

QFI-Investment Risk Management Formula Package

Spring 2025

Exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula package was developed by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not in the formula package.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes. In sources where some equations are numbered and others are not, the page number is provided instead.

Chapter 1 (Black-Scholes formula to be used for questions of other chapters)

P. 7

$$p_0 = Ke^{-rT} \Phi(-d_2(0, T)) - S_0 \Phi(-d_1(0, T))$$

$$(1.3), (1.4) \quad d_1(t_1, t_2) = \frac{\ln(S_{t_1}/K) + (r + \sigma^2/2)(t_2 - t_1)}{\sigma\sqrt{t_2 - t_1}}$$

$$d_2(t_1, t_2) = d_1(t_1, t_2) - \sigma\sqrt{t_2 - t_1}$$

Chapter 3 Risk Measures

P. 79

$$(3.7) \quad ES_\alpha = \frac{(\beta' - \alpha)Q_\alpha + (1 - \beta')E[L|L > Q_\alpha]}{1 - \alpha}$$

P.83

(3.11) (3.12)

$$ES_\alpha = \mu + \frac{\sigma}{1 - \sigma} \phi\left(\frac{Q_\alpha - \mu}{\sigma}\right) = \mu + \frac{\sigma}{1 - \sigma} \phi(z_\alpha)$$

Chapter 6 Copulas

$$P. 169 - 170 \quad F_j(x_j) = \Pr[X_j \leq x_j]$$

$$C(F_1(x_1), \dots, F_m(x_m)) = \Pr[X_1 \leq x_1, \dots, X_m \leq x_m] \quad \text{for } j=1, \dots, m$$

$$C(u_1, u_2, \dots, u_m) = u_1 * u_2 * \dots * u_m$$

$$C(u_1, u_2, \dots, u_m) = \min(u_1, u_2, \dots, u_m)$$

$$C(u, v) = \max(u + v - 1, 0)$$

$$P. 181 - 182 \quad C(u_1, u_2, \dots, u_m) = (u_1^{-\theta} + u_2^{-\theta} + \dots + u_m^{-\theta} - (m-1))^{-\frac{1}{\theta}}, \theta > 0$$

$$C(u_1, u_2, \dots, u_m) = \exp\left\{-((- \ln(u_1))^\theta + (- \ln(u_2))^\theta + \dots + (- \ln(u_m))^\theta)^{\frac{1}{\theta}}\right\}, \theta > 1$$

$$C(u_1, u_2, \dots, u_m) = \Phi_\rho(z_{u_1}, z_{u_2}, \dots, z_{u_m}), -1 \leq \rho \leq 1, z_\alpha = \Phi^{-1}(\alpha)$$

$$\bar{C}(u, v) = \Pr[F_x(X) > u, F_y(Y) > v], \hat{C}(u, v) = \bar{C}(1-u, 1-v)$$

$$\rho(X, Y) = \frac{\text{Cov}[X, Y]}{\sqrt{\text{Var}[X] \text{Var}[Y]}}$$

$$\rho_s(X, Y) = \rho(U, V)$$

$$r = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{r_{x_i} - \bar{r}_x}{s_{r_x}} \right) \left(\frac{r_{y_i} - \bar{r}_y}{s_{r_y}} \right)$$

P. 185 - 189

$$\tau_k = \Pr[(X - X^*)(Y - Y^*) > 0] - \Pr[(X - X^*)(Y - Y^*) < 0]$$

$$= E[\text{sign}((X - X^*)(Y - Y^*))]$$

$$\text{sign}(z) = \{-1, z < 0, 0, z = 0, 1, z > 0\}$$

$$t_k = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{sign}((x_i - x_j)(y_i - y_j))$$

$$C(u, v) = \phi^{-1}(\phi(u) + \phi(v))$$

$$\tau_k = 1 + 4 \int_0^1 \frac{\phi(t)}{\frac{d}{dt} \phi(t)} dt$$

$$\lambda_U = 2 - 2 \lim_{t \rightarrow 0} \frac{\frac{d}{dt} \phi^{-1}(2t)}{\frac{d}{dt} \phi^{-1}(t)}$$

$$\lambda_L = 2 \lim_{t \rightarrow \infty} \frac{\frac{d}{dt} \phi^{-1}(2t)}{\frac{d}{dt} \phi^{-1}(t)}$$

