

# ILA LAM Model Solutions

## Fall 2024

### 1. Learning Objectives:

1. The candidate will understand, evaluate and use stochastic, generalized linear, multi-state, projection and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, and applications.

### Learning Outcomes:

- (1a) With respect to stochastic models:
  - Explain and apply the stochastic modeling methodology, including measurement metrics (e.g., CTE).
  - Describe and apply the theory and uses of real world versus risk neutral assumptions.
  - Describe and apply the techniques of Monte Carlo simulation (including variance reduction and importance sampling).
  - Describe and evaluate Random Number Generator models, and explain their uses, advantages, and theory.
  - Describe and evaluate how stochastic models may be used to understand mortality and policyholder behavior risks and inform the use of reinsurance.
  - Describe the technique of nested stochastic projections and explain why they are needed, and evaluate implementation issues.
- (1b) With respect to generalized linear models:
  - Describe and apply the basic principles of GLMs, and evaluate where GLMs might be useful in a Life Insurance context.
- (1c) With respect to multi-state and transition matrix models:
  - Describe and apply the methodologies for constructing multi-state and transition models in an insurance context.
- (1d) With respect to multi-state and transition matrix models:
  - Describe and apply the modeling methodology in an LTC product context.

### Sources:

Stochastic Analysis of Long-Term Multiple-Decrement Contracts, Actuarial Practice Forum, Jul 2008 (excluding Attachments)

# 1. Continued

## Commentary on Question:

*This question tests the candidates' understanding of stochastic models.*

## Solution:

- (a) Critique the following statements about risks to consider when building a stochastic mortality model:
- A. *Underwriting error reflects the risk in assigning the wrong risk class to each model point.*
  - B. *If the underlying product is interest sensitive, interest rate risk needs to be incorporated when building a stochastic mortality model.*
  - C. *If the insured sample size is large, volatility risk can be ignored.*
  - D. *To model catastrophe risk, the worst possible scenario must be modeled regardless of the complexity of achieving this.*
  - E. *Trend risk is critical and should always be included.*
  - F. *When combining the effect of different mortality risk factors, companies may consider having separate stochastic mortality multiples for different demographic cohorts.*

## Commentary on Question:

*Overall, candidates answered well. Candidates who did not explicitly state true/false, etc received partial marks. Those who only stated true/false etc without any explanations received no marks.*

### A. False/Partially true

Underwriting error reflects the ability to accurately reflect the expected mortality experience and is generally dependent on the underwriting process. The focus is not the source of the risk, but the fact that it exists.

### B. False

Interest rate risk is a financial assumption and doesn't necessarily correlate to non-financial assumptions like mortality.

### C. False

The actual mortality experience will vary around the central value defined as the best estimate. Although mortality volatility likely decreases as the population grows, the risk should not be ignored.

## 1. Continued

D. False

Increased complexity does not imply increased accuracy. A scenario representing appropriate catastrophic event needs to be adjusted so it adequately supports a specific portfolio.

E. False

It depends on the underlying product. If the product is not concerned about trend risk it can be excluded.

F. True

It would be appropriate to consider using different risk multiples for different demographic cohorts.

(b)

(i) Calculate the expected impact on the ending asset balance of a 1-in-10 chance of the best estimate mortality being wrong (underwriting error). Show all work.

(ii) Calculate the expected impact on the ending asset balance of a 1-in-100 chance of the underwriting, volatility, catastrophe, and trend risks all occur. Show all work.

(iii) Identify the risk element that is the least impactful to the mortality result. Justify your response.

### **Commentary on Question:**

*Overall, candidates answered well. Candidates who provided the correct answer without explicitly showing the supporting calculations only received partial marks.*

(i) 1 in 10 event reflects an occurrence of the 90% percentile  
 $40-31$  (UW 90<sup>th</sup> percentile) = 9

(ii) 1 in 100 event reflects an occurrence of the 99% percentile  
 $40-10$  (Cumulative 99<sup>th</sup> percentile) = 30

# 1. Continued

(iii)

PV of Assets = 40

	<b>Underwriting</b>	<b>Volatility</b>	<b>Catastrophe</b>	<b>Trend</b>	<b>Cumulative</b>
99%	24.0	36.0	13.5	34.0	10.0
95%	28.5	37.0	27.0	36.0	18.0
90%	31.0	37.6	30.5	37.0	26.0

Impact = PV of Assets – the ending asset balance by risk element at select percentiles

<b>Impact</b>	<b>Underwriting</b>	<b>Volatility</b>	<b>Catastrophe</b>	<b>Trend</b>	<b>Cumulative</b>
99%	16.0	4.0	26.5	6.0	30.0
95%	11.5	3.0	13.0	4.0	22.0
90%	9.0	2.4	9.5	3.0	14.0

Looking at the impacts of every scenario, the volatility risk has the lowest impact for each risk percentile. As such, it is the risk element that has the least effect on the model.

