

Rainfall Insurance with Derivatives

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Abstract: We discuss rainfall insurance using financial derivatives. Usual modeling is done for temperature related products. We gathered rainfall data in Mexico City over a period of five decades. We show that the time series data is stationary and normally distributed. Thus, we apply the closed form solution proposed by Stephen Jewson in 2003 to value swaps, calls and puts (with and without limits). The model can be used for practical purpose of pricing rainfall derivatives.

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“We all grumble about the weather, but – but – but, nothing is done about it.”

- Mark Twain

“Whilst some people are weather wise, most are otherwise.”

- Benjamin Franklin

Introduction

In the traditional management of natural hazards, governments play an active role. Risk management in natural hazards in developing countries is set as an exclusive domain of the governments. Even in developed countries, government role is extremely large. Reconstruction after Katrina struck New Orleans is a case in point. Most of the money for reconstruction will come directly or indirectly from various federal grants – mostly through FEMA.

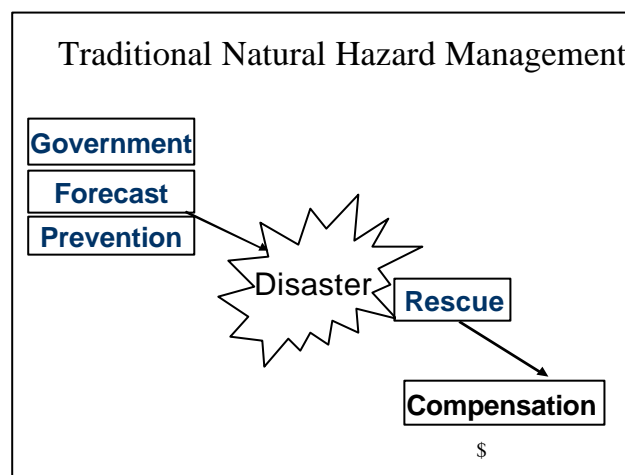
“Disaster Relief” is one of the biggest activities of a government. According to the International Disaster Database (www.md.ucl.ac.be/cred), out of 100 most expensive natural disasters during 1901-2000, Mexico accounts for seven. The losses were mostly uninsured. Therefore, the people covered the losses either directly out of pocket or indirectly by paying additional taxes.

There are two problems with this solution. First, out of pocket payment implies a higher variability of disposable income stream. It is more desirable to have a smoother flow of disposable income. Second, using tax-transfer mechanism can be an inefficient way of paying for losses because the tax collection mechanism is typically expensive. A private public partnership solution might be better.

Disaster relief is also the biggest source of political payoffs. If there is an election around the corner, the existing governments seem especially eager to be seen to be spending money on disaster relief.

Traditional model of management of natural hazards in the developing countries have the following scheme (see Figure 1a). The role of the government is to forecast the event, warn the citizens and try to prevent loss of life – and under certain circumstances, prevent certain kinds of property damage. It can also impose stricter building codes to be implemented. Once the disaster strikes, it undertakes the operation of rescue, and usually ad-hoc grant of compensation for the victims.

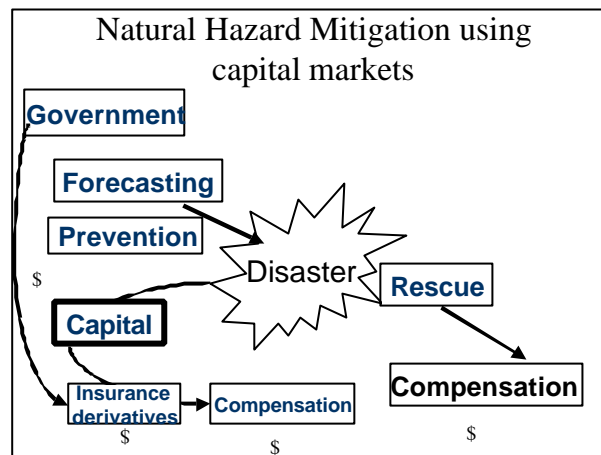
Figure 1a: Traditional Model



A more enlightened approach is to involve the capital market to reduce the financial burden of the victims. Thus, in addition to what the government does, the private sector would play an active role. The role played by disaster relief agencies, in many cases, do not have adequate safeguard in place for managing money. The use of private initiatives can help solve such problems. Take the case of FONDEN in Mexico. It is the government arm for dealing with natural disasters. It was created in 1996 to manage funds for disaster relief. In September 2005, a number of functionaries were

being investigated for inexplicable movement of funds. If FONDEN simply becomes a vehicle for buying insurance cover, such problems can be bypassed.

