T, \sigma_n) \quad (12)$$

where the asset value of the insurer  $I_i$  is denoted as a function of insured RMBS.

Equation (12) implies that a smaller  $V_n^{ins}(t)$  will reduce  $I_i$ , and accordingly lead to a higher strike price  $D_n - I_i(V_n^{ins}(t))$ , which makes  $Put(B_n(t_0), D_n - I_i(V_n^{ins}(t)), r_0, T, \sigma_n)$  become a less out-of-the-money put option. As a result, the put option value will increase, whereas  $V_n^{ins}(t)$  will decrease.

The value of an RMBS when the network reaches equilibrium is denoted as  $V_n^{ins*}$ . The formula for  $V_n^{ins*}$  is:

$$V_n^{ins*} = e^{-rT} * D_n - Put(B_n(t_0), D_n - I_i(V_n^{ins*}), r_0, T, \sigma_n^*) \quad (13)$$

where  $\sigma_n^*$  is the shock regime implied volatility of RMBS  $n$ , whose value is different from  $\sigma_n$ .  $\sigma_n^*$  comes from the Liability Pricing Module, which will be explained in the subsequent section.

Recall that  $I_i(t) = \sum_{n=1}^6 \beta_{i,n} V_n^{ins}(t) + A_i$ . The asset value of insurer  $i$  at equilibrium is as follows:

$$I_i^*(T) = \sum_{n=1}^6 \beta_{i,n} V_n^{ins*}(T) + A_i, i = 1, 2, \dots, 6, n = 1, 2, \dots, 6 \quad (14)$$

## 2. Liability Pricing Module

This module is built to quantify the loss reserves for RMBS, and thus determines the liabilities of insurers. Briefly summarized, the loss reserve of RMBS is estimated based on its underlying asset value distribution and its default probability. During the 2007 Financial Crisis, it was observed that a bond's asset value distribution and default probability not only depend on its own characteristics, but also on the overall market. If a shock that strikes several bonds causes upheaval of the bond market, all the bonds, including those that are not affected by the initial shock, will be negatively impacted.

In order to incorporate market conditions, a two-layer model is established. More precisely, the distribution of the marked-to-market RMBS's underlying asset is derived from the product of two distributions: a Systematic Distribution and an Individual Bond Distribution, both of which follow a lognormal distribution. Additionally, it is assumed that the network will take one year to revert to equilibrium after an initial shock; thus, the time to maturity is fixed at  $T = 1$ . In this section, both the systematic and individual distributions will be presented in the form of  $LN(\mu, \sigma^2)$ .

The Systematic Distribution is a lognormal distribution with mean  $\mu_{sys}$  and variance  $\sigma_{sys}^2$ , denoted as  $LN_{sys}(\mu_{sys}, \sigma_{sys}^2)$ . It is used to measure the robustness of the whole industry under the Black-Scholes framework. Particularly, the change of market robustness is defined as the variance change of the distribution; in other words, when a shock or a rating downgrade occurs, the  $\sigma_{sys}^2$  will change but the  $\mu_{sys}$  will not be affected. On the other hand, the Individual Bond Distribution is written as  $LN_{ind}(\mu_{ind,n}, \sigma_{ind,n}^2)$ , where  $\mu_{ind,n}$  and  $\sigma_{ind,n}^2$  can be different for each RMBS.  $\mu_{ind,n}$  will be affected by the initial shock but  $\sigma_{ind,n}^2$  will not change.

The RMBS Asset Value Distribution, denoted as  $Dis_i^{RMBS}$ , is used to quantify the loss reserves of insurers. This distribution is compounded by  $LN_{sys}(\mu_{sys}, \sigma_{sys}^2)$  and  $LN_{ind}(\mu_{ind,n}, \sigma_{ind,n}^2)$  to comprehensively reflect market conditions and the bond's own characteristics:

$$Dis_n^{RMBS} = LN_{sys}(\mu_{sys}, \sigma_{sys}^2) * LN_{ind}(\mu_{ind,n}, \sigma_{ind,n}^2) \quad (15)$$

Since the product of two lognormal distributed variables still follows a lognormal distribution, the  $Dis_n^{RMBS}$  can be denoted as  $LN_n(\mu_n, \sigma_n^2)$ , and is the distribution of  $B_n$ .

The total liability of the insurer  $i$  at time  $t$  can be written as:

$$L_i(t) = R_n(t) + l_i \quad (16)$$

where  $R_n(t)$  is the loss reserve for RMBS  $n$  at time  $t$ .  $l_i$  represents all other liabilities and is assumed to be constant.

For RMBS  $n$ , the face value of its liability is a fixed value denoted as  $D_n$ , and its marked-to-market underlying asset value, which is denoted as  $B_n(t)$ , follows a lognormal distribution  $LN_n(\mu_n, \sigma_n^2)$ . With these two factors being incorporated,  $R_n(t)$  is defined as the tail value-at-risk (TVaR) given the bond default probability  $p_n$ . At time  $t_0$  (right before the shock occurs),  $\sigma_n$  is derived from the Asset Pricing Module and  $p_n$  is imported from historical data.  $R_n(t_0)$ , as well as  $L_i(t_0)$  can then be calibrated. When a shock strikes the network system, the value of  $\sigma_{sys}$  will increase. Meanwhile, for some or even all RMBS, the corresponding  $\mu_{ind,n}$  value will decline. Accordingly, the parameter values  $\mu_n$  and  $\sigma_n^2$  will change. By introducing the Asset Closeness factor  $r$ , a mechanism is developed so that  $\sigma_n$  goes from pre-shock regime volatility to an equilibrium implied volatility  $\sigma_n^*$ , and the default probability converges to  $p_n^*$ . The details of the mechanism are described in the next section. When the system achieves equilibrium at time  $T$ , loss reserve  $R_n(T)^*$  is calibrated as the TVaR given the default probability  $p_n^*$  under the lognormal distribution  $LN_n(\mu_n^*, \sigma_n^{*2})$ .