- (c) Evaluate whether lapse and mortality should be modeled together or separately. Show all work.

**Commentary on Question:**

*Overall, candidates answered well. Candidates who showed the correct calculations and correctly concluded there is no diversification impact from modeling separate or together received full marks.*

	<b>Lapse</b>	<b>Mortality</b>	<b>Lapse + Mort</b>	<b>Impact without Diversification</b>	<b>Impact with Diversification</b>
99%	37.1	10.0	7.1	32.9	32.9
95%	37.5	18.0	15.5	24.5	24.5
90%	37.8	26.0	23.8	16.2	16.2

PV of Assets = 40

Impact without Diversification = (PV of Assets - Lapse) + (PV of Assets - Mortality)

Impact with Diversification = PV of Assets - (Lapse + Mortality)

Impact of each independent shock is the same if modeled together. Therefore, there is no diversification impact.

## 1. Continued

- (d) Recommend which of the following reinsurance structures the company should use for this block. Justify your response
- Excess Loss Reinsurance
  - Multi-year Stop Loss Reinsurance
  - Experience Refund Reinsurance

### **Commentary on Question:**

*Overall, candidates answered well. Candidates who provided either Excess Loss or Multi-year Stop Loss with supporting justification received full marks. Few candidates suggested experience refund received no marks.*

#### Excess Loss Reinsurance

Generally pays out all death claims in excess of a set retention limit

#### Multi-year Stop Loss Reinsurance

Calculated on a cumulative basis over the life of the contract. The reinsurer will pay above a certain amount.

#### Experience Refund Reinsurance

Requires the insurer to pay the reinsurer a premium first, and then if A/E ratios on mortality are better, then they will receive a refund.

The portfolio has high mortality risk and bad A/E. Experience refund is not appropriate as the insurer would likely lose on the agreement. Excess reinsurance or multi-year stop loss is more appropriate, both can be used depending on actual term and cost of the contract.

## 2. Learning Objectives:

2. The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.
3. The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

### Learning Outcomes:

(2b) Describe and evaluate the following actuarial modeling best practices:

- Model risk management
- Model validation techniques and methods
- Best practices for assumptions governance
- Application of Actuarial Standards of Practices
- Reliance on expert judgment in actuarial modelling

### Sources:

Assumption Governance, The Actuary, Jan 2021

LAM-157-F23: Reflection of Inflation, Interest Rates, Stock Market Volatility, and Potential Recession on Life Insurance Business, American Academy of Actuaries, 2022

ASOP 56: Modeling, sections 3 & 4

LAM-140-19: Asset Adequacy Analysis Practice Note, 2017, questions: 3, 5, 10-16, 18-20, 27, 29-31, 39, 42-60, 65-68, 71-82, 85 & 89

### Commentary on Question:

*Commentary listed underneath question component.*

### Solution:

(a) Critique the following statements:

- A. *Long term care products are not impacted by inflation.*
- B. *In a high inflation and rising interest rate environment, lapse rates will be higher for term products and whole life products with no guarantees.*
- C. *Inflation accompanied by higher interest rates will have a negative impact on many insurance company assets whether the company's statutory accounting is on a market value or a book value basis.*

## 2. Continued

### **Commentary on Question:**

*Overall candidates performed well on this portion of the candidates. For statement B, one common error was to talk about UL and Variable products that would be affected and not term and whole life products with no guarantees which the question explicitly asks for.*

### **Statement A**

- This statement is generally false
- Most LTC products have Cost of Living Adjustments (COLA)
- Most of these COLAS have caps or limits. It is likely the cap or limit is triggered in a high inflation environment. If the cap is not triggered by inflation, then the product will not be impacted
- If there is no COLA then the product is not really impacted
- High inflation can also impact family budgets and the overall affordability of a LTC product

### **Statement B**

There are both true and false statements in this item.

- High inflation can also impact family budgets and the overall affordability of a LTC product
- Term product is not interest rate sensitive and may not be affected by high inflation
- Policyholders are less likely to value a policy with a fixed payout/face amount if they perceive inflation would erode the value of that future payout. Whole Life and Term products have fixed payouts/face amounts.
- The fact the Whole Life product has no guarantee can make the product more attractive in this environment
- Guarantees are perceived to be less attractive in high inflation and rising interest rate environments
- Higher lapses could also be driven by availability of new money products that offer better returns than existing portfolio rate products for Whole Life
- If a current product has a market value adjustment, the lapse concern would be reduced

### **Statement C**

- Generally, in a high inflation and rising interest rates environment, companies will likely (can) face problems if they have to report their assets on a market-value basis rather than a book value basis
- High withdrawals and lapses due to a rise in interest rates when market values of the assets are low can be an issue because the insurance company may take losses if it needs to sell assets to. This would impact assets on MV and BV basis

## 2. Continued

- (b)
- (i) Critique your manager's recommendation.
  - (ii) Describe three relevant considerations when setting the inflation assumption.