P. 193 - 194

$$\tau_k = \frac{\theta}{\theta + 2}, \lambda_U = 0, \lambda_L = 2^{\frac{-1}{\theta}}$$

$$\tau_K = \frac{\theta - 1}{\theta}, \lambda_U = 2 - 2^{1/\theta}, \lambda_L = 0$$

$$\phi(u) = \frac{(u^{-\theta} - 1)^\delta}{\theta^\delta}$$

Chapter 8 Market Risk Models

P. 233

$$dS_t = \mu^* S_t dt + \sigma S_t dW_t, \sigma > 0$$

$$\frac{S_{t+k}}{S_t} \sim \text{LogN}(k(\mu^* - \frac{\sigma^2}{2}), \sqrt{k}\sigma), k > 0$$

$$\Rightarrow \text{Log}(\frac{S_{t+k}}{S_t}) \sim N(k(\mu^* - \frac{\sigma^2}{2}), k\sigma^2)$$

$$Y_{t,h} \sim N(h\mu, h\sigma^2)$$

P. 235

$$e^{Y_{t,h}} = \frac{S_t}{S_{t-h}} \sim \text{LnN}(h\mu, \sqrt{h}\sigma)$$

P. 236

$$(8.1) \quad (8.2) \quad Y_t = \mu + \sigma_t \varepsilon_t$$

$$\sigma_t^2 = a_0 + a_1(Y_{t-1} - \mu)^2 + b\sigma_{t-1}^2, a_0, a_1, b > 0$$

P. 238

$$E_0[\sigma_t^2] = a_0 \left(\frac{1 - (a_1 + b)^{t-1}}{1 - (a_1 + b)} \right) + (a_1 + b)^{t-1} \sigma_1^2$$

For

$$a_1 + b \neq 1$$

(8.6)

$$\lim_{t \rightarrow \infty} a_0 \left(\frac{1 - (a_1 + b)^{t-1}}{1 - (a_1 + b)} \right) + (a_1 + b)^{t-1} \sigma_1^2 = \frac{a_0}{1 - (a_1 + b)}$$

P. 244, 246

$$(8.9) \quad (8.10) \quad (8.11) \quad (8.12) \quad Y_t | \{\rho_t = 1\} \sim N(\mu_1, \sigma_1^2)$$

$$Y | \{\rho_t = 2\} \sim N(\mu_2, \sigma_2^2)$$

$$\Pr[\rho_t = 1 | \rho_{t-1} = 1] = p_{11}$$

$$\Pr[\rho_t = 1 | \rho_{t-1} = 2] = p_{21}$$

$$\Pr[\rho_t = 2 | \rho_{t-1} = 1] = p_{12} = 1 - p_{11}$$

$$\Pr[\rho_t = 2 | \rho_{t-1} = 2] = p_{22} = 1 - p_{21}$$

$$\pi_1 = \frac{p_{21}}{p_{12} + p_{21}}$$

$$\pi_2 = \frac{p_{12}}{p_{12} + p_{21}}$$

P. 250

$$(8.13) \quad l(\theta) = \sum_{i=1}^n \text{Ln} f(x_i | x_1, \dots, x_{i-1}; \theta)$$