Figure 1b: Model of Disaster Management using Capital Markets



The model that includes private sector does not exclude government involvement in natural disaster. It adds capital market solutions in certain critical areas.

Problems with standard weather related insurance

The main problem is the of asymmetric information. Those involved in a production activity will always know more about their risk than any agency at the other end of the contract (whether it is the government or private insurance agency). If households are small (as it tends to be, for developing countries), it is nearly impossible for the counter-party obtain enough information to fully understand the risk. Thus, mistakes will be inevitable; bad loans will be made; bad insurance contracts will be written. Asymmetric information problems create dual problems of adverse selection and moral hazard. When adverse selection occurs, the lender or the insurer has not properly

assessed or classified the risk of their customer. Those who are more risky take out the loan with little intent to pay it back or those who are offered insurance decide that the insurance is under-priced and they are getting a good deal by purchasing it.

Moral hazard occurs after a loan is taken or after the insurance contract is taken out. It involves a change in behavior so that the customer represents more risk than what was believed to be the case. In the case of borrowers of funds, they may decide to use the loan for consumption rather than an income generating activity. Those who are insured may change their behavior in a way that increases the risks beyond what the insurer believed they would be when the insurance was developed.

There are traditional ways of monitoring. However, the administrative cost of monitoring is prohibitively high when each customer is small. This can be mitigated (under certain conditions) through collective action among neighbors who know already know one another. Social networks become important. This element lies at the heart of micro-finance as in Grameen bank. It is also an important element in micro-insurance as an alternative to crop insurance.

Crop Insurance Experience

To make money selling crop insurance premiums collected (P) must cover: payments (I) and administrative costs (A). In other words, for profitability, we need

$$P > (I + A) \text{ or } (I + A)/P < 1.$$

The actual country experience is listed in Table 1. The striking feature of the crop insurance programs in all countries in the list – regardless of whether they are developing or developed – is that the critical ratio $(I + A)/P > 1$ – the exact opposite of what is needed for the program to be actuarially viable.

Table 1: Crop insurance experience around the world

Country	Time Period	(I + A)/P
Brazil	1975-81	4.57
Costa Rica	1970-89	2.80
Japan	1947-77	2.60
Mexico	1980-89	3.65
Philippines	1981-89	5.74
USA	1980-89	2.42

Source: Hazell, P. B. R. 1992. "The Appropriate Role of Agricultural Insurance in Developing Countries." *Journal of International Development* 4: 567-581.

Mexico's FONDEN

Recently, Mexico's natural disaster fund FONDEN has started working towards using Cat Bonds to manage earthquake risks (see, the presentation of FONDEN in OECD <http://www.oecd.org/dataoecd/55/22/33884645.pdf>). In fact, in October 2005, FONDEN awarded the first contract of a Cat Bond. Given that risks of flooding is far greater (in terms of severity and frequency), our proposal is to suggest the possible creation of a market that would help alleviate the financial constraint by making funds available when needed using weather derivatives. Weather derivatives can circumvent the problems of traditional insurance due to adverse selection and moral hazard.

Using Weather Derivatives

Weather derivatives have been used over the past decade in mostly developed countries. There are certain advantages that weather derivatives can bring to developing countries.

Weather derivatives are financial contracts which payoffs depends on climate variables like temperature, rain, snow, wind speed or any weather variable that may be measured by a third (independent) party. Independent party could be governmental

agency or a private company with reputation. The payoff is related to an index over which the insured *does not* have any control.

Weather derivatives have several important elements. (1) The payoffs for weather derivatives do not depend on direct losses suffered by the insured. (2) They give the opportunity to cover a position against the effects of weather volatility that occurs in a very different geographical area than the one in which the owner of the contract is. (3) Weather derivatives provide the owner the possibility of covering against the effects of *volume* volatility. Other derivatives normally cover for *price* fluctuations.

Historically, weather derivatives have been used in the context of variation in temperature. The first weather derivative contract was closed during 1997 between Aquila Energy and Enron Corp. This contract was made over temperature variation. This is the weather variable over which the majority of weather contracts are written today. In fact, according to Weather Risk Management Association, over 90 percent of all weather insurance products are temperature related. But the importance of rainfall insurance through derivatives is rising.