**Commentary on Question:**

*Several candidates talked about level of inflation, grading of inflation, mean reversion etc. instead of the various sources of inflation, product considerations, etc.*

- (i) Simply because the inflation rate is currently at 6%, does not mean that this is the number that should be used as the rate expenses are expected to increase, even temporarily. Basing the assumption on a recent observation is a headline risk. Need to use multiple sources of inflation including historical values.
  - (ii) Should also consider other inflation metrics as the overall inflation rate has many expenditure categories and some may have a more significant impact on the insurance company.
- Much of the expenses of insurance companies are salaries, which are not necessarily directly tied to inflation. The actuary should consider obtaining and analyzing employment cost index (ECI) data. Obtaining and analyzing employment cost index (ECI) data
  - Using several sources for the inflation rate (not just one) is another approach when setting the inflation assumption
  - Policyholder behavior is impacted by the economy (inflation) and should be considered. If inflation and interest are high, some products may be less attractive, driving more surrenders.
  - With greater uncertainty as to future expense levels, policyholder behavior, and/or asset earnings, it may be prudent for the actuary to consider increasing margins when performing cash flow projections, PBR determinations or asset adequacy testing.
- (c) Propose how AAT could be used to better serve the company. Justify your answer.

**Commentary on Question:**

*This part of the question was misunderstood by most candidates. They only elaborated on the how AAT can help with ALM and did not talk about assumptions setting.*



## 2. Continued

- AAT can be used to inform management of actual or possible problems that may arise due to the underlying characteristics or current management of the business.
- While it is critical to comply with regulations and satisfy stakeholders, the effort that is spent on assumptions should be driven by their usefulness in solving actuarial problems and managing business
- Valuing compliance higher than better predicting the future only ensures the accurate implementation of a suboptimal assumption, which is clearly not an ideal outcome.

A compliance-focused approach is not sustainable in today's actuarial landscape. The number of requirements and stakeholders that must be satisfied is growing.

(d) With respect to *ASOP 56, Modeling*:

- (i) Describe two considerations when assessing if the structure of the PBR model is appropriate for AAT.
- (ii) Describe two key topics to discuss when meeting with the consulting firm to understand the model.

### **Commentary on Question:**

*The candidate needs to address ASOP 56 guidance in the context of use of the AAT model for PBR and the issues with the structures and not general guidance of ASOP 56.*

- (i)
  - ASOP 56 section 3.1.4 Model Structure—The actuary should assess whether the structure of the model (including judgments reflected in the model) is appropriate for the intended purpose.
  - Which provisions and risks specific to a business segment, contract, or plan, or interactions, are material and appropriate to reflect in the model;
  - Whether the form of the model is appropriate, such as a projection model (deterministic or stochastic), statistical model, or predictive model; This may be different between AAT and PBR
  - Whether the use of the model dictates a particular level of detail (for example, whether grouping inputs will produce reasonable output, or whether a certain level of detail in the output is needed to meet the intended purpose

## 2. Continued

(ii)

Understanding the Model—When expressing an opinion on or communicating results of the model, the actuary should understand the following:

- important aspects of the model being used, including but not limited to, basic operations, important dependencies, and major sensitivities;
- known weaknesses or limitations - can be in assumptions, methods or other known limitations of the model that have material implications;
- limitations of data or information, time constraints, or other practical considerations

- (e) Describe two key elements that should be included in the Chief Actuary's disclosure to the Board on the reliance of a model developed by an external consultant.

**Commentary on Question:**

*Candidates performed well on this portion of the question.*

Required Disclosures in an Actuarial Report—When issuing an actuarial report under this standard, the actuary should refer to ASOP Nos. 23 and 41. In addition, the actuary should disclose the following in such actuarial reports:

- extent of reliance on experts - this includes the extent to which the model has been reviewed or validated
- the intended purpose of the model - Possibly include if the model was original the AAT model or they created a new PBR model
- material inconsistencies, if any, among assumptions, and known reasons for such inconsistencies
- unreasonable output resulting from the aggregation of assumptions, if material
- material limitations and known weaknesses
- extent of reliance on models developed by others - Need to disclose the model was developed by an external consultant

### 3. Learning Objectives:

4. The candidate will understand the basic design and function of Economic Scenario Generators and Equity Linked Insurance Models.