Chapter 9 Short-Term Portfolio Risk

$$\begin{aligned} L &\sim N(\mu_L, \sigma_L^2) \\ \text{P. 263} \quad \text{VaR}_\alpha(L) &= \mu_L + z_\alpha \sigma_L \\ \text{ES}_\alpha(L) &= \mu_L + \frac{\phi(z_\alpha)}{1-\alpha} \sigma_L \end{aligned}$$

$$(9.1) \quad \text{VaR}_\alpha(L_{t,h}) = Q_\alpha(L_{t,h}) = -Q_{1-\alpha}(R_{t,h})V(t) = Q_\alpha(-R_{t,h})V(t)$$

$$R_{j,t,h} = \frac{(S_j(t+h) - S_j(t))}{S_j(t)}$$

$$L_{t,h} = -\sum_{j=1}^n a_j S_j(t) R_{j,t,h}$$

$$\begin{aligned} \text{P. 266 - 267} \quad L_{t,h} &\sim N(0, h\sigma_L^2) \\ \sigma_L^2 &= \sum_{k=1}^n \sum_{j=1}^n a_j a_k S_j(t) S_k(t) \rho_{jk} \sigma_j \sigma_k \\ \text{VaR}_\alpha(L_{t,h}) &= z_\alpha \sqrt{h} \sigma_L \\ \text{ES}_\alpha(L_{t,h}) &= \frac{\phi(z_\alpha)}{1-\alpha} \sqrt{h} \sigma_L \end{aligned}$$

$$V(t+h) - V(t) \approx \frac{\partial V}{\partial S}(S(t+h) - S(t)) + \frac{1}{2} \frac{\partial^2 V}{\partial S^2}(S(t+h) - S(t))^2 + \dots + \frac{\partial V}{\partial t} h$$

$$L_{t,h} = V(t) - V(t+h) \approx -\Delta(t)S(t)R_{t,h}$$

$$\begin{aligned} \text{P. 268 - 269} \quad \sigma_L &= |\Delta(t)| S(t) \sigma_S \\ \text{VaR}_\alpha(L_{t,h}) &\approx z_\alpha \sqrt{h} \sigma_L = z_\alpha \sqrt{h} |\Delta(t)| S(t) \sigma_S \\ \text{ES}_\alpha(L_{t,h}) &\approx \frac{\phi(z_\alpha)}{1-\alpha} \sqrt{h} \sigma_L = \frac{\phi(z_\alpha)}{1-\alpha} \sqrt{h} |\Delta(t)| S(t) \sigma_S \end{aligned}$$

$$\Gamma_j(t) = \frac{\phi(d_1(t))}{S_j(t) \sigma \sqrt{T-t}}$$

$$\text{P. 273} \quad V(t) - V(t+h) \approx -\Delta_j(t) S_j(t) R_{j,t,h} - \frac{1}{2} \Gamma_j(t) S_j(t)^2 R_{j,t,h}^2$$

$$L_{t,h} \approx -\Delta_j(t) S_j(t) R_{j,t,h} - \frac{1}{2} \Gamma_j(t) S_j(t)^2 R_{j,t,h}^2$$

$$\begin{aligned}
VaR_\alpha &\approx h\mu_L + z_\alpha \sqrt{Var[L_{t,h}]} \\
&= -h\frac{1}{2}\Gamma_j(t)S_j(t)^2\sigma_j^2 + z_\alpha \sqrt{\Delta_j(t)^2 S_j(t)^2 \sigma_j^2 h + \frac{1}{2}\Gamma_j(t)^2 S_j(t)^4 \sigma_j^4 h^2} \\
(9.14) \quad (9.15) \quad ES_\alpha &\approx \mu_L h + \frac{\phi(z_\alpha)}{1-\alpha} \sqrt{Var[L_{t,h}]} \\
&= \frac{-h}{2}\Gamma_j(t)S_j(t)^2\sigma_j^2 + \frac{\phi(z_\alpha)}{1-\alpha} \sqrt{\Delta_j(t)^2 S_j(t)^2 \sigma_j^2 h + \frac{1}{2}\Gamma_j(t)^2 S_j(t)^4 \sigma_j^4 h^2}
\end{aligned}$$

Chapter 19 Behavioral Risk Management

P. 581

$$EU = \sum_j u(W + x_j) p(x_j)$$

$$EIU = \sum_j (u(W + x_j) - u(W)) p(x_j)$$

P. 583

$$(19.1) \quad v(x^*) = \left\{ \begin{array}{ll} (x^*)^\alpha & x^* > 0 \\ -\lambda((-x^*)^\beta) & x^* \leq 0 \end{array} \right\}$$