Rainfall Risks

Who bears the burden of rainfall risks? Clearly agricultural products are the most important element. Rainfall clearly affects agricultural output. However, many segments of the entertainment industry can also be affected by rainfall. The examples would include golf courses, theme parks and beach resorts. Energy products can also be affected by rainfall indirectly. For example, the high energy price in 2005 was directly a consequence of hurricanes and floods that have affected oil production in the Gulf of Mexico encompassing several countries.

Our focus here is on Mexico City. Thus, interested parties for buying such weather derivatives could be the following industries: (1) Construction companies, (2) Beverage industry (e.g., soft drink, beer), (3) Automobile insurers, (4) Government agencies in charge of drainage and roads maintenance, (5) Tourism and entertainment industry, (6) Park and other public areas. Of course, individuals can also buy them to protect their properties against losses.

Modeling Weather Derivatives: Methodology

We use the methodology proposed by Jewson (2003). In what follows, we recapitulate the basic method of calculating the value of the contract using Jewson's methodology of finding closed form solutions in the case of normally distributed observations. In our case, the normality is extremely well suited for the data (see the data section).

Swaps

The payoff for a (long) swap is given by

$$p(x) = \begin{cases} -L_{\$} & \text{if } x \leq L_1 \\ D(x - K) & \text{if } L_1 \leq x \leq L_2 \\ L_{\$} & \text{if } x \geq L_2 \end{cases}$$

if $x \geq L_2$

where x is the index, D is the tick, K is the strike price, $L_{\$}$ is the limit expressed in currency. L_1 and L_2 are the upper and lower limits (in units of the index). $L_{\$} = D(K - L_1)$, $L_{\$} = D(L_2 - K)$.

A compact form would be to write the equation as

$$p(x) = \max(-L_{\$}, \min(D(x - K), L_{\$}))$$

If there is no limit, we can write it as

$$p(x) = D(x - K)$$

Calls

The payoff for a (long) call is given by:

$$p(x) = \begin{cases} 0 & \text{if } x \leq K \\ D(x - K) & \text{if } K \leq x \leq L \\ L_{\$} & \text{if } x \geq L \end{cases}$$

Similarly a long put is valued as

$$p(x) = \begin{cases} L_{\$} & \text{if } x \leq L \\ D(K - x) & \text{if } L < x \leq K \\ 0 & \text{if } x \geq L \end{cases}$$

Notation

To derive closed form solutions for the expected payoffs for the normal distribution, we note some properties of the normal density and distribution functions. The density of a standard normal distribution (with mean 0 and variance 1) will be denoted by:

$$n(x) = n_x = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

The corresponding accumulated distribution is given by:

$$N(x) = N_x = \int_{-\infty}^x n_y d_y \tag{5.7}$$

The density of a normal distribution with mean μ and standard deviation σ is given by:

$$\frac{1}{s} n \left(\frac{y - m}{s} \right) = \frac{n x'}{s} \text{ where we use } x' = \frac{x - m}{s}$$

Deriving equations for calculating expected payoffs

In this subsection, we derive the expected payoff for each of the instruments: swap, calls and puts. The expected payoff allows us to calculate the actuarially fair price and under a competitive market (without the problem of adverse selection and moral hazard), it is also the long run average payoff. It can also be the arbitrage-free price (Jewson and Zervos, 2003).

The expected payoff for a swap is given by the following expression:

$$\mathbf{m}_{pswap} = \frac{\sum_t^T D(x_{t/T} - K)}{n}$$

The expected payoff for a call is given by the following expression:

$$\mathbf{m}_{pcall} = \frac{\sum_t^T D(\text{Max}(0, x_{t/T} - K))}{n}$$

The expected payoff for a call with limits is given by the following expression:

$$\mathbf{m}_{pcall} = \frac{\sum_t^T \text{Min}\{D(\text{Max}(0, x_{t/T} - K), L_s)\}}{n}$$

The expected payoff for a put is given by the following expression:

$$\mathbf{m}_{pput} = \frac{\sum_t^T D(\text{Max}(0, k - x_{t/T}))}{n}$$

The expected payoff for a put with limits is given by the following expression:

$$m_{put} = \frac{\sum_t^T \text{Min}\{D(\text{Max}(0, K - x_{t/T})), L_{\$}\}}{n}$$

Under normality, the relevant expressions are the following

For a swap:

$$\begin{aligned} m_{swap} &= \frac{1}{s} \int_{-\infty}^{\infty} p(x) n_x dx \\ &= \frac{1}{s} \int_{-\infty}^{\infty} D(x - K) n_x dx \\ &= \frac{D}{s} [s(m - K)] \\ &= D(m - K) \end{aligned}$$