#### Learning Outcomes:

- (4a) With respect to Economic Scenario Generators:
- Describe the need for ESGs and explain the structure of ESG models and components.
  - Describe and apply basic default free interest rate models, including one-factor continuous time models.
  - Assess the propriety of a particular ESG model and related assumptions for particular applications.
- (4b) With respect to Equity-Linked models:
- Describe and apply methods for modeling long-term stock returns and certain guarantee liabilities (GMMB, GMDB, GMAB).
  - Describe and evaluate the Actuarial and Hedging risk metrics for GMAB and GMDB models.
  - Describe and apply methods for modeling Guaranteed annuity options and Guaranteed Minimum Income Benefits (GMIB), and EIA guarantees.

#### Sources:

Investment Guarantees, Hardy, Mary, 2003 - Ch. 2: Modeling Long-Term Stock Returns

LAM-139-19: Simulation of a Guaranteed Minimum Annuity Benefit, Freedman, 2019; Excel Model - Stochastic Simulation of a GMAB Option (Accompanies Simulation of a GMAB)

#### Commentary on Question:

*This question focuses on the various modeling approaches of stock returns, and tests candidates' knowledge via a deterministic scenario VA projection.*

*In general, candidates performed better on parts (a) and (b), while (c) proved to be challenging.*

*Detailed commentaries are found along with each question.*

#### Solution:

- (a) State the pros and cons of using a lognormal (LN) model to model stock returns.

#### Commentary on Question:

*Candidates will be awarded full credit for stating at least 2 pros and 2 cons.*

### 3. Continued

Using a LN model to model stock returns has several pros and cons.

Some of the pros are:

- It is simple to implement, interpret and communicate results.
- It provides tractable form and calculations.
- Results over the short-term are reasonable

Some of the cons are:

- It often fails to capture more extreme price movements over the longer term.
- It does not allow autocorrelation in the data
- It also does not capture stochastic variation in volatility and volatility bunching, which are often associated with severe downward stock price movements under extreme scenarios.

(b) Critique the following statements on alternative stock return models:

- A. *Empirical models and AR(1) models capture both autocorrelation and volatility bunching.*
- B. *Regime-Switching LN models use one parameter set and assume deterministic volatility. Such models are unable to capture more extreme observed behavior.*
- C. *The Wilkie model is a multivariate model. It is designed for short-term applications and ideal for assessing hedging strategies.*

**Commentary on Question:**

*Candidates that are familiar with the long-term stock returns reading generally performed well on this question. The ideal response expects candidates to critique whether each component of the statement is true or false, along with supporting reasons.*

**A:**

This statement is not entirely true.

AR(1) models are able to capture autocorrelation in the data, although in a simple way. However, they do not capture volatility bunching and extreme values that have been identified as features in the monthly stock return data.

Empirical models are not able to capture autocorrelation in the data, as returns in successive periods are assumed to be independent and identically distributed. This assumption of independence also implies that empirical models are unable to capture volatility bunching.

### 3. Continued

**B:**

This statement is not entirely true.

Regime-Switching LN models can model multiple regimes with each regime characterized by a different parameter set. Hence, Regime-Switching LN models with  $K$  regimes ( $K > 1$ ) will have more than one parameter set.

Volatility measure can be modelled differently under each regime, and thus, such models introduce stochastic volatility.

As different regimes can be parameterized with parameters associated with normal or extreme scenarios, such models are able to capture more extreme observed behavior.

**C:**

This statement is not entirely true.

The Wilkie model is a multivariate model which projects several related economic series together.

The model is designed more for long-term applications rather than short-term, which is typically more suited for applications that require projections for 10+ years.

This model is designed to be applied to annual data and cannot be easily adapted to more frequent data. This feature makes it not ideal for hedging strategies such as dynamic hedging, which often times require more frequent transactions and rebalancing activities.

(c)

- (i) Calculate the expected maturity payment for a surviving policyholder. Show all work.
- (ii) Calculate the present value of the expected profit to the insurer at time 0. Show all work

### 3. Continued

#### Commentary on Question:

*Candidates in general struggled with this question.*

- (i) Majority of candidates had troubles distinguishing cash inflows and outflows from a policyholder's perspective, as opposed to the insurer's. A common mistake is that many included more component cashflows than needed (specifically the expenses) as part of the FV projection. Initial and Maintenance Expenses should not be deducted from FV as the insurer should bear these costs, and FV belongs to the policyholder.*
- (ii) Many details from part (i) are involved in correctly getting to the final EPV. Survivorship and PV factors also need to be applied at the correct timing in order to receive full credit. Partial credit was awarded to candidates that have the correct formulaic/idea but incorrect answers due to mistakes made in part (i).*

(i)

The maturity payment for a surviving policyholder is either the GMMB or the Fund Value (FV) at the end of the term (year 7), depending on which one is higher. To determine this amount, both the GMMB and FV at end of year 7 will need to be calculated and compared. See Excel solution file for details.