P. 584

$$w(q) = \frac{q^\gamma}{(q^\gamma + (1-q)^\gamma)^\gamma}$$

P. 585

$$\pi(x_1) = w(F(x_1)),$$

$$\pi(x_j) = w(F(x_j)) - w(F(x_{j-1})) \quad j = 2, 3, \dots, n.$$

QFII-110-15: The Devil is in the Tails: Actuarial Mathematics and the Subprime Mortgage Crisis

P. 9

(4.1)

$$C_{\rho}^{gau}(u,v):=\int\limits_{-\infty}^{\Phi^{-1}(u)}\int\limits_{-\infty}^{\Phi^{-1}(v)}\frac{1}{2\pi(1-\rho^2)^{1/2}}\exp\left\{-\frac{s^2-2\rho st+t^2}{2(1-\rho^2)}\right\}dsdt$$

where

$$u,v\in [0,1],|\rho|<1$$

P.74

$$SD_c(x) = -x^T \mu + c \sqrt{x^T \Sigma x} \quad (2.1)$$

$$VaR_\alpha(x) = -x^T \mu + \Phi^{-1}(\alpha) \sqrt{x^T \Sigma x} \quad (2.2)$$

P.75

$$ES_\alpha(x) = -x^T \mu + \frac{\sqrt{x^T \Sigma x}}{(1-\alpha)} \phi(\Phi^{-1}(\alpha)) \quad (2.3)$$

P.80

$$\mathbb{R}C_i = x_i(-\mu_i + c \frac{(\sum x)_i}{\sqrt{x^T \Sigma x}}) \quad (2.10)$$

$$\mathbb{R}C_i = x_i(-\mu_i + \Phi^{-1}(\alpha) \frac{(\sum x)_i}{\sqrt{x^T \Sigma x}}) \quad (2.11)$$

$$\mathbb{R}C_i = x_i(-\mu_i + \frac{(\sum x)_i}{(1-\alpha) \sqrt{x^T \Sigma x}} \phi(\Phi^{-1}(\alpha))) \quad (2.12)$$

P.84

$$\begin{aligned} \mathbb{R}C_i &= \mathbb{R}(\Pi_i | \Pi) = -E[\Pi_i | \Pi = -VaR_\alpha(\Pi)] \\ &= E[L_i | L(x) = VaR_\alpha(L)] \quad (2.15) \end{aligned}$$

P.85

$$\begin{aligned}
\mathbb{R}C_i &= \mathbb{R}(\Pi_i \mid \Pi) = -\mathbb{E}[\Pi_i \mid \Pi \leq -VaR_\alpha(\Pi)] \\
&= \mathbb{E}[L_i \mid L(x) \geq VaR_\alpha(L)] \quad (2.16)
\end{aligned}$$

P.92

$$\mathbb{R}C_i = \mathbb{E}[L_i] + \frac{\text{Cov}(L, L_i)}{\sigma^2(L)} (VaR_\alpha(L) - \mathbb{E}[L]) \quad (2.17)$$

P.93

$$\begin{aligned}
\mathbb{R}C_i &= \frac{VaR_\alpha(L)}{\sum_{i=1}^n \mathbb{R}C_i} \hat{f}(VaR_\alpha(L)) \\
&= VaR_\alpha(L) \frac{\sum_{j=1}^m K(L^{(j)} - VaR_\alpha(L)) L_i^{(j)}}{\sum_{i=1}^n \sum_{j=1}^m K(L^{(j)} - VaR_\alpha(L)) L_i^{(j)}} \quad (2.18) \\
&= VaR_\alpha(L) \frac{\sum_{j=1}^m K(L^{(j)} - VaR_\alpha(L)) L_i^{(j)}}{\sum_{j=1}^m K(L^{(j)} - VaR_\alpha(L)) L^{(j)}}
\end{aligned}$$