For a call:

$$\begin{aligned} m_{pcall} &= \frac{1}{s} \int_{-\infty}^{\infty} p(x) n_x dx \\ &= \frac{1}{s} \int_k^{\infty} D(x - K) n_x dx \\ &= \frac{D}{s} [s^2 n_{K'} + s(m - K)(1 - N_{K'})] \\ &= Ds n_{K'} + D(m - K)(1 - N_{K'}) \end{aligned}$$

For a put:

$$\begin{aligned}m_{pput} &= \frac{1}{\mathbf{s}} \int_{-\infty}^{\infty} p(x) n_x dx \\ &= \frac{1}{\mathbf{s}} \int_{-\infty}^K D(K-x) n_x dx \\ &= -\frac{D}{\mathbf{s}} [\mathbf{s}^2 (-n_{K'} + \mathbf{s}(\mathbf{m}-K)N_{K'})] \\ &= D\mathbf{s}n_{K'} + DN(K-\mathbf{m})\end{aligned}$$

We use three different values of K – the strike price following the suggestion by Jewson (2003).

$$K_1 = \mathbf{m} + 0.5\mathbf{s}$$

$$K_2 = \mathbf{m} + \mathbf{s}$$

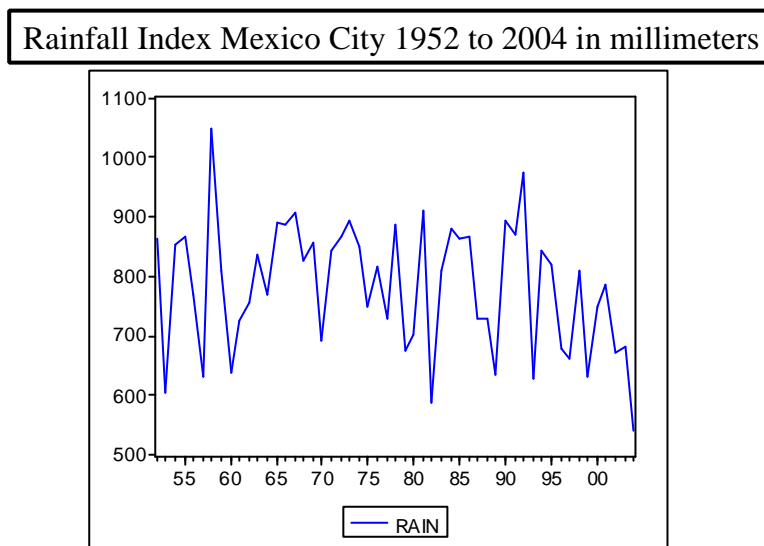
$$K_3 = \mathbf{m} + 1.5\mathbf{s}$$

We use the following methods for determining the prices: (1) Burn Analysis (also called historical analysis) (2) Index Modeling – the method of calculating premiums using a distribution fitted to the data at hand. We report them in the data section.

Data

We collected data on rainfall in Mexico from the City Government files. The data was available on a daily basis since 1933. However, there were too many cases of non-reporting in the first two decades. Thus, we included our data from 1952. Before we can proceed analyzing the data, the first element we have to test is for trends in the data. We perform a standard Augmented Dickey Fuller test for determining if the data has unit roots. The results show the absence of a unit root at 1% level of significance.

Figure 2: Annual Rainfall in Mexico City



Results

First, we start with the descriptive statistics of the original data. Note that unless the data used can be shown to have Normal distribution, our theoretical pricing model will be of no use. We run two tests to check the validity of Normal distribution. The standard Jarque-Bera test statistic shows that we cannot reject the Normality of the distribution for the entire time period. We also examined the validity for each subperiod considered (not reported here). All results point to Normality of the distribution. The same is true when we do a quantile quantile plot against a Normal distribution (see Figures 3 and 4).