FV = 564,950

GMMB = \$600,000

Comparing GMMB and FV at year 7, the GMMB is higher, so the maturity payment for the surviving policyholder will be **\$600,000**.

(ii)

The final answer can be broken down as follows:

EPV of Fee Income: \$123,662

EPV of Initial Expense: (\$25,000)

EPV of Maintenance Expense: (\$1,767)

EPV of Maturity Payment: (\$23,027)

EPV of Profit = Sum of Above = **\$73,868**

#### **4. Learning Objectives:**

3. The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

#### **Learning Outcomes:**

- (3a) With respect to Asset-Liability Models:
- Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
  - Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
    - Major product risks for which ALM can be a useful tool for their management.
    - Using ALM as a means to manage interest rate risk, equity risk, and risks from optionality.
  - Describe how common insurance contracts and variations generate embedded options in an insurer's balance sheet, and assess basic strategies for managing exposures created by such embedded options.
  - Describe and apply the basic concepts of cash flow matching, immunization, duration/convexity matching, segmentation.
  - Describe and apply Key Rate Durations (KRD) and their use in evaluating interest rate sensitivities of portfolios, including understanding the derivation of KDRs, the profiles of KDRs for selected major asset types, and assessing KRDs in a portfolio context.
  - Describe and evaluate the Goldman Sachs' ALM/Strategic Asset Allocation approach for integrating ALM into an enterprise's risk and financial management framework.
  - Describe and evaluate ALM modeling considerations in the context of modeling risk aggregation, dependency, correlation of risk drivers and diversification.

#### **Sources:**

LAM-117-14: Key Rate Durations: Measures of Interest Rate Risk

#### **Commentary on Question:**

*This question was poorly done overall. Most of the students were only able to get less than half of the points in the questions. Either they didn't know how to calculate the key rate duration or they were not able to provide the advantage of using key rate duration over effective duration in associating with the question.]*

## 4. Continued

The goal of the question is for the candidate to understand the three propositions of key rate durations. The candidate is expected to:

- Calculate key rate durations based on effective durations
- Compare effective duration to key rate duration, explaining the disadvantages and advantages
- Describe the key rate duration profiles of assets

### Solution:

(a)

- Determine whether the student calculated the correct key rate durations. Show all work.
- Demonstrate that the two securities have the same effective duration. Show all work.

### Commentary on Question:

Most of the students were able to identify that the two securities should have the same effective duration, although they are not able to calculate the correct key rate duration for security B at each tenor. Full points is only given to if everything is calculated correctly.

$KRD = (\text{Price of zero-coupon bond}/\text{total portfolio value}) * \text{zero-coupon bond's KRD}$

Total portfolio value

= 217.91

t	Weight	Effective Duration	KRD
3	$95.88/217.91=44\%$	3.00	$44\%*3=1.32$
7	$74.09/217.91=34\%$	7.00	$34\%*7=2.38$
15	$47.94/217.91=22\%$	15.00	$22\%*15=3.30$

Key rate duration of year 7 is incorrectly calculated.

Effective duration of security A = 7year

Effective duration of security B =  $1.32+2.38+3.3=7$

Two securities have the same effective duration.



## 4. Continued

- (b)
- (i) Calculate the change in the value of Security A and B under each scenario. Show all work.
  - (ii) Explain why effective duration is often inadequate in measuring a security's interest rate risk exposure. Justify your answer using part (i).
  - (iii) Identify two other advantages of key rate duration over effective duration.

### Commentary on Question:

*Most of the students were able to identify that the price change = key rate duration \* rate change. However, only a few students understand that MV should be used in the calculation instead of Par value.*

*Part ii and Part iii were very poorly done. Most of the students were not able to justify answers in associating with part i) and were only able to identify one advantage of using key rate duration over effective duration.*

(i)

Scenario X:

- Portfolio A is unaffected since the 7-year rate does not change
- Portfolio B =  $-D(3) * 0.20\% * MV + -D(15) * (-0.20\%) * MV$

Scenario Y:

Level shift so can use effective duration

- Portfolios A and B =  $-(\text{EffD}) * (-0.20\%) * MV$

Scenario Z:

- Portfolio A is unaffected since the 7-year rate does not change
- Portfolio B =  $-D(3) * (-0.20\%) * MV + -D(15) * 0.20\% * MV$

Change

Portfolio	Scenario X	Scenario Y	Scenario Z
A	-	1.04	-
B	0.8	3.05	-0.8

## 4. Continued

(ii)

Effective duration predicts the change in price for a parallel yield curve shift. However, the spot curve rarely moves in a parallel fashion even on the infinitesimal level, which makes effective duration not precise enough in many bond portfolio strategies, such as hedging or immunization. When these strategies fail to be effective, often it is not because the yield curve shifts too much, but because the yield curve does not shift in a parallel fashion.