P.157

(5.5)

$$F(t, T_1, T_2) = \frac{1}{(1 + \frac{f_n(t, T_1, T_2)}{n})^{n*(T_2 - T_1)}}$$

(5.6)

$$F(t, T_1, T_2) = e^{-f(t, T_1, T_2)(T_2 - T_1)}$$

P. 160

(5.14)

$$f(0, T, T + \Delta) = r(0, T) + (T + \Delta) * \frac{r(0, T + \Delta) - r(0, T)}{\Delta}$$

P. 161

(5.21)

$$r(0, T_n) = \frac{1}{T_n} \sum_{i=1}^n f(0, T_{i-1}, T_i) * \Delta$$

P. 166

(5.25)

$$\text{Value of FRA at } t = V^{FRA}(t) = V^{fixed}(t) - V^{floating}(t) = N * [M * Z(t, T_2) - Z(t, T_1)]$$

P. 170

(5.34)

$$P_c^{fwd}(0, T, T^*) = \frac{C}{2} * \sum_{i=1}^n P_z^{fwd}(0, T, T_i) + P_z^{fwd}(0, T, T_n)$$

P. 175

(5.40)

$$V^{swap}(t_i; c, T) = 100 - (\frac{C}{2} * 100 * \sum_{j=i+1}^M Z(T_i, T_j) + Z(T_i, T_M) * 100)$$

P. 176

(5.43)

$$c = n^* \left(\frac{1 - Z(0, T_M)}{\sum_{j=1}^M Z(0, T_j)} \right)$$

P. 181

(5.51)

$$f_n^s(0, T, T^*) = n^* \frac{1 - F(0, T, T^*)}{\sum_{j=1}^M F(0, T, T_j)} \quad (\text{This is a correction to the text formula})$$

Chapter 6 Interest Rate Derivatives: Futures and Options

P. 203

(6.4)

$$\text{P\&L from futures at } t = k \cdot \text{contract size} \cdot [P^{\text{fut}}(t, T) - P^{\text{fut}}(t - dt, T)]$$

Handbook of Fixed Income Securities, Fabozzi, Frank J., 2021

Chapter 69 Credit Derivative Valuation and Risk

P. 1693

$$V_{\text{Premium}} = \frac{C(T)}{2} \sum_{i=1}^N \Delta(t_{i-1}, t_i) Z(t_i) (Q(t_i) + Q(t_{i-1}))$$

P. 1964

$$V_{\text{Protection}} = (1 - R) \sum_{n=1}^M Z(t_n) (Q(t_{n-1}) - Q(t_n))$$

P. 1696

$$Q(T) = \exp \left(- \int_0^T h(t) dt \right)$$

P. 1700

$$\text{Spread DV01} = -A(T, S + 1bp) \cdot 1bp + (S - C) \cdot (A(T, S) - A(T, S + 1bp))$$

P. 1704

$$A(\bar{S}, T) \approx \frac{1 - \exp\left(-\left(W + \frac{\bar{S}}{(1-R)}\right)T\right)}{W + \frac{\bar{S}}{(1-R)}} \times \frac{365}{360}$$

P. 1706

$$V^{Index} = \frac{1}{P} \sum_{p=1}^P (C_i(T) - S_p(T)) A_p(T) = (C_i(T) - \bar{S}_i(T)) \bar{A}_i(T)$$

P. 1707

$$Q_i(t) = \frac{1}{P} \sum_{p=1}^P Q_p(t) = N_i(t)$$

Investment Risk Management, Baker & Filbeck, 2015

Chapter 8 Liquidity Risk

P. 144

(8.1)

$$LR_{it} = \frac{\sum_{t=1}^T P_{it} V_{it}}{\sum_{t=1}^T |P_{it} - P_{it-1}|}$$