Figure 3: Summary statistics for rainfall in mm in Mexico City

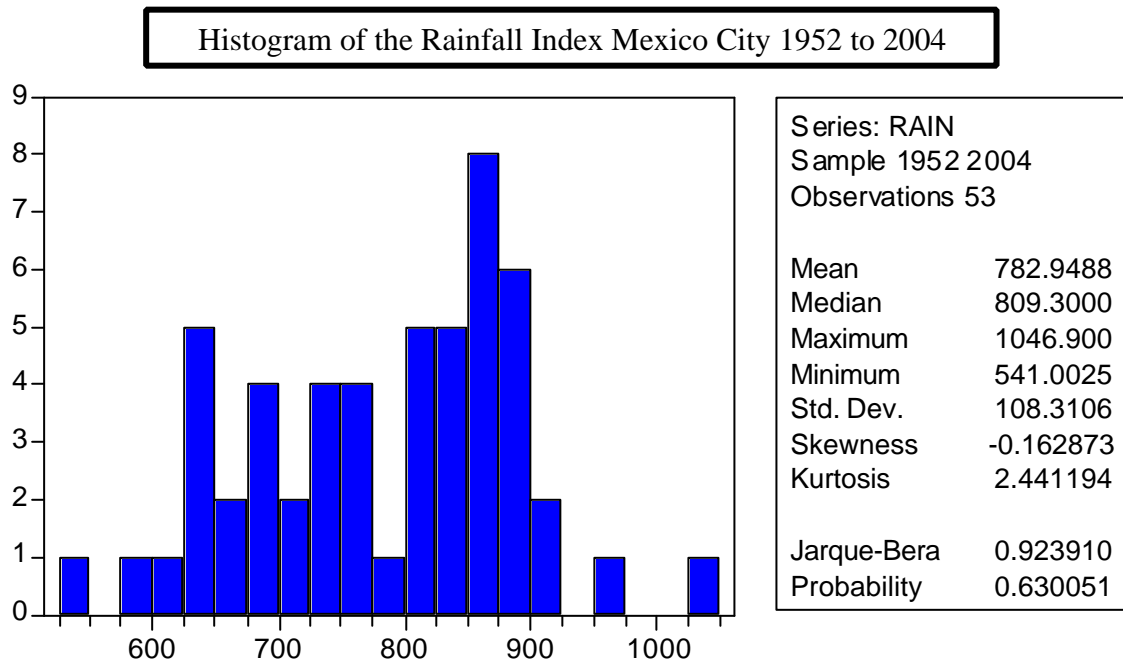
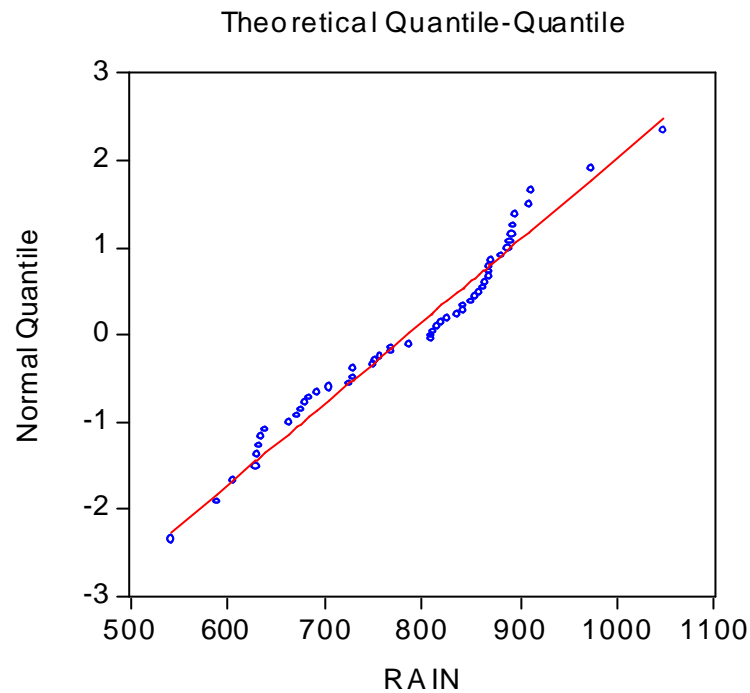


Figure 4: A quantile-quantile plot using theoretical Normal distribution



Next, we divide our sample into three. In the first sample, we take the entire time period (called Series 1 below). In the second, we take a subperiod of the last 30 years (called Series 2 below). Finally, we take a subperiod of 20 years (called Series 3). For each of the series, we perform burn analysis and index modeling with different instruments: swaps, calls (with or without limits) and puts (with or without limits). They are reported in the tables below.