Observations from part (i):

Scenarios 1 and 3 do not affect portfolio A because the 7-year rate does not change, but portfolio B is affected. The portfolios have the same effective duration, but very different interest rate exposures. They behave very differently under non-parallel yield curve shifts. Therefore, effective duration is not useful for non-parallel yield curve shifts. You must understand the sensitivity of each key rate.

(iii)

KRDs recognize that the yield curve movement is driven by multiple market factors. The validity of key rate durations does not depend on any equilibrium model of the yield curve movement. Key rate durations are applicable over a broad range of arbitrary yield curve movements. It is easy to use KRDs to create a replicating portfolio of a bond with embedded options using zero-coupon bonds. Thus, the cash flow of the replicating portfolio correctly represents the instantaneous expected cash flow of the option. Hence, key rate durations can provide valuable insight into option-embedded bond behavior that other measures (such as effective duration) cannot.

- (c) Contrast the interest rate risk profile of these assets, including the sensitivity to rate changes at different points in the yield curve.

### **Commentary on Question:**

*This section was done poorly. Most of the students were not able to compare and contrast the duration difference between these assets. Some students described the asset feature instead of focusing on the impact of different features on duration of each assets.*

### **Callable Corporate Bond**

Callability shortens the effective duration

The callable bonds are more sensitive to the shorter-term key rate changes however a callable bond with a lower coupon rate has less probability of being called which means that the effective duration is relatively higher and the bond also tends to be more sensitive to the longer-term key rate risk

The increase in interest rate risk exposure is not uniform across the yield curve but is more substantial for the long-term rate

## 4. Continued

### **Callable Bond with a Sinking Fund**

The sinking fund reduces the effective duration of the bond

The sinking fund makes the bond more sensitive to the shorter-term key rates

Significantly reduces the long-term rate exposure since much of the portion of the bonds has been sunk over the years

### **European Call Option**

The option is insensitive to the changes of any key rates with a term before the expiration date

The key rate duration for the expiration date is negative

A call option is exposed much more to a curvature yield curve movement than to a parallel movement

### **European Put Option**

Has a positive key rate duration at expiration but negative key rate durations beyond the expiration date

The key rate duration profile of a European put option is almost the mirror image of that of a European call option, but the magnitude of the key rate durations is not the same

The relationship between the key rate durations of the call and the put can be derived from the put/call parity

## 5. Learning Objectives:

5. The candidate will understand the role of the Investment Actuary and the Portfolio Management Process in the Life Insurance company context, as well as the common forms of Fixed income securities and their uses, and the methods and processes used for evaluating portfolio performance and asset allocation.

### Learning Outcomes:

- (5c) Describe and assess the role of and significant considerations related to the design and function of asset allocation strategies.
- (5e) Describe and assess Alternative Investment Portfolios (including real estate) in the context of an insurance company portfolio.
- (5f) Describe and apply methods and processes for evaluating portfolio performance, including performance attribution, sources of earnings analysis on investment income, benchmarks, metrics, and risk adjusted performance appraisals (including total return vs reported earnings).
- (5i) Describe the attributes of US Treasuries, Agency Debt Securities, Municipal bonds, Corporate bonds, Private Money Market securities, Floating Rate Agreements, Agency Mortgage Backed securities, Agency Collateralized Mortgage securities, Interest Rate Swaps and Swaptions, Credit Derivatives and High Yield Bonds, and the markets they are traded in.

### Sources:

Managing Investment Portfolios, Maginn, John L. & Tuttle, Donald L., 3rd Edition, 2007  
- Ch. 5: Asset Allocation (sections 2-4)  
- Ch.12: Evaluating Portfolio Performance (section 4)

Handbook of Fixed Income Securities, Fabozzi, Frank J., 9th Edition, 2021  
- Ch. 10: Corporate Bonds  
- Ch. 22: Agency Mortgage Passthrough Securities  
- Ch. 4: Bond Pricing, Yield Measures and Total Return

Profiles of Alternative Assets in Life Insurance Landscape

### Commentary on Question:

*Commentary listed underneath question component.*

### Solution:

- (a) Describe two differences between TAA and SAA.

### Commentary on Question:

*This question tests candidate's understanding of common asset allocation methods. Most of candidate did well on this question and can explain the key difference between SAA and TAA*

## 5. Continued

Strategic asset allocation (SAA) sets an investor's desired long-term exposures to systematic risk, while Tactical asset allocation (TAA) involves making short-term adjustments to asset-class weights based on short-term predictions of relative performance among asset classes.