Finally, the total liability of the insurer  $i$  after the shock can be expressed as:

$$L_i(T)^* = R_n(T)^* + l_i \quad (17)$$

Therefore, the liability change of an insurer  $i$  can be written as follows:

$$\Delta L_i = L_i(T)^* - L_i(t_0) \quad (18)$$

$\sigma_n^*$  is then applied in the Asset Pricing Module to determine  $V_n^{ins*}$  and  $I_i^*(T)$  as in equations (13) and (14).

### The Mechanism of the Liability Pricing Module

The mechanism of the Liability Pricing Module is illustrated here. All the distribution parameters are presented as relative values for computing convenience, and they are subsequently adjusted by the corresponding market share of the bond. Consequently, the final results will be the same as using the absolute value directly.

Before the shock:

1. The parameter of Systematic Distribution is estimated as  $\mu_{sys} = 1$  and  $\sigma_{sys}^2 = 0.1$ .
2. For each RMBS, the pre-shock regime implied volatility  $\sigma_n$  is calibrated from the Asset Pricing Module, and  $\mu_n$  is set equal to 1. As the result, the RMBS Asset Distribution  $LN_n(\mu_n, \sigma_n^2)$  is known.
3. The RMBS aggregate liability  $L_0 = \sum_{n=1}^6 w_n \cdot D_n$ , where  $w_n$  is the market share of RMBS  $n$ . Based on the compound distribution of  $LN_n(\mu_n, \sigma_n^2)$ ,  $n = 1, 2, \dots, 6$ , the bond market default probability  $p_0$  can be obtained by simulation. As the asset value  $A_n$  of each RMBS is randomly generated, the market becomes default when  $L_0 > \sum_{n=1}^6 A_n$ . After simulating  $N = 1,000,000$  times,  $p_0 = m/N$ , where  $m$  is the total counts that default occurs.

After the Shock:

4. The underlying asset value of one or several RMBS decreases. For illustrative purposes, assume RMBS 1's asset value drops 10% due to the shock.
5. The mean of  $LN_{ind}(\mu_{ind,1}, \sigma_{ind,1}^2)$  decreases from 1 to 0.9. Thus, the value of  $\mu_{ind,1}$  is recalibrated, denoted as  $\mu'_{ind,1}$ .
6. For other RMBS in the market, the mean of  $LN_{ind}(\mu_{ind,n}, \sigma_{ind,n}^2)$  drops from 1 to  $(1 - 0.1 * r^{\frac{1}{w_n}})$ , where  $r$  is the Asset Closeness Factor. Accordingly, the value of  $\mu_{ind,n}$  is recalculated and is noted as  $\mu'_{ind,n}$ .
7. RMBS Asset Distribution changes, as  $LN_n(\mu'_n, \sigma_n'^2) = LN_{sys}(\mu_{sys}, \sigma_{sys}^2) * LN_{ind}(\mu'_{ind,n}, \sigma_{ind,n}^2)$ .
8. Accordingly, a series of evolved RMBS Asset Distribution is obtained as  $LN_n(\mu'_n, \sigma_n'^2)$ , then an evolved bond market default probability  $p'_0$  is calculated, subject to  $VaR(p'_0) = \sum_{n=1}^6 w_n \cdot D_n$ .
9. An updated Systematic volatility  $\sigma'_{sys}$  is recalibrated, based on the equation  $L_0 = VaR(p'_0)$ .
10. RMBS Asset Distribution is evolved as  $LN_n(\mu'_n, \sigma_n'^2) = LN_{sys}(\mu_{sys}, \sigma_{sys}'^2) * LN_{ind}(\mu'_{ind,n}, \sigma_{ind,n}^2)$ .
11. Back to step 8 to recalculate the bond market default probability, and repeat the loop from step 8 to step 10 until  $p'_0$  converged to equilibrium default probability  $p_0^*$ , and the corresponding equilibrium Asset Distribution is  $LN_n(\mu_n^*, \sigma_n^{*2})$ .
12. Calculate Tail Value at Risk (TVaR) given  $p_0^*$  under  $LN_n(\mu_n^*, \sigma_n^{*2})$ , as the loss reserve for RMBS.
13.  $\sigma_n^*$  is embedded in the Asset Pricing Module to quantify the RMBS market value after shock.

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Conference: Sovereign Debt and Defaults after the Financial Crisis of 2007-2008, November 28-29, 2011, the Federal Reserve Bank of Atlanta.

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IMF. Conference on Operationalizing Systemic Risk Monitoring. Part IV. "The Use of Network Analysis to Assess Systemic Risk"

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MITACS. Workshop on Financial Networks and Risk Assessment

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