Table 2: Burn analysis and Index modeling, 1952-2004

Series 1							
Burn Analysis				Modeling Index			
	σ				σ		
Swap	0.5	1	1.5	Swap	0.5	1	1.5
K	837.13	891.31	945.50	K	837.13	891.31	945.50
P	54,182.38	108,364.76	162,547.15	P	54,182.38	108,364.76	162,547.15
	σ				σ		
Call	0.5	1	1.5	Call	0.5	1	1.5
K	837.13	891.31	945.50	K	837.13	891.31	945.50
P	19,647.72	5,281.17	2,428.04	P	21,434.55	4,717.12	3,175.63
	σ				σ		
Put	0.5	1	1.5	Put	0.5	1	1.5
K	728.77	674.58	620.40	K	728.77	674.58	620.40
P	24,022.81	9,684.20	2,384.01	P	21,434.55	4,717.12	3,175.63
	σ				σ		
Call with limit	0.5	1	1.5	Call with limit	0.5	1	1.5
K	837.13	891.31	945.50	K	837.13	891.31	945.50
P	16,904.03	4,232.37	2,401.54	P	17,656.36	5,154.32	2,900.49
	σ				σ		
Put with limit	0.5	1	1.5	Put with limit	0.5	1	1.5
K	728.77	674.58	620.40	K	728.77	674.58	620.40
P	21,165.32	9,050.59	2,384.01	P	17,656.36	5,154.32	2,900.49

Table 3: Burn analysis and Index modeling, 1975-2004

Series 2							
Burn Analysis				Modeling Index			
	σ				σ		
Swap	0.5	1	1.5	Swap	0.5	1	1.5
K	814.56	868.75	922.93	K	814.56	868.75	922.93
P	-	-	-	P	-	-	-
	54,182.38	108,364.76	162,547.15		54,182.38	108,364.76	-162,547.15
	σ				σ		
Call	0.5	1	1.5	Call	0.5	1	1.5
K	814.56	868.75	922.93	K	814.56	868.75	922.93
P	22,107.55	6,741.00	1,661.65	P	21,434.55	9,027.87	3,175.63
	σ				σ		
Put	0.5	1	1.5	Put	0.5	1	1.5
K	706.20	652.02	597.83	K	706.20	652.02	597.83
P	22,329.46	7,838.44	2,218.85	P	21,434.55	9,027.87	3,175.63
	σ				σ		
Call with limit	0.5	1	1.5	Call with limit	0.5	1	1.5
K	814.56	868.75	922.93	K	814.56	868.75	922.93
P	20,167.07	6,606.61	1,661.65	P	17,656.36	7,899.00	2,900.49
	σ				σ		
Put with limit	0.5	1	1.5	Put with limit	0.5	1	1.5
K	706.20	652.02	597.83	K	706.20	652.02	597.83
P	19,552.96	7,471.31	2,218.85	P	17,656.36	7,899.00	2,900.49

Table 4: Burn analysis and Index modeling, 1985-2004

Series 3							
Burn Analysis				Modeling Index			
	σ				σ		
Swap	0.5	1	1.5	Swap	0.5	1	1.5
K	809.13	865.23	921.34	K	809.13	865.23	921.34
P	-	-	-	P	-	-	-
	56,101.37	-112,202.75	-168,304.12		56,101.37	-112,202.75	-168,304.12
Call	σ			Call	σ		
	0.5	1	1.5		0.5	1	1.5
K	809.13	865.23	921.34	K	809.13	865.23	921.34
P	23,259.68	7,150.88	2,572.10	P	22,193.70	9,347.61	3,288.10
Put	σ			Put	σ		
	0.5	1	1.5		0.5	1	1.5
K	696.93	652.02	597.83	K	696.93	640.83	584.73
P	22,197.01	7,838.44	2,218.85	P	22,193.70	9,347.61	3,288.10
Call with limit	σ			Call with limit	σ		
	0.5	1	1.5		0.5	1	1.5
K	809.13	865.23	921.34	K	809.13	865.23	921.34
P	20,077.44	6,773.72	2,572.10	P	17,998.86	8,079.13	2,973.29
Put with limit	σ			Put with limit	σ		
	0.5	1	1.5		0.5	1	1.5
K	696.93	652.02	597.83	K	696.93	640.83	584.73
P	19,400.62	7,471.31	2,218.85	P	17,998.86	8,079.13	2,973.29

Conclusion

In many cases illustrated by Tables 2, 3 and 4 we observe that the two types of analysis do not necessarily yield the same premium for the options discussed: swaps, calls with and without limits and puts with and without limits. The results differ because in the case of index modeling we force a distribution to our data. However, the results do not differ by large amounts in most cases. Therefore, we consider our results to be robust.

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