- (b) Critique the following statements regarding asset allocation:
- A. *We should never add an asset class to our portfolio if the Sharpe ratio of the new asset class is lower than the Sharpe ratio of the existing portfolio.*
  - B. *Measuring growth based on a money-weighted return basis will always be materially different from using a time-weighted return basis.*
  - C. *Yield-to-maturity considers interest-on-interest, and it assumes that the coupon payments can be reinvested at an interest rate equal to the coupon rate.*
  - D. *Prepayments on a mortgage only occur when rates decrease.*
  - E. *If the default rate on a corporate bond is higher than its credit spread, it would always be preferable to invest in treasuries rather than that particular corporate bond.*

### **Commentary on Question:**

*This question tests candidate's understanding of different portfolio performance measurement metrics and various asset classes including mortgages and corporate bonds. Simply answering True or False will not earn partial credit; Candidates must clearly state the limitations of the statements and propose revisions to receive full credit. While candidates generally performed well on the Statement A, B, C and D, many struggled with Statement E. Candidates failed to recognize that after accounting for recovery rates, the actual default loss rate could be lower than the credit spread, meaning the corporate bond could still be a worthwhile investment.*

### Statement A

This statement is not correct in all cases, and it depends on the correlation of the new asset class's rate of return with the current portfolio's rate of return. Even though the Sharpe ratio of the new class is lower, it could still be added to achieve a mean-variance improvement if the new asset class's Sharpe ratio exceeds the product of the existing portfolio's Sharpe ratio and the correlation of the new asset class's rate of return with the current portfolio's rate of return (i.e.,  $\text{Sharpe Ratio (new)} > \text{Sharpe Ratio (existing portfolio)} * \text{Correlation (Return of new portfolio, Return of existing portfolio)}$ ).

## 5. Continued

### Statement B

This statement is not correct in all cases.

The money-weighted return (MWR) is sensitive to the size and timing of external cash flows to and from the account, while the time-weighted return (TWR) is unaffected by these flows.

Under “normal” conditions, these two return measures will produce similar results. However, when external cash flows occur that are large relative to the account’s value, and the account’s performance is fluctuating significantly during the measurement period, then the MWR and the TWR can differ materially.

### Statement C

This statement is not always true.

The statement is true when the coupon rate equals to the yield-to-maturity.

However, when yield-to-maturity does not equal to the coupon rate, the yield-to-maturity assumes that the coupon payments can be reinvested at an interest rate equal to the yield-to-maturity.

### Statement D

This statement is not correct.

Even though prepayments from refinancing or turnover could increase when interest rates are declining or low; some degree of turnover will take place even in high interest rate environments due to seasonality or housing affordability, (i.e., household income/housing prices).

Seasoning and defaults can also cause prepayments, and these categories of prepayments do not always depend on the interest rate environment.

### Statement E

The statement is not always true.

This decision will depend on the recovery rate and whether the default rate is on a "default loss rate basis". If some non-zero recovery rate is assumed, then the default loss rate could be substantially lower than the default rate. Thus, it is possible that the default rate is higher than the credit yield spread, but the default loss rate is still lower than the credit yield spread.

- (c) Your investment team is seeking to increase yield and is investigating alternative assets. Critique the following statements:
- A. *As it is outside the expertise of an insurance company and it is hard to model, real estate investment trusts should be avoided.*
  - B. *Private equity should be managed using tactical asset allocation.*

## 5. Continued

- C. *Collateralized loan obligations have minimal to no risk, given there is sufficient diversification in the underlying pooled loans.*
- D. *Energy transition infrastructure is a safe investment vehicle for insurance companies to invest in.*

### **Commentary on Question:**

*The question is designed to assess candidates' knowledge of various alternative assets, including commercial real estate, private equity, collateralized loan obligations (CLOs), and energy transition infrastructure. Candidates performed reasonably well and were awarded credits for discussing features of these alternative assets. Common items that could lead to partial point deduction included:*

- A. *Failing to recognize that Real Estate Investment Trusts (REITs) can be modeled similarly to equity and debt securities.*
- B. *Overlooking the illiquid nature of private equity investments.*
- C. *Not explicitly listing the risks associated with CLOs or providing adequate explanations for these risks.*
- D. *Failing to identify energy transition infrastructure as a safer alternative to traditional infrastructure investments while offering higher returns.*

### Statement A – False

A real estate investment trusts (REIT) can be invested in to gain exposure to commercial real estate (CRE)  
Equity REITs can be publicly traded and are legally required to pay out 90% or more of their taxable income.  
As REITs are publicly traded, they can be modeled like Equities and debt securities.