(8.2)

$$ML_t = \sum_{i=1}^N \frac{(P_{it} - P_{it-1})^2}{V_{it}}$$

(8.3)

$$HH_i = \frac{(P_{i\max} - P_{i\min}) / P_{i\min}}{V / N * P_{avg}}$$

P. 145

(8.4)

$$TR_{it} = \frac{\sum_{i=1}^N P_{it} * Q_{it}}{P_{avg} * V}$$

(8.5)

$$PCTR_{it} = \frac{|\% \Delta P|}{TR_{it}}$$

(8.6)

$$ILLIQ_{it} = \frac{|R_{it}|}{\sum_{i=1}^N P_{it} * Q_{it}}$$

P. 146

(8.9)

$$VR_i = \frac{VAR(R_{it})}{K * VAR(r_{it})}$$

P. 147

(8.10)

$$MR_i = \frac{1}{K} \sum_{k=1}^K \left| \frac{P_{ik} - P_{ik-1}}{P_{ik}} \right| * 100$$

Chapter 25 Futures

P. 488

$$N_f = \frac{N_A}{qS_0}, \quad (25.4)$$

P. 490

$$N_f = \frac{hN_A}{qS_0}, \quad (25.9)$$

P. 494

$$p = \frac{i_t^f - i_t^{\text{pre}} \left(\frac{d_1}{B} + \frac{d_2}{B} \right)}{\left(i_t^{\text{post}} - i_t^{\text{pre}} \right) \frac{d_2}{B}}. \quad (25.15)$$

Credit-Risk Modelling, Bolder

Chapter 1

$$(1.3) \quad \text{VaR}_\alpha(L) = \inf\left(x : \mathbb{P}(L \leq x) \geq 1 - \alpha\right)$$

$$(1.6) \quad \text{VaR}_\alpha(L) = F_L^{-1}(1 - \alpha)$$

Chapter 2

$$(2.26) \quad \widehat{\mathbb{E}(L_N)} = \frac{1}{M} \sum_{m=1}^M L_N^{(m)}$$

$$(2.27) \quad \widehat{\sigma(L_N)} = \sqrt{\frac{1}{M-1} \sum_{m=1}^M \left(L_N^{(m)} - \widehat{\mathbb{E}(L_N)}\right)^2}$$

$$(2.28) \quad \widehat{\text{VaR}_\alpha(L_N)} = \tilde{L}_N(\lceil \alpha \cdot M \rceil)$$

$$(2.29) \quad \widehat{\mathcal{E}_\alpha(L_N)} = \frac{1}{M - \lceil \alpha \cdot M \rceil + 1} \sum_{m=\lceil \alpha \cdot M \rceil}^M L_N^{(m)} \quad \text{This is a correction to an error in the text.}$$

$$(2.45) \quad \mathbb{E}\left((\mathbb{D}_N - Np)^2\right) = Np(1 - p)$$

$$(2.60) \quad f_{\mathbb{D}_N}(k) = \mathbb{P}(\mathbb{D}_N = k) = \frac{e^{-N\lambda}(N\lambda)^k}{k!}$$

$$(2.61) \quad F_{\mathbb{D}_N}(m) = \mathbb{P}(\mathbb{D}_N \leq m) = \sum_{k=0}^m \frac{e^{-N\lambda}(N\lambda)^k}{k!}$$

Chapter 3

$$(3.10) \quad \mathbb{E}(\mathbb{D}_N) = N\bar{p}$$

$$(3.11) \quad \text{var}(\mathbb{D}_N) = N\bar{p}(1 - \bar{p}) + N(N - 1)\text{var}(p(Z))$$

$$(3.17) \quad f_Z(z) = \frac{1}{B(\alpha, \beta)} z^{\alpha-1} (1 - z)^{\beta-1}$$

$$(3.18) \quad B(\alpha, \beta) = \int_0^1 t^{\alpha-1} (1 - t)^{\beta-1} dt$$

$$(3.19) \quad B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$(3.20) \quad \Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$$

$$(3.21) \quad \mathbb{E}(Z) = \frac{\alpha}{\alpha+\beta}$$

$$(3.22) \quad \text{var}(Z) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$(3.26) \quad \rho_{\mathcal{D}} = \rho(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}) = \frac{1}{\alpha+\beta+1}$$