### Statement B – False

Private Equities are very illiquid and long term.  
TAA is more appropriate when short term adjustments are made.  
The goal for insurance companies is to maintain a level of liquid assets and fulfilling funding commitment for PE direct investments.

### Statement C – False

Collateralized loan obligations (CLOs) have following risks:  
Credit Risk: Given the profile of the underlying loans, they can default.  
Interest Rate Risk: CLOs with fixed interest rates will decline in value when interest rate rise.  
Prepayment risk: Arises if loans are paid off before scheduled dates.  
Liquidity risk: Lower CLO tranches have higher illiquidity.

## **5. Continued**

Statement D – Partially True

Insurance companies have financed infrastructure classes. Energy transition infrastructure is used to produce renewable, environment-friendly energy, which is a much safer alternative asset category that offer same benefits as traditional energy infrastructure to provide entry to a high growth market. However as real assets, energy transition infrastructure investment still poses some risks from sustaining operations and financing. The main risks are operating risk, technology risk, technology risk, political risk etc.



## 6. Learning Objectives:

5. The candidate will understand the role of the Investment Actuary and the Portfolio Management Process in the Life Insurance company context, as well as the common forms of Fixed income securities and their uses, and the methods and processes used for evaluating portfolio performance and asset allocation.

### Learning Outcomes:

- (5b) Describe and evaluate how a company's objectives, needs and constraints affect investment strategy and portfolio construction (including capital, funding objectives, risk appetite and risk return tradeoff, tax and accounting, accounting considerations, and constraints such as regulation, rating agency ratings and liquidity).
- (5f) Describe and apply methods and processes for evaluating portfolio performance, including performance attribution, sources of earnings analysis on investment income, benchmarks, metrics, and risk adjusted performance appraisals (including total return vs reported earnings).
- (5h) Describe and apply conventional yield metrics used in bond performance evaluation.

### Sources:

Handbook of Fixed Income Securities, Fabozzi, Frank J., 9th Edition, 2021 - Ch. 4: Bond Pricing, Yield Measures and Total Return (pp. 76-94);

LAM-154-23: Ch. 7 (sections 7.2-7.5 & 7A) of Derivatives Markets, McDonald, 3rd Edition

### Commentary on Question:

*Commentary listed underneath question component.*

### Solution:

- (a)
  - (i) Calculate the total return of each bond on a bond-equivalent basis. Show all work.
  - (ii) Recommend which bond to purchase, based on your calculation from (i). Justify your answer.
  - (iii) Critique the option of purchasing a different security with higher yield-to-maturity.

## 6. Continued

### **Commentary on Question:**

*Candidates struggled with part (i). The most common mistakes included valuing the bonds at time of issue or maturity and using these values to calculate an annual return over the entire term to maturity. For part (ii), most candidates were able to recommend the correct bond based on their answers in part (i). Candidates also performed well on part (iii) and were able to adequately critique the option by explaining the multiple risks associated with higher YTM bonds.*

The primary objective of this question is to calculate the total return of each bond on a bond-equivalent basis. This can be achieved either by using formulas or simulating cash flows—both approaches will be demonstrated here. The key assumption is that the bond will be sold after 2 years. The bond's value at that point ( $t=2$ ) will consist of:

- **Coupon payments and interest-on-interest:** This includes the coupon payments received in the first 2 years and the reinvested interest, calculated using the 5% reinvestment rate.
- **Present value (PV) at  $t=2$  of future cash flows:** This includes the PV of all remaining coupon payments and the maturity value, discounted using the projected annual yield on comparable bonds.

Once the value of each bond at  $t=2$  is determined, the total return is calculated using the bond's initial purchase price. This total return is expressed as a semi-annual return over the 2 years, which is then multiplied by 2 to arrive at the bond-equivalent yield (BEY).

Please see the excel solution for detailed calculations

- (b)
- (i) Calculate the modified duration of the bond.
  - (ii) Calculate the convexity of the bond.
  - (iii) Estimate the change in market value of a 1% increase in interest rates using the information calculated in (i) and (ii).

Show all work.

## 6. Continued

### **Commentary on Question:**

*Candidates generally performed well when calculating the bond's duration. Common mistakes included using an incorrect interest rate or assuming semi-annual coupon payments. Partial credit was awarded in these cases. For part (ii), many candidates struggled with deriving the correct formula for convexity, leading to a higher frequency of errors. Candidates performed well in part (iii), where most candidates correctly applied the calculated duration and convexity from parts (i) and (ii).*

The primary objective of this question is to calculate the modified duration and convexity of the bond. A key assumption is that when the bond's par value equals its market value, the yield is equal to the coupon rate. For part (iii), candidates are required to use both duration and convexity to calculate the change in the bond's market value. It is important to note that when interest rates rise, the market value of bonds will always decrease.

Please see the excel solution for detailed calculations