$$(3.27) \quad \mathbb{E}(Z) = \bar{p} = \frac{\alpha}{\alpha+\beta}$$

$$(3.35) \quad p_1^{-1}(z) \equiv y = \frac{1}{\sigma_1} \left(\ln\left(\frac{z}{1-z}\right) - \mu_1 \right)$$

$$(3.36) \quad p_2^{-1}(z) \equiv y = \frac{\Phi^{-1}(z) - \mu_2}{\sigma_2}$$

$$(3.39) \quad f_{p_1(Z)}(z) = \frac{1}{z(1-z)\sqrt{2\pi\sigma_1^2}} \exp\left[-\frac{(\ln(\frac{z}{1-z}) - \mu_1)^2}{2\sigma_1^2}\right]$$

$$(3.52) \quad \mathbb{P}(\mathbb{D}_N = k) = \int_{\mathbb{R}_+} \frac{e^{-\lambda(s)} \lambda(s)^k}{k!} f_S(s) ds$$

$$(3.53) \quad f_S(s) = \frac{b^a e^{-bs} s^{a-1}}{\Gamma(\alpha)}$$

$$(3.54) \quad \mathbb{P}(\mathbb{D}_N = k) = \frac{\Gamma(a+k)}{\Gamma(k+1)\Gamma(a)} q_1^a (1 - q_1)^k, \quad q_1 = \frac{b}{b+1}$$

$$(3.81) \quad \sigma(p_n(S)) = p_n \sqrt{\frac{\omega_1^2}{a}}$$

$$(3.83) \quad \mathbb{E}(\mathbb{I}_{\{X_n \geq 1\}} \mathbb{I}_{\{X_m \geq 1\}}) = p_n p_m (\omega_0^2 + 2\omega_0 \omega_1 + \omega_1^2 \mathbb{E}(S^2))$$

$$(3.84) \quad \mathbb{E}(S^2) = 1 + \frac{1}{a}$$

$$(3.85) \quad \mathbb{E}(\mathbb{I}_{\{X_n \geq 1\}} \mathbb{I}_{\{X_m \geq 1\}}) = p_n p_m \left(1 + \frac{\omega_1^2}{a}\right)$$

$$(3.86) \quad \rho(\mathcal{D}_n, \mathcal{D}_m) = \left(\frac{\omega_1^2}{a}\right) \left(\frac{p_n p_m}{\sqrt{p_n(1-p_n)} \sqrt{p_m(1-p_m)}}\right)$$

$$(3.95) \quad \sigma(p_n(\mathbf{S})) = p_n \sqrt{\sum_{k=1}^K \frac{\omega_{n,k}^2}{a_k}} \quad \text{This is a correction to an error in the text.}$$

$$(3.99) \quad \rho(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}) = \left(\sum_{k=1}^K \frac{\omega_{n,k} \omega_{m,k}}{a_k}\right) \left(\frac{p_n p_m}{\sqrt{p_n(1-p_n)} \sqrt{p_m(1-p_m)}}\right)$$

Chapter 4

$$(4.1) \quad y_n = aG + b\epsilon_n$$

$$(4.5) \quad Y_n = a \cdot G + \underbrace{\sqrt{1-a^2}}_b \epsilon_n$$

$$(4.7) \quad y_n = \sqrt{a} \cdot G + \sqrt{1-a} \epsilon_n$$

$$(4.18) \quad p_n(G) = \Phi\left(\frac{\Phi^{-1}(p_n) - \sqrt{\rho}G}{\sqrt{1-\rho}}\right)$$

$$(4.22) \quad \rho(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}) = \frac{\mathbb{P}(\mathcal{D}_n \cap \mathcal{D}_m) - p_n p_m}{\sqrt{p_n p_m (1-p_n)(1-p_m)}}$$

$$(4.23) \quad \mathbb{P}(\mathcal{D}_n \cap \mathcal{D}_m) = \Phi(\Phi^{-1}(p_n), \Phi^{-1}(p_m); \rho)$$

$$(4.30) \quad \rho(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}) = \frac{\overbrace{\Phi(\Phi^{-1}(p_n), \Phi^{-1}(p_m); \rho)}^{\mathbb{P}(\mathcal{D}_n \cap \mathcal{D}_m)} - p_n p_m}{\sqrt{p_n p_m (1-p_n)(1-p_m)}}$$

$$(4.31) \quad \rho(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}) = \frac{\Phi(\Phi^{-1}(\bar{p}), \Phi^{-1}(\bar{p}); \rho) - \bar{p}^2}{\bar{p}(1-\bar{p})}$$

$$(4.44) \quad \text{var}(\mathbb{D}_N | G) = \frac{p(G)(1-p(G))}{N}$$

$$(4.46) \quad F(x) = \mathbb{P}(G \leq -p^{-1}(x))$$

$$(4.49) \quad F(x) = h(b(x))$$

$$(4.50) \quad h(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{\mu^2}{2}} du \quad \text{This is a correction to an error in the text}$$

$$(4.51) \quad b(x) = \frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}$$

$$(4.55) \quad F'(x) = \sqrt{\frac{1-\rho}{\rho}} e^{\frac{(\Phi^{-1}(x))^2}{2} - \frac{b(x)^2}{2}}$$

$$(4.69) \quad y_n = \sqrt{\frac{\nu}{W}} (\sqrt{\rho}G + \sqrt{1-\rho}\epsilon_n)$$

$$(4.72) \quad \mathbb{E}\left(\frac{1}{W}\right) = \frac{1}{\nu-2}$$

$$(4.73) \quad \text{cov}(y_n, y_m) = \rho \left(\frac{\nu}{\nu-2}\right)$$

$$(4.74) \quad \rho(y_n, y_m) = \rho$$

$$(4.81) \quad y_n = \sqrt{V} (\sqrt{\rho}G + \sqrt{1-\rho}\epsilon_n)$$

$$(4.87) \quad p_n = F_{\mathcal{NV}}(K_n)$$

$$(4.103) \quad y_n = \underbrace{a_{n,K+1}\epsilon_n}_{\text{Idiosyncratic}} + \underbrace{\sum_{k=1}^K a_{n,k}Z_k}_{\text{Systematic}}$$

$$(4.107) \quad \text{cov}(y_n, y_m) = a_n^T a_m$$

$$(4.108) \quad p_n(Z) = \Phi \left(\frac{\Phi^{-1}(p_n) - a_n^T Z}{a_{n,K+1}} \right)$$

$$(4.110) \quad y_n = \sqrt{h(V)} \left(a_{n,K+1} \epsilon_n + \sum_{k=1}^K a_{n,k} Z_k \right)$$

$$(4.113) \quad \rho(y_n, y_m) = a_n^T a_m$$

$$(4.115) \quad p_n(Z, V) = \Phi \left(\frac{\sqrt{\frac{1}{h(V)}} F_{\mathcal{NV}}^{-1}(p_n) - a_n^T Z}{a_{n,K+1}} \right)$$

$$(4.116) \quad y_n(k) = \sqrt{h(V)} \left(\underbrace{\sqrt{a}G + \sqrt{(1-a)b_k}R_k}_{\text{Systematic element}} + \underbrace{\sqrt{(1-a)(1-b_k)}\epsilon_n}_{\text{Idiosyncratic element}} \right)$$

$$(4.120) \quad \rho(y_n(k), y_m(j)) = a + \mathbb{I}_{k=j}(1-a)\sqrt{b_k b_j}$$

$$(4.123) \quad p_n(G, R_k, V) = \Phi \left(\frac{\sqrt{\frac{1}{h(V)}} F_{\mathcal{NV}}^{-1}(p_n) - \sqrt{a}G - \sqrt{(1-a)b_k}R_k}{\sqrt{(1-a)(1-b_k)}} \right)$$